Wind Farm Layout Optimization

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ABSTRACT

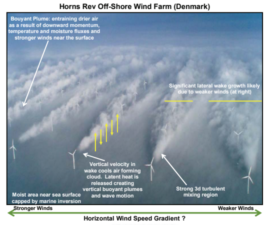
We study the problem of choosing the optimal layout of individual turbines within the boundaries of a wind farm subject to proximity constraints and maximizing energy production. We present a mathematical model for wind turbine placement based on the wind distribution at the wind farm site. At present, this model considers the problem in two dimensions and can utilize estimates of the distribution of wind speed and direction at turbine hub height. Ideally, such distributions would be representative of the climatology of a potential wind farm site, if the required instrumentation is in operation. Alternatively, the data can be derived from simulation via the application of numerical weather prediction. In this case, we use the WRF-ARW community model configured to provide detailed wind estimates at an arbitrary target site.

The layout model takes into consideration the hard constraints on turbine placement imposed by budget and turbine operational limitations. The objective is to maximize energy production subject to energy losses incurred due to wake effects between turbines. As opposed to existing approaches based on heuristic and global search methods, we describe an exact search method based on Branch and Bound that provides optimality guarantees. If given enough time, our solving method is guaranteed to find the optimal solution and if terminated early it provides a feasible solution together with an optimality gap. Existing methods find progressively better solutions but have no way of determining how far the best solution found so far is from the optimal and hence, provide solutions that may be arbitrarily bad.

BACKGROUND

- Wake turbulence from turbines reduces wind speed downstream (e.g., second turbine on the lower left)
- Downstream turbines might produce 20-30% less power than first row turbines (i.e. with free wind stream)
- Power efficiency can vary largely by wind direction, yet industry standards follow an unrealistic assumption of a single, prevailing direction,
  - 5-9 rotor diameters apart in the prevailing wind direction
  - 3-5 diameters apart in the direction perpendicular to the prevailing winds
- Additional constraints that impact layout (e.g., shipping lanes for off-shore, property rights for on-shore) need to be considered

Horns Rev Off-Shore Wind Farm (Denmark)



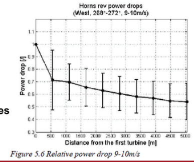


Figure 5.6 Relative power drop 9-10m/s

(Courtesy: Sandia National Laboratories and NREL)

GOALS

- **Given:**
  - A site, including possible turbine locations and the area of planned wind farm
  - Wind speed distribution for each interval of directions
  - Project budget and costs (e.g., number of desired turbines)
  - Turbine specifications (e.g., radius, height, power curve, cut-in/cut-out speeds)
  - Turbine proximity constraints
- Select a set of turbine locations satisfying constraints
- Choose locations of turbines to maximize power production

WAKES AND POWER

- Power is a function of wind speed, which is reduced by wake effects
  - $\delta V_{wdir,p}$ - the fraction of the original free stream wind speed, U_{wdir}, that has been lost due to wakes from upstream turbines
- Power taking wake effects into account:
  - Ignoring operational losses (multiplicative factors)

$$P' = \sum_p \sum_{\langle wdir,U,frac\rangle} frac * \frac{\rho}{2}\pi R_t^2 * ((1-\delta V_{wdir,p}) * U)^3$$

  - Or we can use the turbine power curve for more realistic estimate

$$P = \sum_p \sum_{\langle wdir,U,frac\rangle} frac * P_t\left((1-\delta V_{wdir,p}) * U\right)$$

- Use the Risø Model, a simplified analytical model widely used in industry for computing wakes (suitable within optimization)
- For each location, p2, and wind direction, wdir:
  - $I_{p2,wdir}$ – the set of all upstream locations that can affect wind at p2 when wind is coming from wdir

    If the angle between wind direction and the vector between the two turbine locations $\theta < \theta_{thresh}$, a threshold angle 15-20°
  - $\delta V_{wdir,p2,p1}$ – single wake speed deficit caused by a turbine at location $p1 \in I_{p2,wdir}$ to a turbine at p2

    A function of the angle θ, the distance between the two locations, turbine radius R, turbine thrust coefficient C_T, wake decay constant, α

$$\delta V_{wdir,p1,p2} = 1 - \frac{V_{wdir,p2}}{U_{wdir,0}} = f(\theta_{wdir,p1,p2}) * (1-\sqrt{1-C_T}) * \left(\frac{R}{R+\alpha * dist(p1,p2)}\right)^2 \qquad f(\theta) = \frac{1+\cos(9\theta)}{2}$$

  - $\delta V_{wdir,p2}$ - speed deficit caused by the multiple wake effects

$$\delta V_{wdir,p2} = 1 - \frac{V_{wdir,p2}}{U_{wdir,0}} = \sqrt{\sum_{pi \in I_{p2,wdir}} (\delta V_{wdir,pi,p2})^2}$$

OPTIMIZATION

- Discretize area of planned wind farm into potential turbine locations {p}
- Binary decision variables $\{x_p\}$: equal to 1 if a turbine is placed at location p, 0 otherwise
- Maximize power:

$$\max P' = \sum_p \left\{ x_p * \sum_{\langle wdir,U,freq\rangle \in speeddistr} \left\{ freq * \frac{\rho}{2}\pi R_t^2 U^3 * \left(1 - \sqrt{\sum_{pi \in I_{p,wdir}} \left(x_{pi} * \delta V^2_{wdir,pi,p}\right)}\right)^3 \right\}\right\}$$

- Budget constraint:

