

# An application of a Retrospective Optimal Interpolation (ROI) to WRF

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## Introduction

- Retrospective Optimal Interpolation (ROI) is a new scheme introduced by Song et al. (2009) and Song and Lim (2009). The formulation of ROI is similar with that of Optimal Interpolation (OI), but ROI iteratively assimilates an observation set at a post analysis time into a prior analysis, possibly providing the high quality reanalysis data. It is possible that ROI method assimilate the data at post analysis time using perturbation method (Errico and Raeder, 1999) without adjoint model.
- In previous study, ROI method is applied to Lorenz 40-variable model (Lorenz, 1996) to validate the algorithm and to investigate the capability. Thus, It is required to apply this ROI method into a more realistic and complicated model framework such as WRF.

$$\mathbf{x}_{(n)}^{roi} = \mathbf{x}_{(n-1)}^{roi} + \mathbf{W}_{(n)}^{roi} [\mathbf{y}_n^{obs} - \mathbf{H}_n(\mathbf{M}_n(\mathbf{x}_{(n-1)}^{roi}))]$$

$$\mathbf{P}_{(n)}^{roi} = [\mathbf{I} - \mathbf{W}_{(n)}^{roi} \mathbf{H}_n \mathbf{M}_n] \mathbf{P}_{(n-1)}^{roi}, \quad \mathbf{P}_{(1)}^{roi} = \mathbf{B}$$

$$\mathbf{W}_{(n)}^{roi} = \mathbf{P}_{(n-1)}^{roi} \mathbf{M}_n^T \mathbf{H}_n^T [\mathbf{H}_n \mathbf{M}_n \mathbf{P}_{(n-1)}^{roi} \mathbf{M}_n^T \mathbf{H}_n^T + \mathbf{R}_n]^{-1}$$

$$\frac{\partial f}{\partial \mathbf{x}} \delta \mathbf{x} = \frac{f(\mathbf{x}' + \alpha \delta \mathbf{x}) - f(\mathbf{x}')}{\alpha}, \quad \alpha \ll 1$$

$$\mathbf{H}_n \mathbf{M}_n \mathbf{P}_{(n-1)}^{roi} \mathbf{M}_n^T \mathbf{H}_n^T \rightarrow [\mathbf{H}_n \mathbf{M}_n [\mathbf{s}_{(n)}^1 \ \mathbf{s}_{(n)}^2 \ \dots \ \mathbf{s}_{(n)}^k]] [\mathbf{H}_n \mathbf{M}_n [\mathbf{s}_{(n)}^1 \ \mathbf{s}_{(n)}^2 \ \dots \ \mathbf{s}_{(n)}^k]]^T$$

$$\mathbf{S}_{(n)} \mathbf{S}_{(n)}^T \rightarrow \mathbf{H}_n \mathbf{M}_n \mathbf{s}_{(n)}^k = \frac{1}{\alpha_k} [\mathbf{H}_n (\mathbf{M}_n (\mathbf{x}_{(n)}^{roi} + \alpha_k \mathbf{s}_{(n)}^k)) - \mathbf{H}_n (\mathbf{M}_n (\mathbf{x}_{(n)}^{roi}))]$$

## Objective of This Work

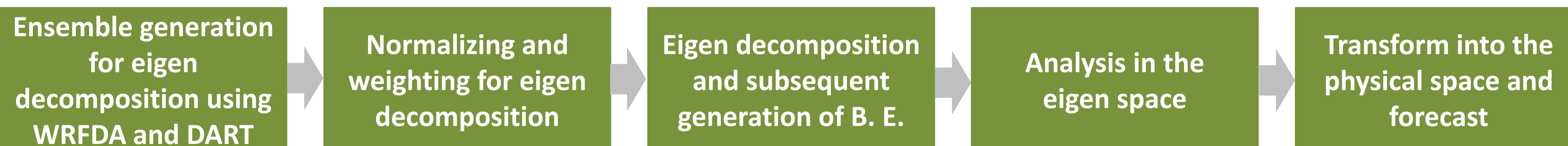
- Development of WRF-ROI system.
- Application of a reduced-rank formulation of ROI instead of a reduced-resolution method to overcome huge computational costs.
- Evaluation on the performance of WRF-ROI system through the assimilation using column data with assumption of a perfect model.

