Conservative and Accurate Data Remapping for Coupling WRF and CAM

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1. Abstract

- The two-way coupling of WRF and CAM for regional climate modeling requires transferring data (such as wind velocities, temperature, moisture, etc.) between the grids of WRF and CAM. These grids are in general non-matching, and the data transfer should be both physical conservative and numerically accurate.
- Our method contains two parts: the common refinement and the L-2 minimization, which are shown as follows:

1. The former computes the grid intersections and provides the necessary data structures for the latter. We project the grids of WRF and CAM onto a plain surface using their corresponding map projection methods. Then common refinement method will locate the edge intersection points of two grids, and determine sub-facets to create a new mesh based on the union of the intersection points and vertices of the original two grids within the overlapping area.
2. After obtaining the common refinement mesh, we use a weighted-residual formulation, which minimizes the L-2 norm of the error, to complete the data transfer. The L-2 minimization method satisfies physical conservation and at the same time is as accurate as interpolation.

2. L2 Minimization Algorithm

•This method is to minimize L2 norm of error over whole domain:

$$E = \sqrt{\int_\Omega (g-f)^2 dx} \quad , \qquad (1)$$

where f is the source function and g is the target function.

•Then E is minimized if

$$\partial \int_\Omega (g-f)^2 dx / \partial g_i = 0 \quad , \quad \text{for } i = 1, \cdots, n \qquad (2)$$

•Using finite element shape functions, we have $f = \sum_{i=1}^{m} \phi_i f_i$ **and** $g = \sum_{i=1}^{n} \psi_i g_i$ **, where** ϕ_i **and** ψ_i **are shape functions at source control points and target control points, respectfully.**

•From equation (2), we can get:

$$\frac{\partial \int_\Omega (g-f)^2 dx}{\partial g_i} = \frac{\partial \int_\Omega (\sum_{j=1}^{n} \psi_j g_j)^2 dx}{\partial g_i} - 2\frac{\partial \int_\Omega \sum_{j=1}^{n} \psi_j g_j f dx}{\partial g_i} + \frac{\partial \int_\Omega f^2 dx}{\partial g_i}$$

$$= 2\sum_{j=1}^{n} \int_\Omega \psi_i \psi_j dx g_i - 2\int_\Omega \psi_i f dx \quad , \quad i = 1, \cdots, n \qquad (3)$$

•Equations (3) are actually a $n \times n$ **linear system Mx=b :**

$$\begin{bmatrix} \int_\Omega \psi_1\psi_1 ds & \int_\Omega \psi_1\psi_2 ds & \cdots & \int_\Omega \psi_1\psi_n ds \\ \int_\Omega \psi_2\psi_1 ds & \int_\Omega \psi_2\psi_2 ds & \cdots & \int_\Omega \psi_2\psi_n ds \\ \cdots & \cdots & \cdots & \cdots \\ \int_\Omega \psi_n\psi_1 ds & \int_\Omega \psi_n\psi_2 ds & \cdots & \int_\Omega \psi_n\psi_n ds \end{bmatrix} \cdot \begin{bmatrix} g_1 \\ g_2 \\ \cdots \\ g_n \end{bmatrix} = \begin{bmatrix} \int_\Omega \psi_1 (\sum_{i=1}^{m} \phi_i f_i) ds \\ \int_\Omega \psi_2 (\sum_{i=1}^{m} \phi_i f_i) ds \\ \cdots \\ \int_\Omega \psi_n (\sum_{i=1}^{m} \phi_i f_i) ds \end{bmatrix} \qquad (4)$$

In the above linear system, M is the *consistent mass matrix*, and b is *load vector*. And this data transfer method will be referred to L2 *minimization*.

•In order to solve linear system (4), key question is how to integrate the entries of mass matrix and load vector. In our method, we construct common-refinement mesh, so that $\int_\Omega \psi_i \psi_j ds$ **and** $\int_\Omega \psi_i (\sum_{k=1}^{m} \phi_k f_k) ds$ **are discretized to every sub-element of common-refinement mesh.**

3. Construct Common-Refinement Mesh

•Definition: A common refinement of two meshes is a mesh composed of elements that subdivide the elements of both input meshes simultaneously, or, simply put, the intersections of the elements of the input meshes.

