A SOM-PCA Model for Inverting Shallow-Water Acoustic Data

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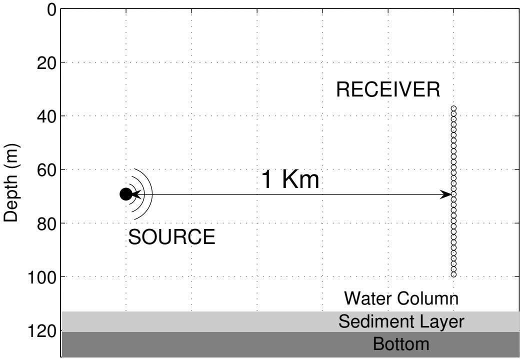
Outline

Shallow-water acoustic tomography (SWAT)

Geometry description of the environment



- The seafloor geoacoustic properties are supposed known
- Find the sound speed profile $\mathbf{c}$ from received data
  - Forward model (WAPE+NLBC) $\mathbf{p} = G(\mathbf{c}),\ f = 500\,\mathrm{Hz}$
  - Inversion processus: variational method

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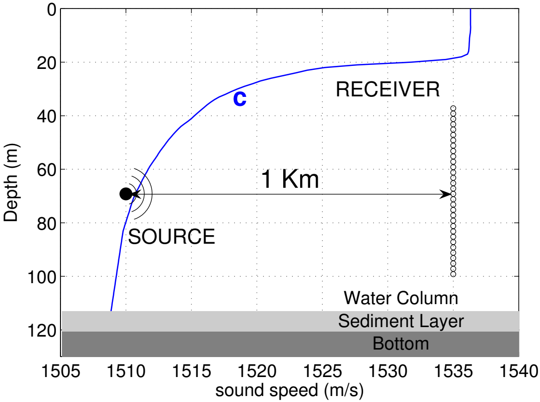

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Variational method

- Minimizing a cost function (accurate phase) (Collins et al. 1992)

$$J_o(\mathbf{c}) = \min_{\lambda} ||\mathbf{p}_{obs} - \lambda \mathbf{p}||^2 \Leftrightarrow J_o(\mathbf{c}) = \mathrm{tr}\mathbf{R} - \frac{\mathbf{p}^T\mathbf{R}\mathbf{p}}{\mathbf{p}^T\mathbf{p}}$$

  - $\mathbf{p} = G(\mathbf{c})$ predicted vector at the VRA positions
  - $\mathbf{R} = \mathbf{p}_{obs}\mathbf{p}_{obs}^{\dagger}$ spatial covariance matrix
  - $\mathbf{R}$ approximated from measured acoustic signals $s_j(t),\ j = 1, \ldots, 32$ (Hermand and Gerstoft 1996)

- The gradient of $J_o(\mathbf{c})$ - Adjoint by YAO (Thiria et al. 2006 and Nardi et al. 2009)

$$\nabla_c J_o = \mathbf{G}^{\dagger} \nabla_p J_o$$

where $\mathbf{G} = \frac{\partial G}{\partial x}$: linear tangant, $\mathbf{G}^{\dagger}$ its adjoint and $\nabla_p J_o = -\frac{\mathbf{R}\mathbf{p}}{||\mathbf{p}||^2} + \frac{(\mathbf{p}^{\dagger}\mathbf{R}\mathbf{p})\mathbf{p}}{||\mathbf{p}||^4}$.

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Variational method - Regularization

- SWAT inverse problem is ill-posed
- Measurements are noisy

$$s(t) = s(t) + e(t),\ e(t) \text{ Gaussian noise}$$

- Regularization:
  - Background term $\mathbf{c} \hookrightarrow \mathcal{N}(\mathbf{c}_b, B)$ (Bayesian formalism)

$$(\mathbf{c} - \mathbf{c}_b)B^{-1}(\mathbf{c} - \mathbf{c}_b)$$

  - Dimensional reduction by Emperical PCA

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Regularization - Background terme

- Cost function with background term :

$$J(c) = J_o(c)$$

  - c_b background (a priori information)
  - B covariance matrix of the background error : primary role
    - Consistency of increments between neighboring points
    - Consistency of increments between variables
- Gradient of J

$$\nabla_{c} J = \mathbf{G}^{\dagger} \nabla_{p} J_o + B^{-1}(c - c_b)$$

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$$J(c) = J_o(c) + \underbrace{\frac{1}{2}(c - c_b)^T B^{-1}(c - c_b)}_{\text{distance to the background } c_b}$$

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Regularization - Empirical PCA

- $\mathcal{A}$ set of celerity profiles locally representative of the problem
- Basis of PCA

$$\mathbf{c} = \mathbf{c}_b + \sum_{i=1}^{q} a_i U_i, \; q \ll M$$

where U_i Eigenvectors of the empirical variance covariance matrix of data of $\mathcal{A}$

