EVIDENCE FOR CLUSTERING OF TEMPERATURES AT HIGH LEVELS BASED ON EXTREME VALUE THEORY

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“Recipe for Disaster: The Formula That Killed Wall Street”

Felix Salmon (*Wired Magazine*: 23 Feb. 2009):

$$\Pr[T_A < 1, T_B < 1] = \phi_2(\phi^{-1}(F_A(1)), \phi^{-1}(F_B(1)), \gamma)$$

- **Gaussian copula**

-- Statistical model for joint dependence

Marginal distributions are transformed to Gaussian (or uniform)

-- Proved inadequate for financial applications

What about for climate extremes?

Outline

(1) Background / Motivation

(2) Extremal Index for Clustering at High Levels

(3) First-order Gaussian Process

(4) Application to Daily Maximum Temperature

(5) Discussion

(1) Background / Motivation

- **Heat waves**

-- Meteorological phenomenon

Extreme event

Persistent (e. g., associated with blocking)

-- Statistical perspective

Extreme Value Theory

Concept of Extremal Index

Measure of clustering at high levels

(2) Extremal Index for Clustering at High Levels

- Extreme Value Theory (EVT)

-- Consider sequence of observations $\{X_1, X_2, \ldots\}$

Assumed independent (*for now*) and identically distributed (IID)

-- Extremal Types Theorem for $\max\{X_1, X_2, \ldots, X_n\}$

Approximate generalized extreme value (GEV) dist. for large n

-- Analogue for excess over high threshold $X_t - u$, given $X_t > u$

Approximate generalized Pareto (GP) dist. for high threshold u

- **EVT under temporal dependence**

-- Consider stationary process $\{X_1, X_2, . . . \}$

That is, possibly temporally dependent

-- Weak memory (Observations far apart in time)

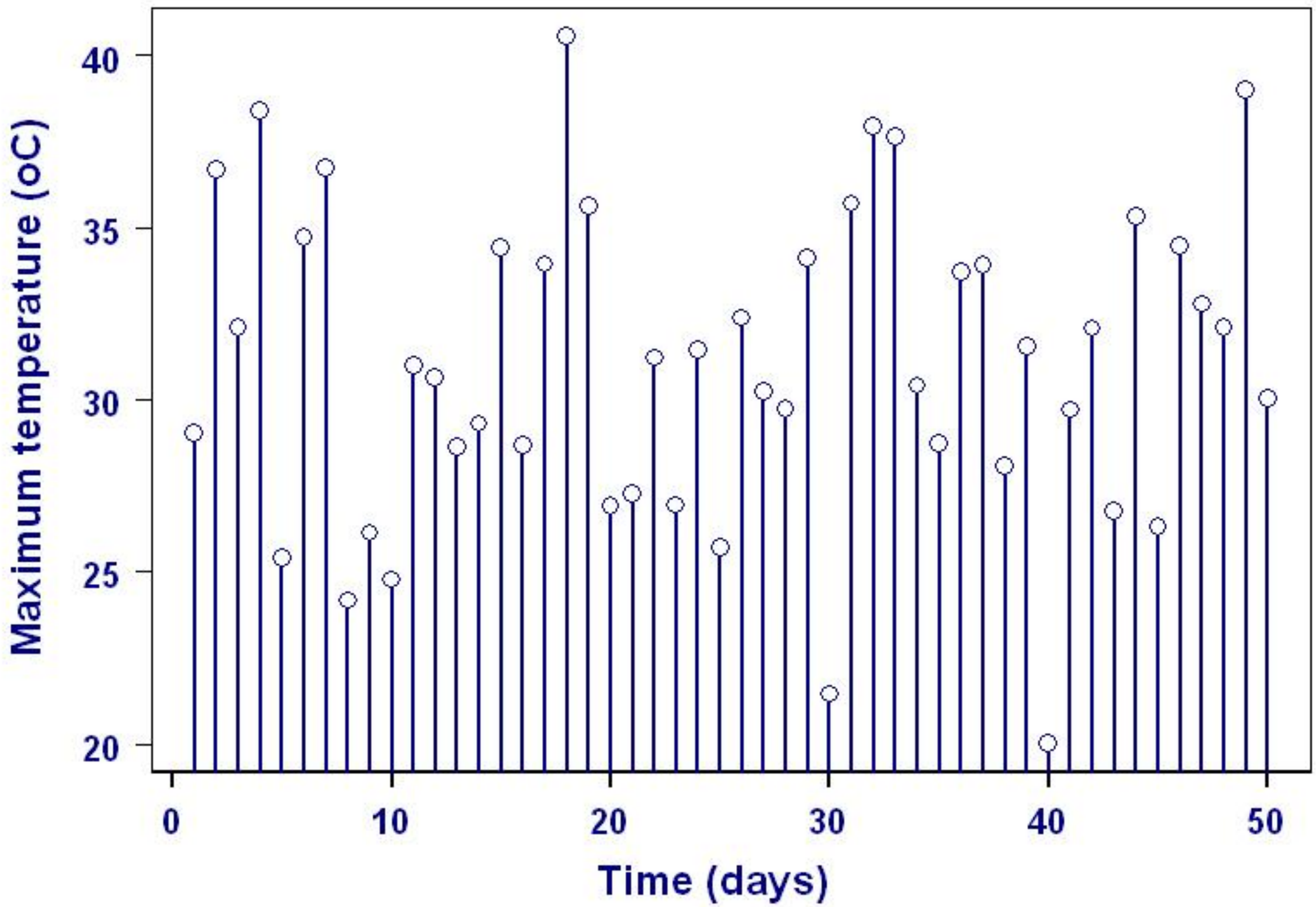
EVT still holds if dependence weakens at fast enough rate

