Spectral and morphing ensemble Kalman filters

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Outline

Introduction: The Ensemble Kalman Filter (EnKF)

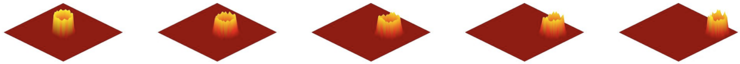
Get an approximate forecast covariance from an ensemble of simulations, then use it in the Bayesian update

- by sample covariance
  - converges to optimal filter in large ensemble limit and gaussian case (Mandel et al., 2009b)
  - adjusts the state by linear combinations of ensemble members
- localized sample covariance
  - tapered sample covariance: better approximation for small ensembles using assumed covariance distance
  - other localized filters (Ensemble adjustment, LETKF,...)
  - still restricted to linear combinations locally
- probability distributions not too far from gaussian needed for proper operation

See the book by Evensen (2009) for references.

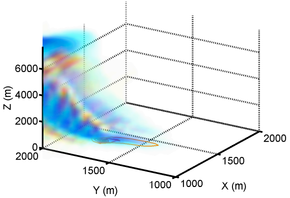
Morphing EnKF (Beezley and Mandel, 2008)

- Moving coherent features: **need also position correction**
- Replace the state by a deformation of a reference field + a residual
  - by automatic registration: multiscale optimization
  - also related to advection field found in radar analysis
- run EnKF on the extended states: **closer to gaussian**
- recover ensemble members from the deformation and residual fields
- basically, **replace linear combinations by morphs**:

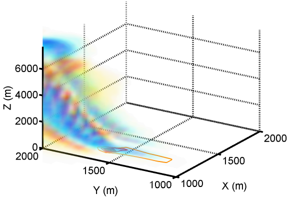


  - Intermediate states from a linear combination of deformation fields and residual fields
  - **tricky: the right kind of combination to avoid ghosting**

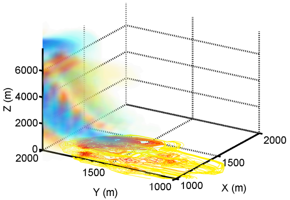
WRF-Fire (Mandel et al., 2009a)



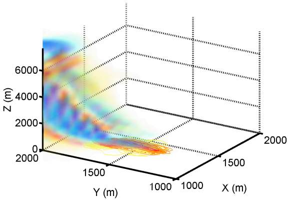
Data source



No assimilation



Standard EnKF



Morphing EnKF

Some related work on position correction and alignment

- error model with position of features (Davis et al., 2006a,b) and distortion (Hoffman et al., 1995; Marzban et al., 2009; Marzban and Sandgathe, 2010; Nehrkorn et al., 2003)
- global low order polynomial mapping (Alexander et al., 1998)
- alignment as a pre-processing step to additive correction (Lawson and Hansen, 2005; Ravela et al., 2007; Aonashi and Eito, 2010)
- 1D morphing to improve 12-hour forecasts (Beechler et al., 2010)

Optimal statistical interpolation by FFT

Find the analysis u^a from the forecast u^f by balancing the state error with the covariance Q and the data error with the covariance R:

$$\left\| u^f - u^a \right\|^2_{Q^{-1}} + \| Hu^a - d \|^2_{R^{-1}} \to \min_{u^a}$$

$$\Longleftrightarrow u^a = u^f + K \left(d - Hu^f \right), \quad K = QH^T \left(HQH^T + R \right)^{-1}$$

- Standard: covariance Q drops off by distance
- but Green's function $Q = \Delta^{-1}$, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, drops off OK: use the Laplacian for covariance (Kitanidis, 1999)
- Δ has **no directional bias**, $\Delta^{-\alpha}$ is the covariance of a homogeneous isotropic random field. Power law spectrum, eigenvalues $C(m^2 + n^2)^{-\alpha}$; **larger** $\alpha \Rightarrow$ **smoother functions**
- Δ is **diagonal** after FFT: **fast implementation**, at least when $H = I$ (all state observed); generalizations also exist.

Data assimilation with high-resolution weather fields in seconds on a laptop, not a supercomputer.