## Reduced-rank Formulation of ROI

- If we obtain a similar quality of analysis by analyzing fewer control variables, we could reduce the computational costs for implementing ROI. Eigen-decomposition of background error covariance can concentrate ROI analyses on the error variances of governing eigenmodes by transforming the control variables into eigenvectors.
- The total energy norm (Ehrendorfer et al., 1999; Errico, 2000) is used to normalize the background and analysis field. The total energy norm,  $W$ , is defined by where  $V$  indicates the domain volume,  $A$  the domain lower surface,  $\mathbf{x} = (U, T, q, p_{surf})$ ,  $T_r$  is a reference temperature (300 K) and  $p_r$  a reference pressure (1000 hpa). The four terms on the r.h.s. of the following equation represent the kinetic, potential, moist and surface-pressure components of the total energy, respectively. In this work, we only use the kinetic and potential energy components. We also plan to attach the rest terms (moist, surface-pressure components) in the near future.

$$\mathbf{x}_1^T \mathbf{W} \mathbf{x}_2 = \int_V \left\{ \mathbf{U}_1 \cdot \mathbf{U}_2 + \frac{c_p}{T_r} T_1 T_2 + \frac{\epsilon(z) L_v^2}{c_p T_r} q_1 q_2 \right\} dV + \int_A \left\{ \frac{RT_r}{p_r} p_{surf1} p_{surf2} \right\} dA$$

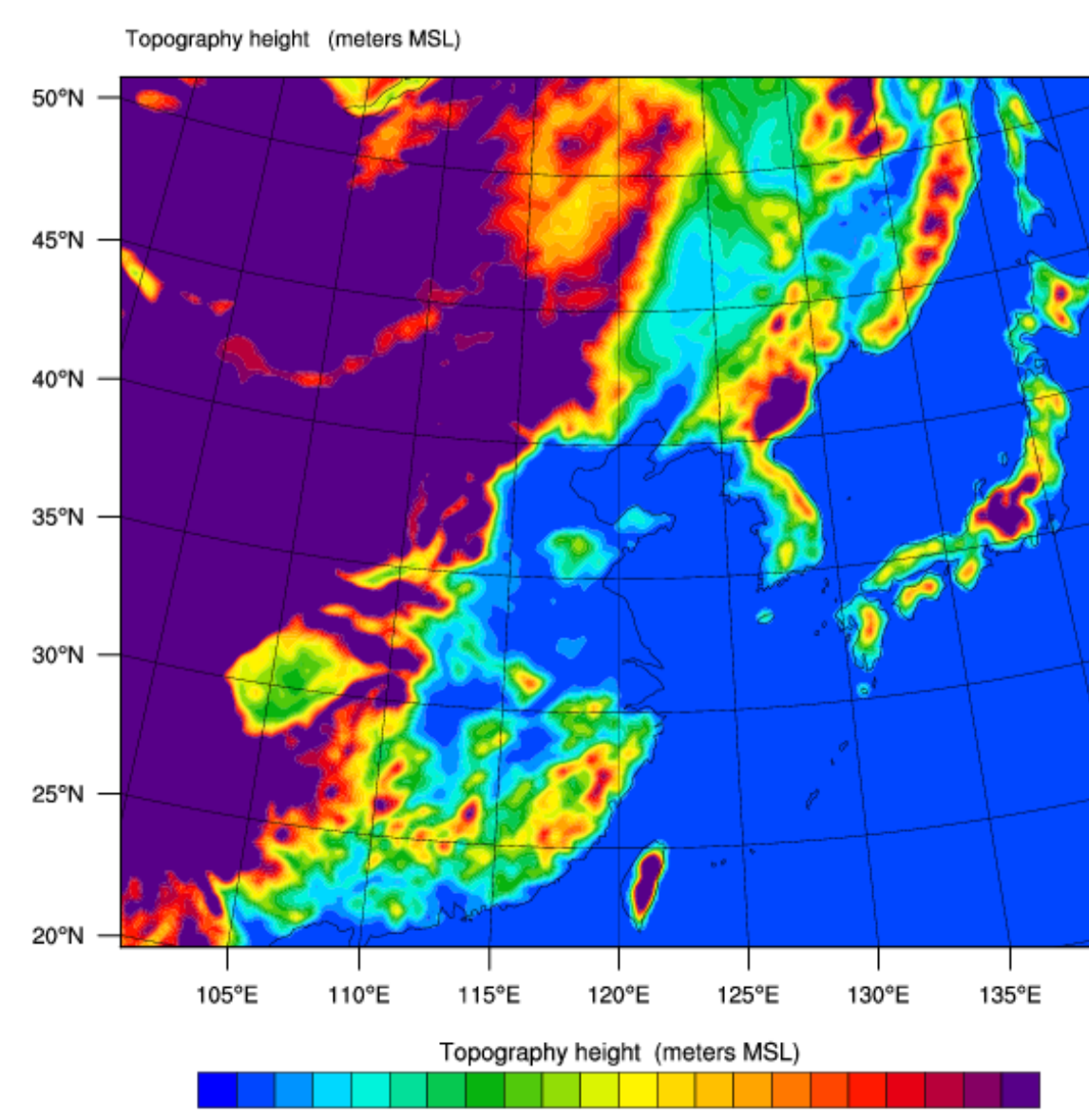
## Flow chart of reduced-rank formulation of ROI