-- Clustering at high levels (Observations close together in time)

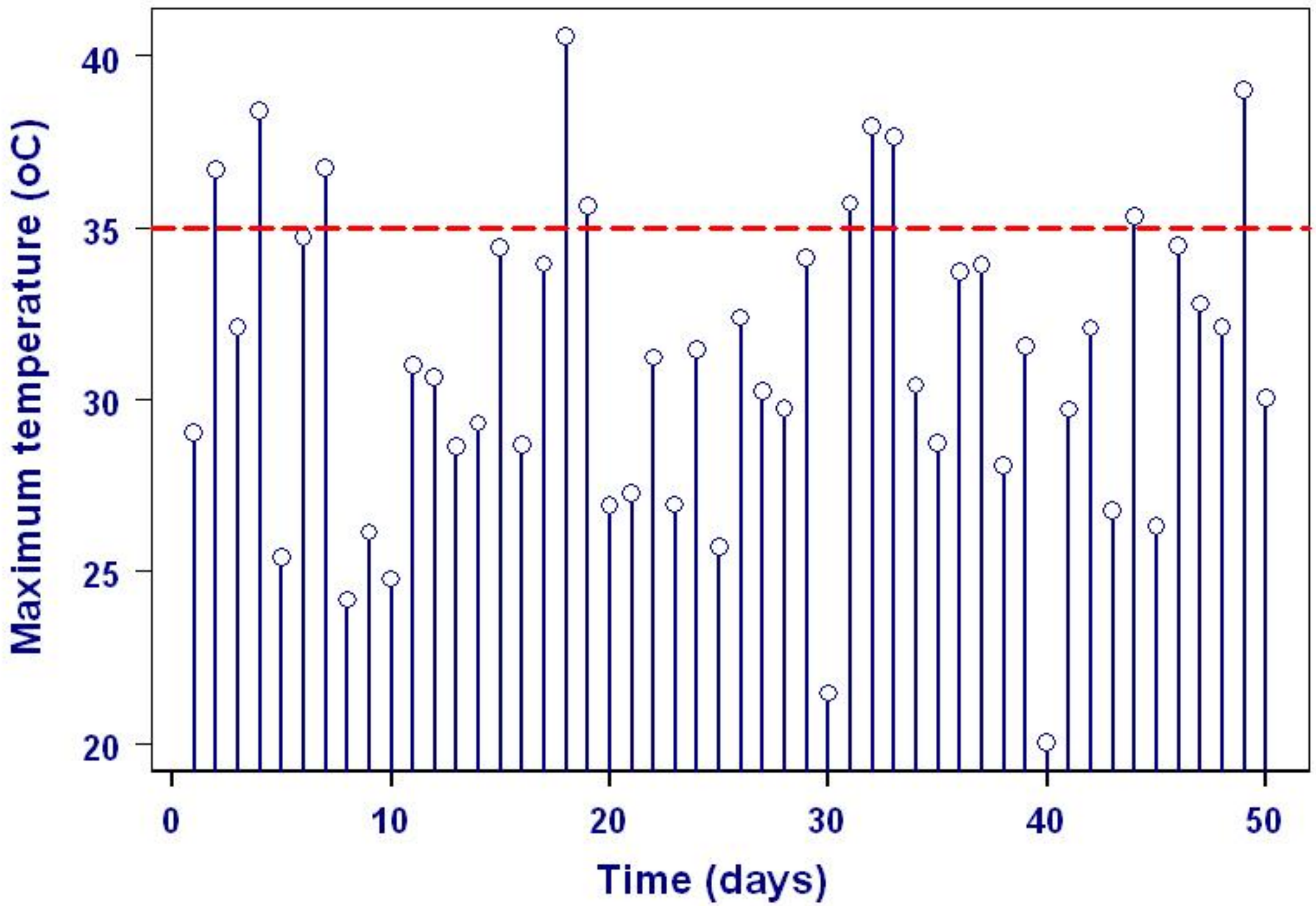
EVT still holds with adjustment to normalizing constants