- Cost function without background term

$$J(\mathbf{a}) = J_0(\mathbf{c}_b + \textstyle\sum_{i=1}^{q} a_i U_i)$$

- New control vector $\mathbf{a}$ of reduced dimension q, $q \ll M$
- Regularization by reducing the dimension q $(q = 3 \ldots)$
- No criterion for the choice of q (Filter useful information)

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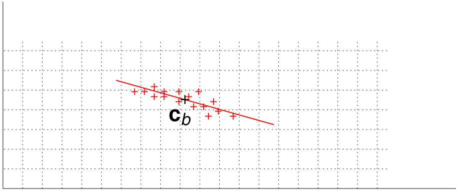
Regularization by probabilistic PCA

- PPCA is a probabilistic interpretation of empirical PCA
- Introduces a random vector $\eta \hookrightarrow \mathcal{N}(0, I_q)$
- $\mathbf{c} = \mathbf{W}\eta + \mathbf{c}_b + \varepsilon$, with $\varepsilon \hookrightarrow \mathcal{N}(0, \kappa^2 I_M)$

(by likelihood maximization)

$\Downarrow$

- $\mathbf{c} \hookrightarrow \mathcal{N}(\mathbf{c}_b, B)$ where $B = \mathbf{W}\mathbf{W}^T + \kappa^2 I_M$



Regularization by probabilistic PCA

- PPCA parameters are estimated from the data set $\mathcal{A}$
  - $\mathbf{c}_b$ the data mean
  - $\mathbf{W} = U(L^{1/2} - \kappa^2 I_q)$, $U = (U_1, \ldots, U_q)$ and L : diagonal matrix $L = (\lambda_i)$
  - $\kappa^2 = \frac{1}{M-q} \sum_{i=q+1}^{M} \lambda_i$, where λ_i : the eigenvalues
- κ^2 can be negligible for $q < M$ large enough ($\kappa^2 \approx 0$)
- New cost function

$$J(\eta) = J_o(\mathbf{W}\eta + \mathbf{c}_b) + \frac{1}{2}\eta^T \eta$$

- Circumvent the problem of estimating B^{-1} and q
- The components of η are non correlated
- Provides a better preconditioning for the minimization process

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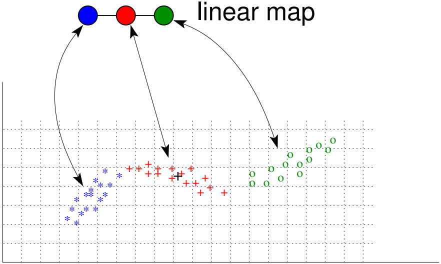
Global optimization



- Mixture of gaussian
- Local cost function

$$J_i(\mathbf{c}) = J_0(\mathbf{c}) + (\mathbf{c} - \mathbf{c}_{b,i})^T B_i^{-1} (\mathbf{c} - \mathbf{c}_{b,i}), \quad \mathbf{c} \hookrightarrow \mathcal{N}(\mathbf{c}_{b,i}, B_i)$$

- Local cost function by PPCA

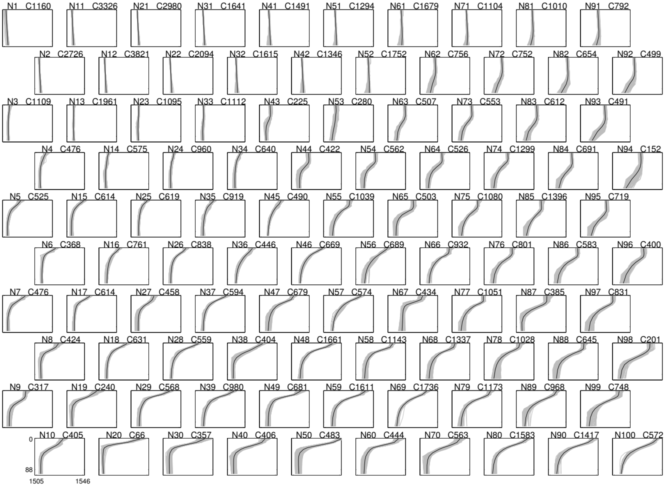
$$J_i(\eta) = J_o(\mathbf{W}_i \eta + \mathbf{c}_{b,i}) + \eta^T \eta, \quad \eta \hookrightarrow \mathcal{N}(0, I_{q_i})$$

- Groups and neighborhood $\Leftarrow$ topological maps

Global optimization - Topological maps

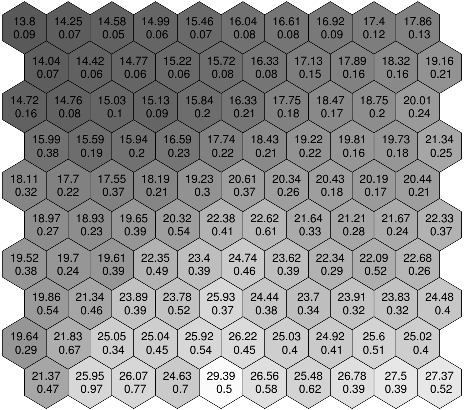
- Training set $\mathcal{A}$:
  - Medwin formula $c(z, T(z), S(z))$
  - $T(z), S(z)$ simulated by Mercator Ocean
  - Area : Mediterranean (Elba Island)
  - period: 4 years (2003-2006)
  - Each profile is a daily average
  - $N = 1461$ profiles of celerity
- Topological maps : hexagonal : 10×10