Spectral diagonal estimation of covariance

Sample covariance is a bad approximation for small ensembles: low rank causes spurious long-range correlations. Instead,

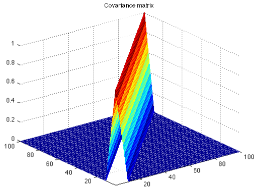
- transform the members into the spectral space
- compute the diagonal of the sample covariance
- fast matrix-vector operations in the spectral space

Orthogonal wavelets approximate weather states well (Fournier, 2000). Spectral diagonal approximation of the covariance:

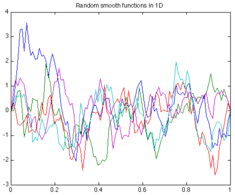
- by **Fourier transform** (Berre, 2000): *homogeneous in space*
- by **wavelets** (Deckmyn and Berre, 2005; Fournier and Auligné, 2010; Pannekoucke et al., 2007): *localized*

Assumes that spectral modes are uncorrelated. Unlike classical tapered covariance, **provides automatic tapering and fast multiplication by the inverse by FFT or fast wavelet transform**.

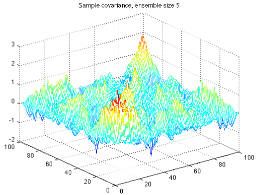
Automatic tapering by FFT diagonal estimation



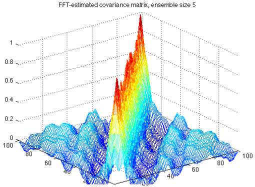
Given covariance



Ensemble of 5 random functions



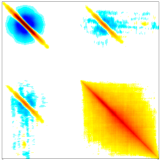
Sample covariance



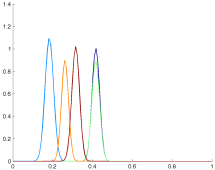
FFT estimation

From Mandel et al. (2010b)

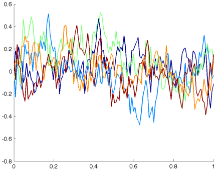
Covariance estimation, 2 variables



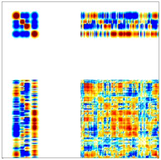
Covariance, sample of 1000



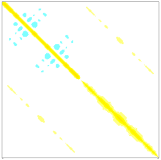
Variable 1, sample of 5



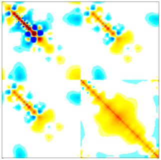
Variable 2, sample of 5



Covariance, sample of 5



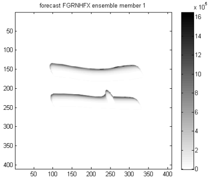
FFT estimation, sample of 5



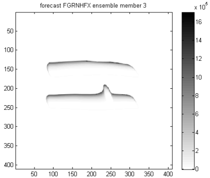
Wavelet estimation, sample of 5

Estimation by FFT results in a distribution that is homogeneous in space, smearing the distribution across the domain. Wavelet estimation keeps the spatial structure, while filtering out spurious long-distance correlations.

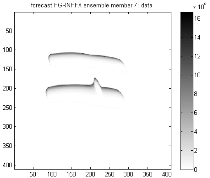
FFT EnKF for wildland fire simulation



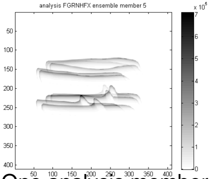
One forecast member



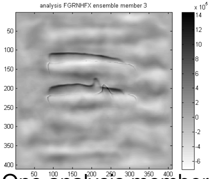
Another forecast member



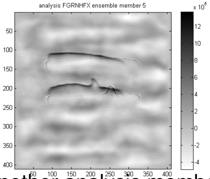
Data



One analysis member with sample covariance



One analysis member with FFT estimation



Another analysis member with FFT estimation

Data assimilation for WRF-Fire by the morphing EnKF with ensemble size 5. Standard sample covariance results in ghosting, while FFT estimated covariance gives interpolation between the forecast and the data. From Mandel et al. (2010c).

Conclusion

- Spectral EnKF can operate succesfully with a very small ensemble (5-10 members)
- It can deal with position adjustment in combination with morphing EnKF.
- Observation on the whole domain or subrectangle.
- The base algorithm is the same for FFT and for orthogonal wavelets.
- In progress:
  - Spectral EnKF in the case of multiple variables
  - Wavelet EnKF to improve data assimilation for wildland fires, precipitation (Mandel et al., 2010b), and epidemics simulation (Krishnamurthy et al., 2010; Mandel et al., 2010a)
  - Assimilation of time series of point data

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