Consider *cluster* (or *hot spell*): $X_{t+1} > u, . . ., X_{t+k} > u$

Starts at time $t + 1$, length k days ($k \geq 1$), where u is high threshold



Maximum temperature (oC)
40
35
30
25
20
0
10
20
30
40
50
Time (days)



Maximum temperature (oC)
Time (days)
40
35
30
25
20
0
10
20
30
40
50

- **Condition for *no* clustering at high levels**

$$\Pr\{X_{t+k} > u \mid X_t > u\} \to 0 \text{ as } u \to \infty, \; k = 1, 2, \ldots$$

- **Clustering at high levels (*Extremal Index* θ, 0 ≤ θ ≤ 1)**

(i) θ = 1

No clustering at high levels (Independent or Gaussian process)

(ii) 0 ≤ θ < 1

Clustering at high levels

***Interpretations*: (i) Mean Cluster Length ≈ 1 / θ**

(ii) Effective sample size (proportion)

- **Estimating Extremal Index**

(i) ***Intervals Estimator*** **(Ferro-Segers 2003)**

Does not require identification of clusters

Consider “interexceedance times” between exceedances of threshold

Coefficient of variation of these interexceeance times converges to simple function of Extremal Index θ

Resample (Bootstrap) interexceedance times to obtain confidence interval for Extremal Index θ

(ii) Runs

Approach typically used in "declustering"

Runs parameter $r = 1, 2, \ldots$

$r = 1$ corresponds to defining clusters as run of consecutive exceedances of threshold

$r = 2$ corresponds to terminating a cluster if two or more consecutive days fall below threshold

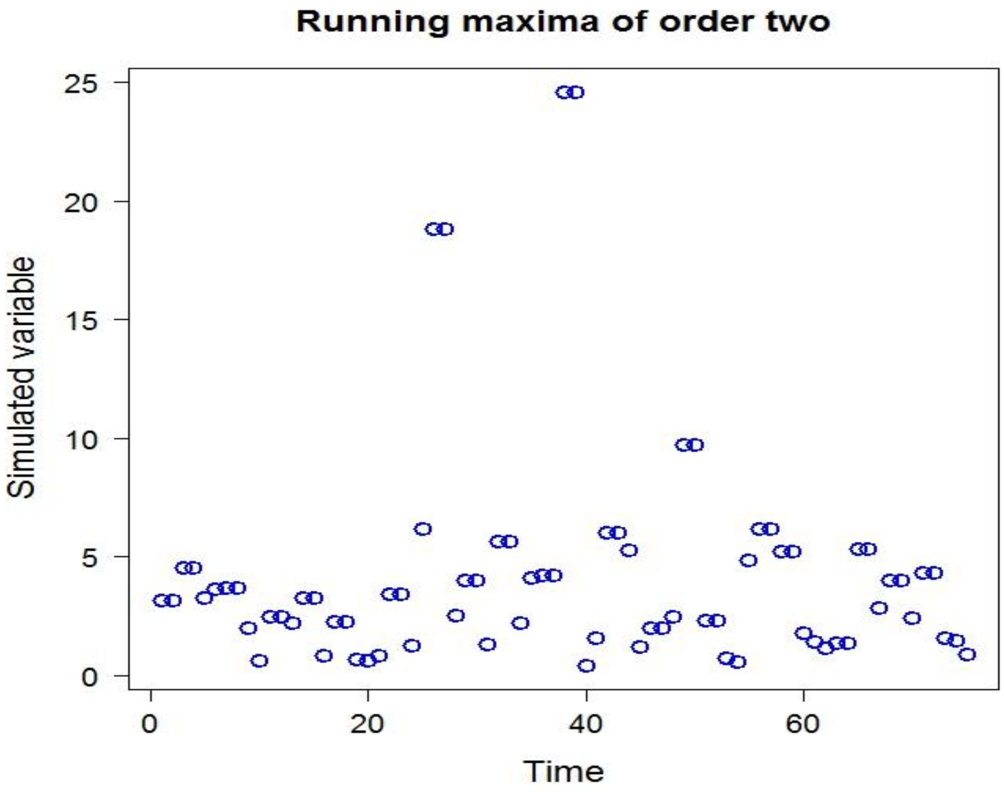
Identify clusters based on some value of r