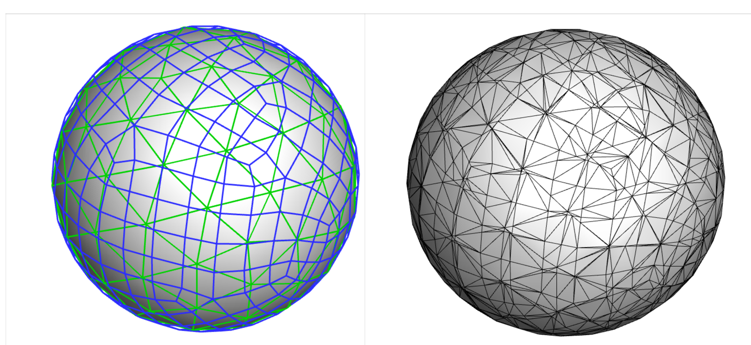

Figure 1: the left sphere shows two input meshes (blue and green), and the right sphere shows the common refinement mesh that triangulates the intersection cells.

Highlight

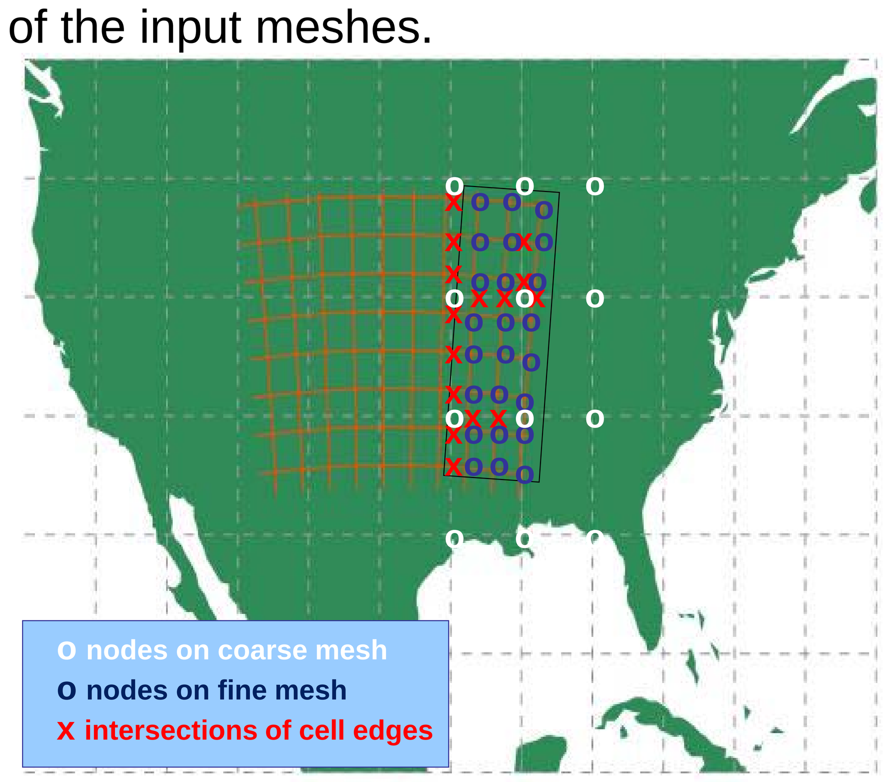

Figure 2: Common-refinement mesh for highlighted overlapped area

•Construct the common-refine mesh:

1. ***Map projection:*** Using the same map projection method as WRF's to transform CAM's grid from spherical coordinate to Cartesian coordinate, because in general CAM's grid are regular latitude-longitude grid using spherical coordinate, while WRF's grid are rectangular mesh in Cartesian coordinate.
2. **Face-to-face intersection:** loop through CAM's grid and WRF's grid simultaneously to compute intersection for every pair of source element and target element.
   a) *Define a pair of seed element.*
   b) *Locate point projection:* find local natural coordinate of target element's vertices within source element. For triangle, natural coordinate is just the barycentric coordinate. For rectangle, natural coordinate is parameterized along two cell sides.
   c) With the natural coordinate of vertices of both source element and target element, we can locate a pair of edges that might have intersections and calculate *edge-to-edge intersection*.
3. **Organize and triangularize the intersection area we get from step 2 to construct common-refinement mesh.**

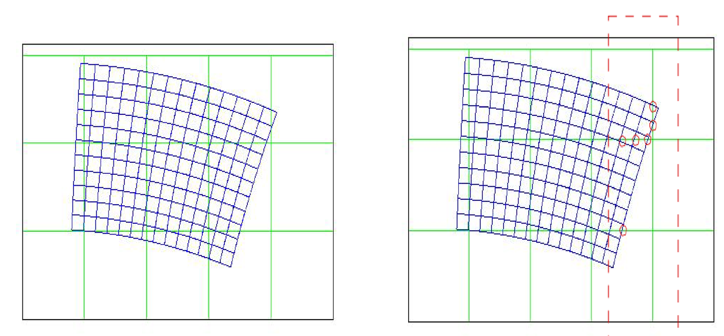

Figure 3: The left graphic are two original input grids, the right graphic shows the face-to-face intersection.

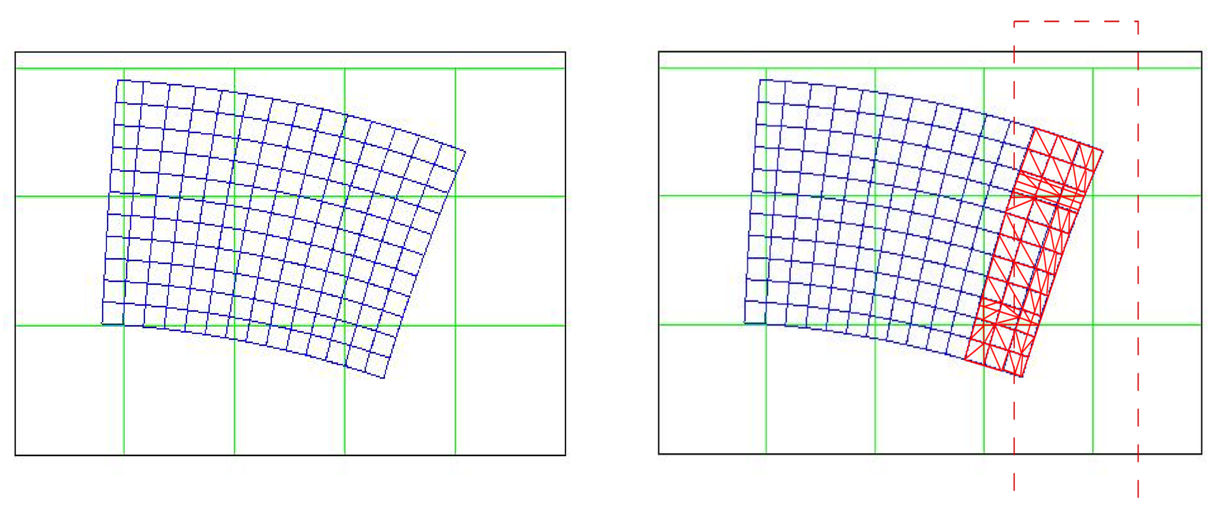

Figure 4: Same as figure 3 but with illustration of triangularization .

