Fractal Characteristics of Simulated and LMA-Detected Lightning Flashes

Blake J. Allen[1], Edward R. Mansell[2], and Eric C. Bruning[3]

[1]School of Meteorology, University of Oklahoma, Norman, OK
[2]NOAA / National Severe Storms Laboratory, Norman, OK
[3]Atmospheric Sciences Group, Texas Tech University, Lubbock, TX

Corresponding author address: blake.allen@ou.edu

Abstract

The fractal dimension of lightning has been studied using both models and observations in the past, but both types of studies suffered from limitations that are now avoidable. Due to a lack of computing power and model sophistication, the fractal dimension of simulated lightning has previously only been studied using either two-dimensional models or unidirectional three-dimensional (3D) models with extremely simplified charge distributions. Also, due to a lack of modern observing networks, previous studies of the fractal dimension of observed lightning were limited to analyzing photographs of flashes in which channels exited the cloud.

In this study, the fully 3D bidirectional lightning model of Mansell et al. (2002) has been used to simulate lightning flashes within a small, short-lived simulated storm with a realistic charge distribution. The model dynamics were run at 250m, while the lightning resolution was made as fine as 25m. The fractal characteristics of these flashes have been analyzed by calculating the correlation dimension using the method originally described in Grassberger and Procaccia (1983). In addition, the fractal characteristics of lightning flashes detected in 3D by the Oklahoma Lightning Mapping Array (OK-LMA) during a small central Oklahoma storm described in Bruning et al. (2007) have also been analyzed using the aforementioned method.

The Simulated and observed flashes have been compared and the relationship between correlation dimension and model resolution has been analyzed and used to inform the tuning of parameters in the lightning model.

The Correlation Dimension

The correlation dimension is calculated using the correlation integral:

$$C\left(r\right) \equiv \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \theta\left(r - \left|\vec{X}_i - \vec{X}_j\right|\right)$$

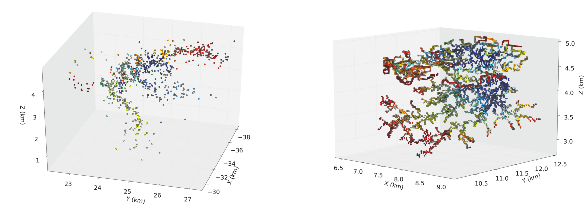
$N =$ number of points $\theta =$ Heaviside Step Function
$r =$ threshold distance $\vec{X} =$ point position

If a dataset has a fractal character, the correlation integral should obey a power law:

$$C\left(r\right) \propto r^{\nu}$$

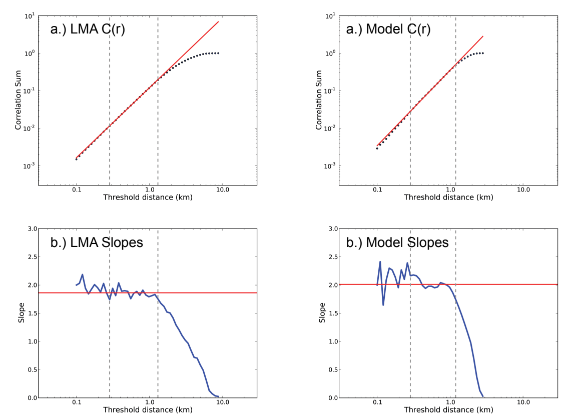
Therefore, the correlation dimension, ν, corresponds to the slope of a line fitted to the fractal scaling region in a log-log plot of C(r) vs. r.

Example LMA Flash **Example 25m Model Flash**



Above: Spatial plot of source points from an LMA-detected flash (left) and a modeled flash (right). Blue source points occurred the earliest in the flash and red points occurred the latest.

Below: a.) Log-log plot of C(r) vs. r for the LMA flash (left) and the modeled flash (right). The Solid red line corresponds to a linear least squares fit over the scaling region of each flash. b.) Semi-log plot of forward-differenced slopes between each point vs. r for the LMA flash (left) and modeled flash (right). In all plots the dashed gray lines denote the fractal scaling regions of the flashes where the fits were performed.



References

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Correlation Dimension Results

Data Set	Correlation Dimension	Standard Deviation	Number of Flashes
25.00m model run	1.96	0.04	23
27.78m model run	1.95	0.05	27
31.25m model run	1.95	0.04	22
35.71m model run	1.92	0.07	22
41.67m model run	1.88	0.06	22
50.00m model run	1.85	0.07	24
62.50m model run	1.75	0.06	21
83.33m model run	1.62	0.08	23
125.0m model run	1.39	0.10	28
LMA	1.67	0.14	21

Application of Results to the Model

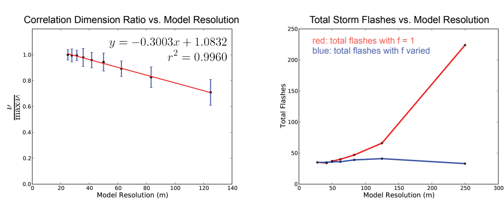
In addition to fractal dimension, model resolution affects flash rate. Also, both flash rate and fractal dimension are sensitive to the critical electric field threshold for propagation, E_{crit}, in the equation that governs the probability of adding adjacent points to the flash:

$$p_i\left(E\right) = \begin{cases} \frac{1}{F}\left(E_i - fE_{crit}\right)^{\eta} & \text{for } E_i > fE_{crit} \\ 0 & \text{for } E_i \leq fE_{crit} \end{cases}$$

E_{crit} is held constant in the model, but its effective value can be varied by setting the factor f to a value between 0 and 1. The correlation dimension data from this study may be useful for specifying this value (see plots below).

Below left: a line was fitted to a plot made using the model data in the table above. The equation of the line was then used to calculate values of f for various resolutions, which were then used for additional model runs of the same storm as the one analyzed in the correlation dimension calculations.

Below right: The total number of lightning flashes in the original model runs (red, with f = 1 for all resolutions), and in the runs with f varied as discussed above (blue) .



Conclusions

The fractal character of lightning produced by the model is similar to that of LMA-detected lightning, but the correlation dimension is very sensitive to model resolution. The results of the flash rate tests shown are preliminary and further testing needs to be done with other simulated storms, but these early results show that the equation found above may be useful as a way of setting the factor f.