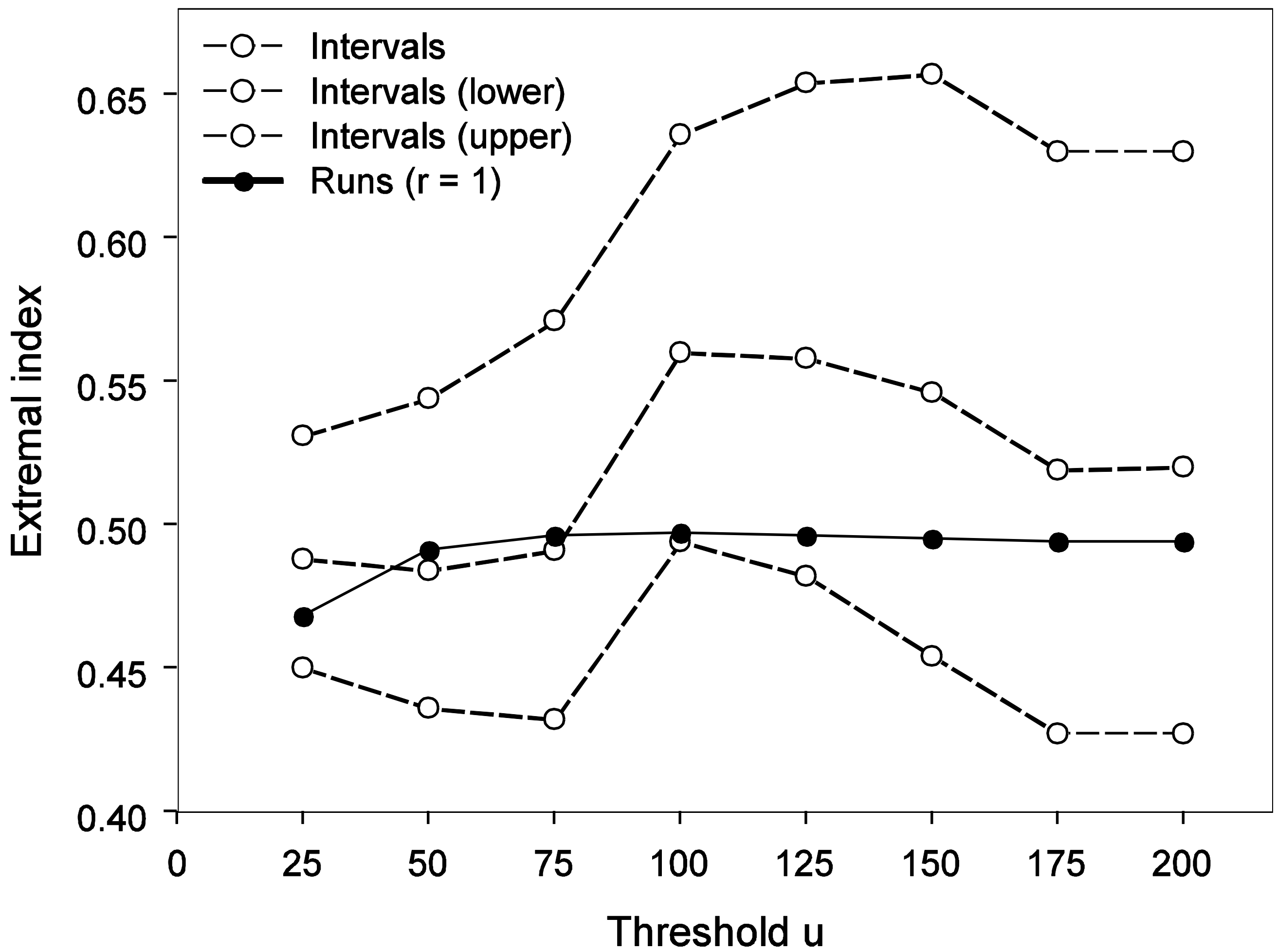
Then estimate θ by: 1 / (Mean Cluster Length)