Global optimization - Profiles distribution



Distribution of celerity profiles in an hexagonal topological map of dimension : 10×10

Global optimization - Sea Surface Temperature



Sea Surface Temperature average
and mean deviation by node

Global optimization - Random walk

- Global optimization by random walk
- Sampling according to the a posteriori probability

$$\sigma_i = k\rho_i L_i, \; i = 1, \ldots, 100$$

- k normalization constante
- ρ_i a priori information (surface temperature)
- L_i the likelihood (optimization of the local cost function)

Global optimization - Random walk

- The a priori ρ_i

$$\rho_i = \exp(-\frac{|SST_i - SST_0|}{\theta_i})$$

  - SST_i the surface temperature average of the i-th group
  - θ_i the mean deviation of the surface temperature of i-th group
  - SST_0 the surface temperature of the desired profile

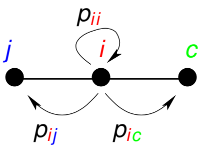
- The likelihood L_i

$$L_i = C \exp(-\frac{J_i(\eta_i^*)}{\tau}), \quad C \text{ constante}$$

  - η_i^* local optimum
  - τ is "the system energy" $0.01 \leq \tau \leq 0.5$

Global optimization - Random walk

- Random walk $\equiv$ transition rules p_{ij}



- p_{ij} transition rules of the a priori ρ_i
- Transition rules of the a posteriori $\sigma_i = k\rho_i L_i$:
  4. Accept $i \to j$ with the probability

$$\min(\frac{L_j}{L_i}, 1)$$

- Define p_{ij}
  1. Equiprobable choice of a neighbor j of i
  2. Accept $i \to j$ with the probability $\min(n_i/n_j, 1)$
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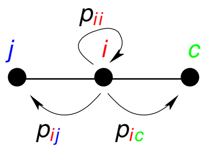
Global optimization - Random walk



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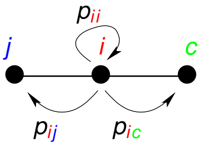

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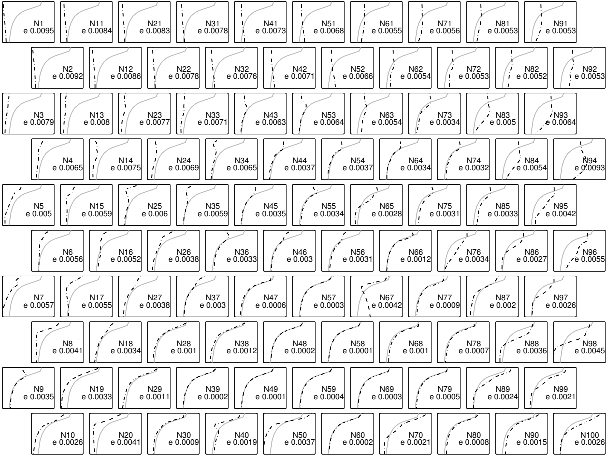
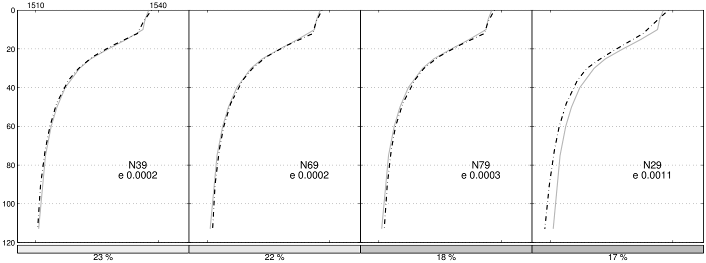
Inversion results



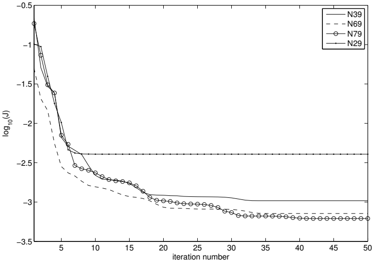
Result of the inversion of the celerity profile in N79

$$\mathbf{c}^* \in N79$$

Inversion results



The first four celerity profiles chosen by the random walk



Conclusion

- The Probabilistic PCA provides a better preconditioning for the local minimization process
- It allows a dimensional reduction and the introduction of the background term into the cost function
- The use of oceanic models allows enlarge of the dataset
- Classification is done by topological maps
- The local PPCA method is applied on various groups obtained by the classification
- The choice of desired celerity profile is done by a random walk

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Thank you!