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An Explicit Time-Difference Scheme with an Adams-Bashforth Predictor and a Trapezoidal Corrector

Sajal K. Kar[1,2]

1- I.M. Systems Group, Inc. (IMSG), 2- National Centers for Environmental Prediction/Environmental Modeling Center (NCEP/EMC)

Adams-Bashforth trapezoidal (ABt) scheme:

For the generic tendency equation

$$\frac{\partial \Psi}{\partial t} = F(\Psi),$$

the proposed ABt scheme is defined via two steps:

Predictor step,

$$\frac{\Psi^{(n+1)*} - \Psi^{n}}{\Delta t} = \tfrac{1}{2}(3F^{n} - F^{n-1}).$$

Corrector step,

$$\frac{\Psi^{n+1} - \Psi^{n}}{\Delta t} = \tfrac{1}{2}[F^{(n+1)*} + F^{n}].$$

Here Ψ is an arbitrary prognostic variable, which is a function of space and time (t); $F^{(n+1)*} \equiv F[\Psi^{(n+1)*}]$. The superscripts n-1, n, and n+1 denote the three time level indices, while Δt is the time step. The superscript (n+1)* denotes a provisional index for the time level n+1.

The stability and phase-change characteristics of the ABt scheme applied to the **the oscillation equation**

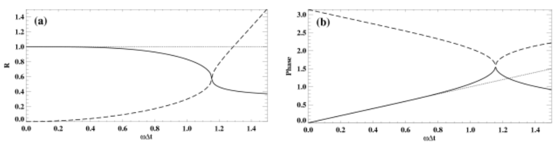
$$\frac{\partial \Psi}{\partial t} = i\omega\Psi,$$

and **the friction equation**

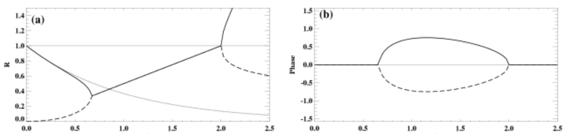
$$\frac{\partial \Psi}{\partial t} = -\kappa\Psi,$$

are displayed below in terms of the amplitude and phase of the amplification factor. The ABt scheme is stable for $|\omega\Delta t| < 1.287$ and $0 < \kappa\Delta t \le 2/3$.

Stability diagrams for the oscillation equation



Stability diagrams for the friction equation



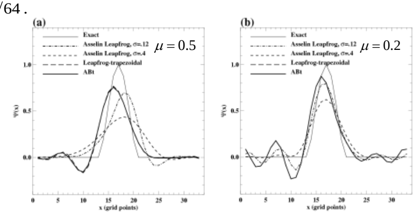
In the numerical tests to follow, we compare the ABt scheme with the Asselin time-filtered leapfrog scheme (Robert 1966, Asselin 1972) and the leapfrog-trapezoidal scheme (Kurihara 1965).

PS For a comprehensive write-up of this work please contact:
Dr. Sajal K. Kar
I.M. Systems Group, Inc., W/NP2 Rm 207, WWBG
5200 Auth Road, Camp Springs, MD 20746-4304
Sajal.Kar@noaa.gov

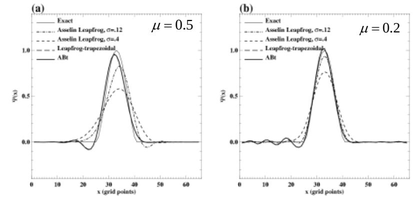
Application of the ABt scheme to a linear advection problem

For an arbitrary variable $\Psi(x,t)$ transported in the x-direction by a fluid with a constant velocity c, the *one-dimensional linear advection equation* is governed by $\partial_t\Psi + c\partial_x\Psi = 0$. The exact solution is $\Psi(x,t) = \Psi^0(x-ct)$, for an initial distribution of Ψ given by $\Psi(x,0) = \Psi^0(x)$. A 4th order centered scheme is used for $\partial_x\Psi$. The domain is periodic with a grid size Δx. The advective-Courant number is denoted by $\mu = |c|\Delta t/\Delta x$. The filter parameter (σ) of the time-filtered leapfrog scheme is varied between 0.12 and 0.4. An initially specified normal-mode distribution of Ψ is translated three times around the domain by separate numerical time integrations using the ABt scheme, the time-filtered leapfrog scheme, and the leapfrog-trapezoidal 'Kurihara' scheme. Within stability limits, μ is first set at the value of 0.5 and then at 0.2. For the first set of integrations, $\Delta x = 1/32$. For the second set of integrations, the grid resolution is doubled to $\Delta x = 1/64$.