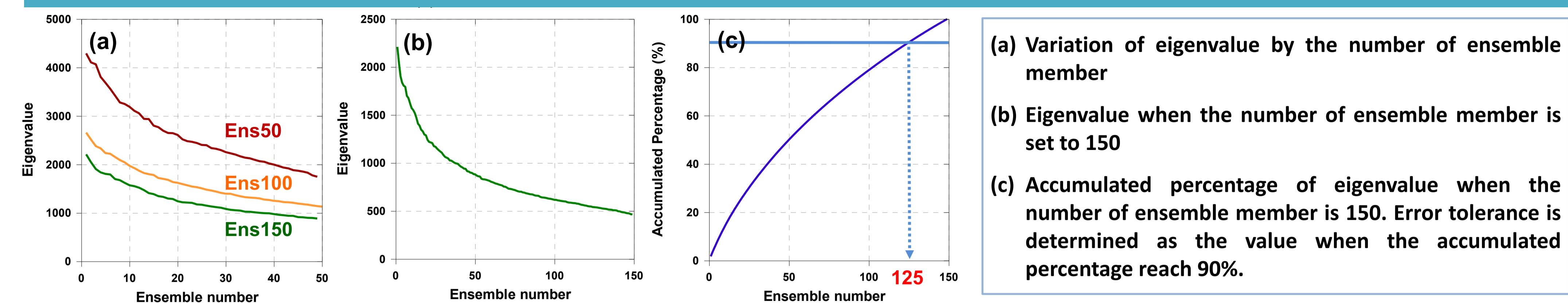
## Model Configuration

- WRF v3.2 is used in this study.
- True value is provided by the simulation starting from 24 hours before the model setup with the parameterizations for cumulus, PBL, and microphysics which are different from the model setup shown in the Table.
- Potential temperature profile at the grid point (100, 87) is used for the observation, and the data after 1 hour from analysis time is only applied in this work for the simplicity.

variable	configuration
Grid number	200 x175
Grid distance	20 km
Time step	120 s
Cumulus	Kain-Fritsch scheme
PBL	YSU scheme
Microphysics	WSM 6-class
Input data	NCEP FNL data
Integrated time	2003.08.05.00 UTC- 2003.08.08.00 UTC
Analysis time	2003.08.05.12 UTC