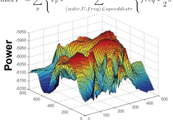
$$\sum_p x_p * cost_p \le B$$

- Proximity Constraint:

$$x_{p1} + x_{p2} \le 1, \quad \forall p1,p2: \; p1 != p2, \; dist(p1,p2) \; < \; distcutoff$$

- Integer Problem – only binary variables
- Linear constraints – multiple knapsack problem
- Nonlinear Objective (length of expression $O(m^2)$ if m is number of locations

$$\max P' = \sum_p \left\{ x_p * \sum_{\langle wdir,U,freq\rangle \in speeddistr} \left\{ freq * \frac{\rho}{2}\pi R_t^2 U^3 * \left(1 - \sqrt{\sum_{pi \in I_{p,wdir}} \left(x_{pi} * \delta V^2_{wdir,pi,p}\right)}\right)^3 \right\}\right\}$$



With eight turbines placed, the resulting power when adding one more turbine at any location, (x,y)

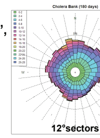
See Related Presentation (7B.5 Energy)

SOLUTION

- Cannot use standard MILP software e.g. CPLEX
- MINLP solvers geared towards non-linear constraints
- Here: General-purpose Branch-and-Bound
- Local upper bound on the objective:
  - Given partial layout with k turbines allocated, and remaining budget enough for s turbines
  - For the s non-allocated turbines, estimate power given current layout at all feasible locations and choose the best s locations
  - For each allocated turbine, estimate speed deficits for each wind direction by including the s least interfering feasible locations and compute power
- Branch heuristic
  - Choose the feasible location with best estimated power given current layout
- CPLEX
  - Encodes and enforces the linear constraints
  - Leave the objective as a free variable – no objective estimate from LP relaxation
  - The resulting ILP problem is easy
    - Multiple feasible solutions at the root and at every search node
    - local Upper Bound cuts at every search node
    - global Lower Bound Cuts every time a better solution is found
- Constraint programming
  - Can encode any kind of constraint or expression
  - Does not use the LP substructure of the problem
  - Using a Depth-First-Search with Restarts (no GAP information)

RESULTS

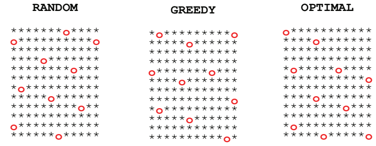
- Hypothetical wind farm at "Cholera Bank", off the southeastern coast of New York State, [40° 26.6'N, 73° 32.5'W]
- 180 days of wind simulations
- Off-shore Nordex-N80 turbines (2.5MW)
  - 40m blade radius
  - Operating range: 4 – 25 m/sec



12°sectors

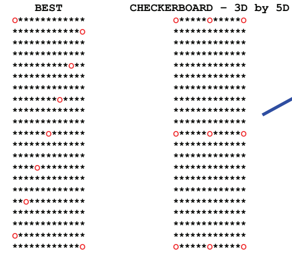
Example 1: Square Grid of 12x12 Sites

Rated kW	25000	25000	25000
Nowake power kW	8671.36	8671.36	8671.36
Nowake/rated	0.3468544	0.3468544	0.3468544
Solution	RAND	GREEDY	BEST
Wake power kW	6757.19	6642.54	7316.25
Wake power kWh/year	5.92E+07	5.82E+07	6.41E+07
Wake/no wake	0.779254	0.766033	0.843726
Wake/rated	0.270288	0.265702	0.29265
Mininum turbine power kW	510.785	497.714	543.893
Maximum turbine power kW	772.42	776.559	867.136



Example 2: Nine Turbines on 13x21 Grid

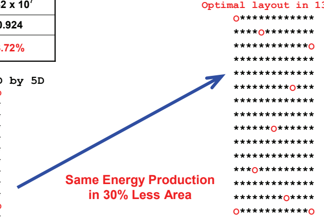
Rated kW	22500	22500
No wake kW	7804.22	7804.22
No wake/rated	0.347	0.347
Solution	Checkerboard	Best
Wake kW	6955.6	7214.62
Wake kWh/yr	6.09×10^7	6.32×10^7
Wake/nowake	0.891	0.924
Improvement over checkerboard	0	3.72%



Example 3: Nine Turbines on Different Grids

Area (sq. miles)	160000	230400	268800	307200	460800	384000
Locations	11x11	13x13	13x15	13x17	13x21	13x25
Checkerboard	2.5Dx2.5D	3Dx3D	3Dx3.5D	3Dx4D	3Dx5D	3Dx6D
Checkerboard kW	6537.4	6708.44	6781.86	6852.94	6955.6	7031.14
Best kW	6634.53	6817.89	6964.94	7023.3	7214.62	7230.18
Improvement	1.49%	1.63%	2.70%	2.49%	3.72%	2.83%

Optimal layout in 13x15



Same Energy Production in 30% Less Area

ON-GOING WORK

- Resolve MIP model not scaling for large numbers of locations (e.g., large offshore farms)
  - Too many feasible solutions
- Explore alternative methods
  - Integrate Linear & Constraint Programming
    - Encapsulate the LP proximity constraints as a single custom constraint in CP
    - Use the LP relaxation to propagate the constraint
  - NLP models – use (x,y) coordinates of each turbine as decision variables
    - Constrained non-linear model
    - Non-smooth, non-differentiable
- Explore alternative methods
- Potential refinements
  - Introduce more accurate wake calculation
  - Leverage 3d wind data
  - Support multiple types of turbines