1st Set:

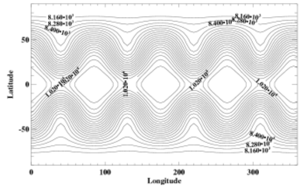


2nd Set:

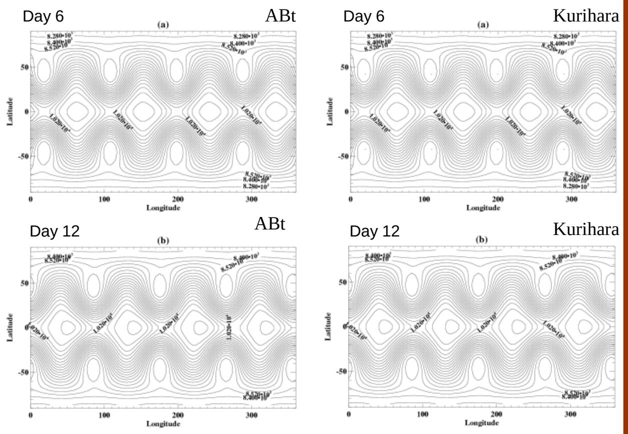


Application of the ABt scheme in a nonlinear shallow-water model

A global, nonlinear, shallow-water, grid-point model (Kar et al. 1994) is integrated in time using the ABt scheme and the leapfrog-trapezoidal scheme, the latter used to obtain a reference solution. The horizontal finite-difference scheme for the model is based on the second-order mass-conserving and partial fourth-order energy and potential-enstrophy conserving scheme on the staggered C grid (Arakawa and Lamb, 1981). A zonal polar filter is employed at the high latitudes. A Rossby-Haurwitz wave-number 4 initial condition (Phillips 1959) is used with the mean-depth of the shallow water, $h_0 = 8\times10^3\,\mathrm{m}$. The model is configured with a uniform latitude-longitude grid resolution of $\Delta\varphi = 4^\circ$ and $\Delta\lambda = 5^\circ$. The model has been integrated for 12 days with $\Delta t = 640\,\mathrm{s}$ for both schemes. The initial free-surface height, followed by the day-6 and day-12 forecasts of the free-surface height made by the ABt scheme and the leapfrog-trapezoidal scheme are displayed below and in the next panel.



Initial global field of free-surface height (m) for the Rossby-Haurwitz wave number 4. The contour interval is 120 m.



Summary

An explicit time-difference scheme with an Adams-Bashforth predictor and a trapezoidal corrector, thus named **the ABt scheme**, has been proposed. Linear computational stability analyses of the proposed scheme were carried out for the oscillation and the friction equation. The ABt scheme was applied to a one-dimensional linear advection problem and the numerical solutions were compared to those obtained using the Robert-Asselin time-filtered leapfrog and the leapfrog-trapezoidal scheme (Kurihara 1965). The proposed scheme was also implemented in a global, nonlinear, grid-point shallow-water model. The numerical time-integration solutions were then obtained for the Rossby-Haurwitz wave-number 4 initial condition and compared to the same obtained using the leapfrog-trapezoidal 'Kurihara' scheme. Numerical results obtained from the linear and nonlinear models seem to indicate that the ABt scheme is conditionally stable with accuracies comparable to the Kurihara scheme.

The proposed scheme is uniquely suitable for implementation into a predictor-corrector type two-time-level fully-implicit semi-Lagrangian scheme for 3D hydrostatic/nondrostatic models, in which (i) the advection terms are treated with a two-time-level semi-Lagrangian scheme; (ii) the gravity/sound wave terms are treated with a two-time-level implicit trapezoidal scheme; and (iii) the *nonlinear* (leftover from the semi-implicit linearization) terms are treated with the ABt scheme proposed here.

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