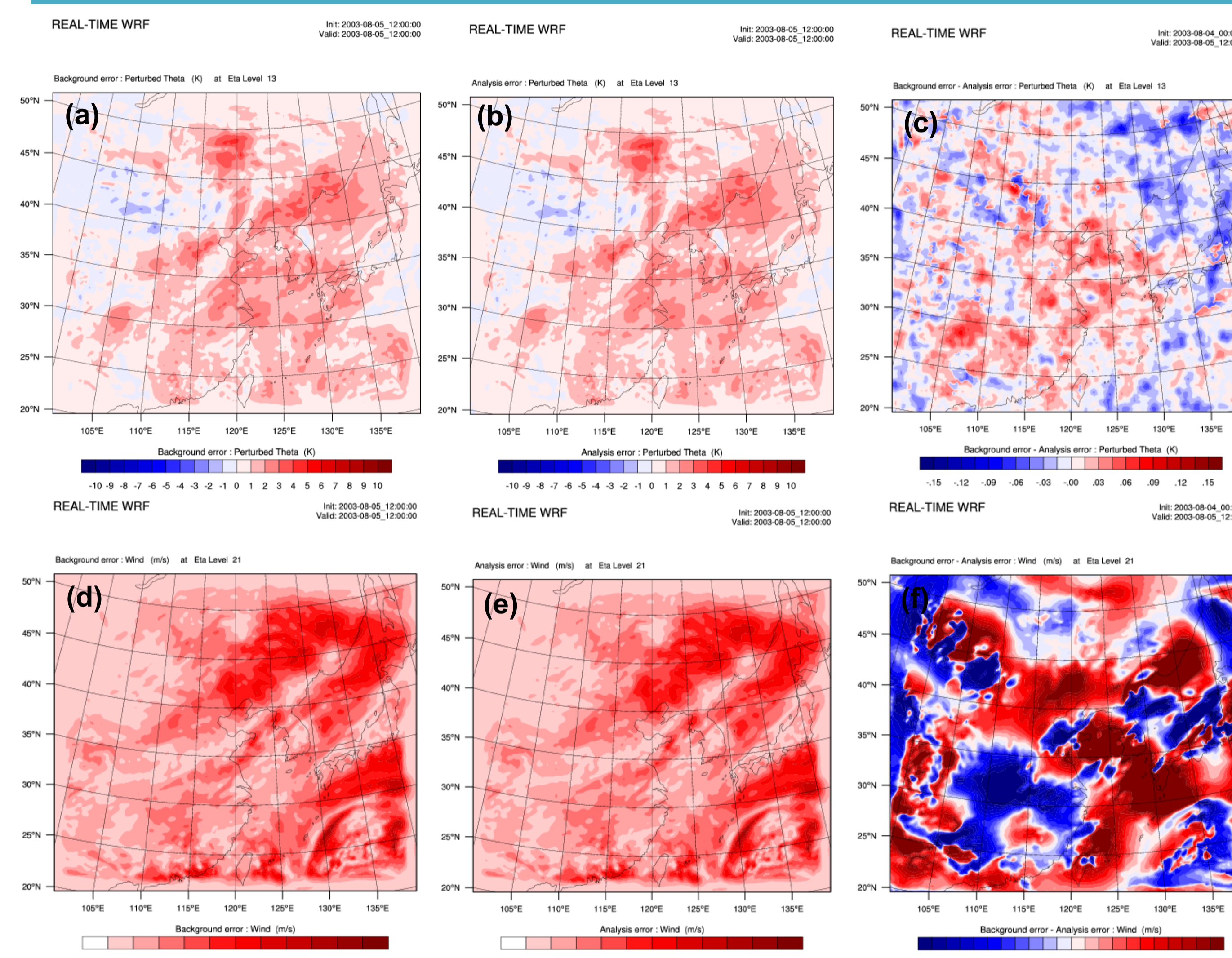


## Decision of Error Tolerance



- Variation of eigenvalue by the number of ensemble member
- Eigenvalue when the number of ensemble member is set to 150
- Accumulated percentage of eigenvalue when the number of ensemble member is 150. Error tolerance is determined as the value when the accumulated percentage reach 90%.

## Analysis Results



- Background error of perturbed potential temperature
- Analysis error of perturbed potential temperature
- Difference between (a) and (b). The positive indicates the reduced error by analysis
- Background error of wind field
- Analysis error of wind field
- Difference between (a) and (b).

- Small differences are found between background and analysis errors, which is attributed to insignificant effect due to the single column assimilation.
- Assimilation of only potential temperature affects initial wind field as well as potential temperature.

## Difference between CTRL and Analysis Experiments

(a)	Theta	Wind	(b)	Theta	Wind
Total area	0.001024	-0.00147	Total area	0.001507	-0.00025
21x21	0.000321	-0.00841	21x21	-0.00201	-0.01481
41x41	0.004874	0.000146	41x41	0.003172	-0.0002
61x61	0.004949	0.002825	61x61	0.003556	0.002023
81x81	0.005246	0.002746	81x81	0.003563	0.001879

(c)	Theta	Wind	(d)	Theta	Wind
Total area	0.001575	0.00056	Total area	0.0023	-0.00092
21x21	0.001724	-0.01823	21x21	0.023587	0.004063
41x41	0.004999	-0.01187	41x41	0.010927	0.012362
61x61	0.003942	-0.00669	61x61	0.007029	0.002726
81x81	0.002793	-0.00261	81x81	0.004282	0.00024

- Difference between background error and analysis error
- Difference in forecast error between CTRL and analysis experiments after 1 h
- Same as (b) except after 3 h
- Same as (b) except after 6 h

- Errors from analysis experiments are overall smaller than those from CTRL and background experiments, but the order of error is obviously small.

## Summary and Further Work

- With the application of ROI algorithm into WRF, the rank-reduced method is simultaneously used in the assimilation to reduce the computational cost.
- Through the experiments assimilating single column potential temperature, the analysis and forecast results are improved with the reduced error, but the magnitude of this improvement is not quite large.
- Further assimilations will be performed with moisture field as well as wind and temperature based on the realistic observation system. Various sensitivity experiments are additionally necessary to characterize the ROI method.