4. Implementation

1. **We compute the intersection between source mesh and target mesh based on common refinement data structure.**
2. **For every sub-element** t_s **(which is a triangle) of common-refinement mesh, we compute** $\int_{t_s} \psi_i \psi_j ds$ **and** $\int_{t_s} \psi_i \phi_k ds$ **for mass matrix and load vector, respectfully.**
   - ϕ_j **and** ψ_i **are shape functions of parent source element and target element, respectfully.**
   - **Integrals are computed by using quadrature rule in** t_s**. Currently we use 6 points quadrature rule.**
3. **By summarizing and organizing the integrals we get in the second step, we get linear system (4), and it can be solved by multiple numerical methods.**

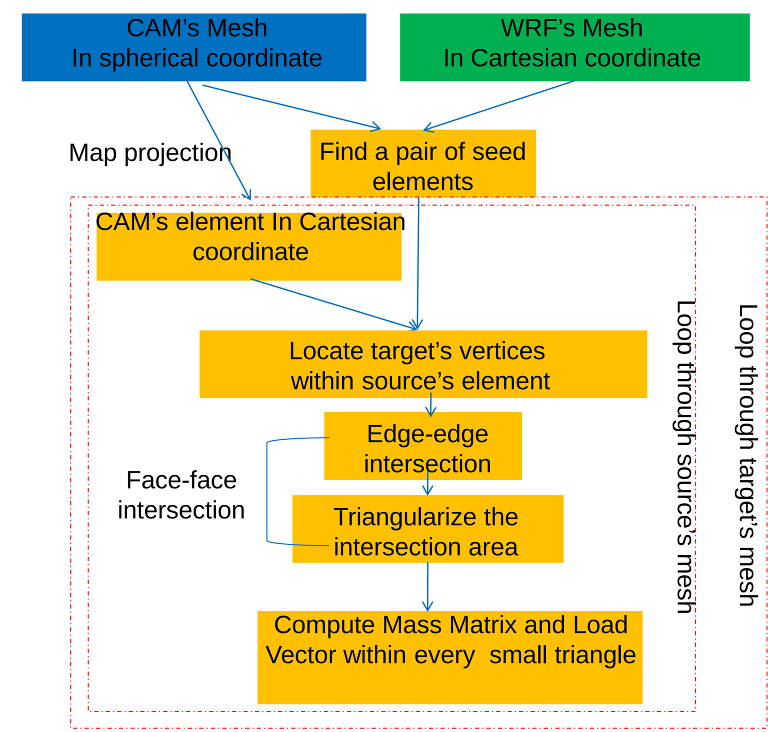

Figure 5: Summary diagram of the implementation

5. Advantages of C-R based Algorithm

•Strict physical conservation is naturally inherent from L2 minimization.

•Accuracy: since shape functions of source elements and target elements, then provided the degrees of the quadrature rule are sufficiently high, the entries of mass matrix and load vector can be exactly computed over sub-element. With common-refinement discretization, L2 minimization method is at least as accurate as interpolation.

•Code efficiency: our method is optimal in time and space, with linear complexity.

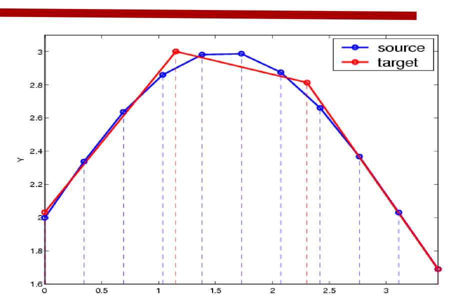

Figure 6: 1-d illustration for L2 minimization method

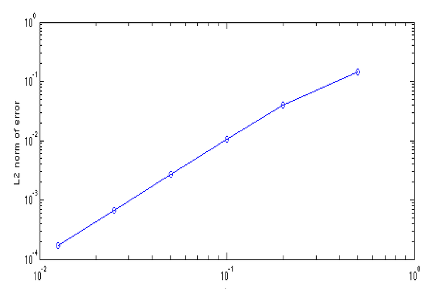

Figure 7: Convergent rate of error is 2nd order

6. Summary

1. The whole algorithm has been implemented.
2. Physical convergence, accuracy and code efficiency have been verified using multiple cubic functions. With bilinear shape functions, the convergent rate of error is 2nd order. And higher order of accuracy can be achieved with higher order shape functions.
3. The method is being integrated into WRF/CAM coupling system.
4. Next step, we will test the method with real case.

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