- **Artificial example (Running maxima of order two)**

$Y_k = \max\{X_k, X_{k+1}\}, k = 1, 2, \ldots, \quad \{X_1, X_2, \ldots\}$ **IID**

Then extremal index $\theta = 0.5$ for $\{Y_k\}$ time series





Running maxima of order two

(3) First-Order Gaussian Process

- **AR(1) Process (zero mean, unit variance)**

$$X_t = \phi X_{t-1} + \varepsilon_t , \qquad \varepsilon_t \sim N(0, 1 - \phi^2)$$

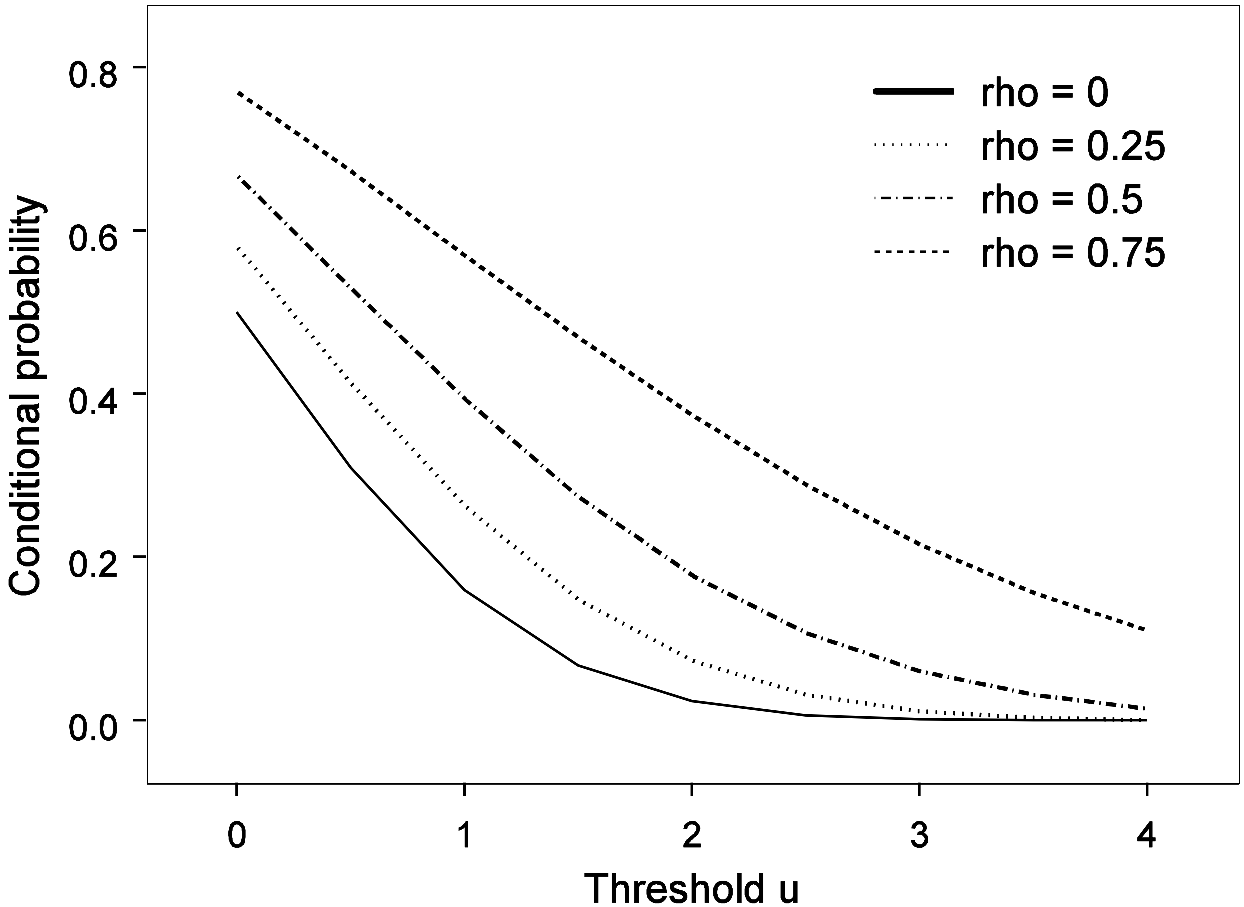
-- Autocorrelation function

$$\mathrm{Corr}(X_t , X_{t+k}) = \phi^k, \; k = 1, 2, \ldots$$

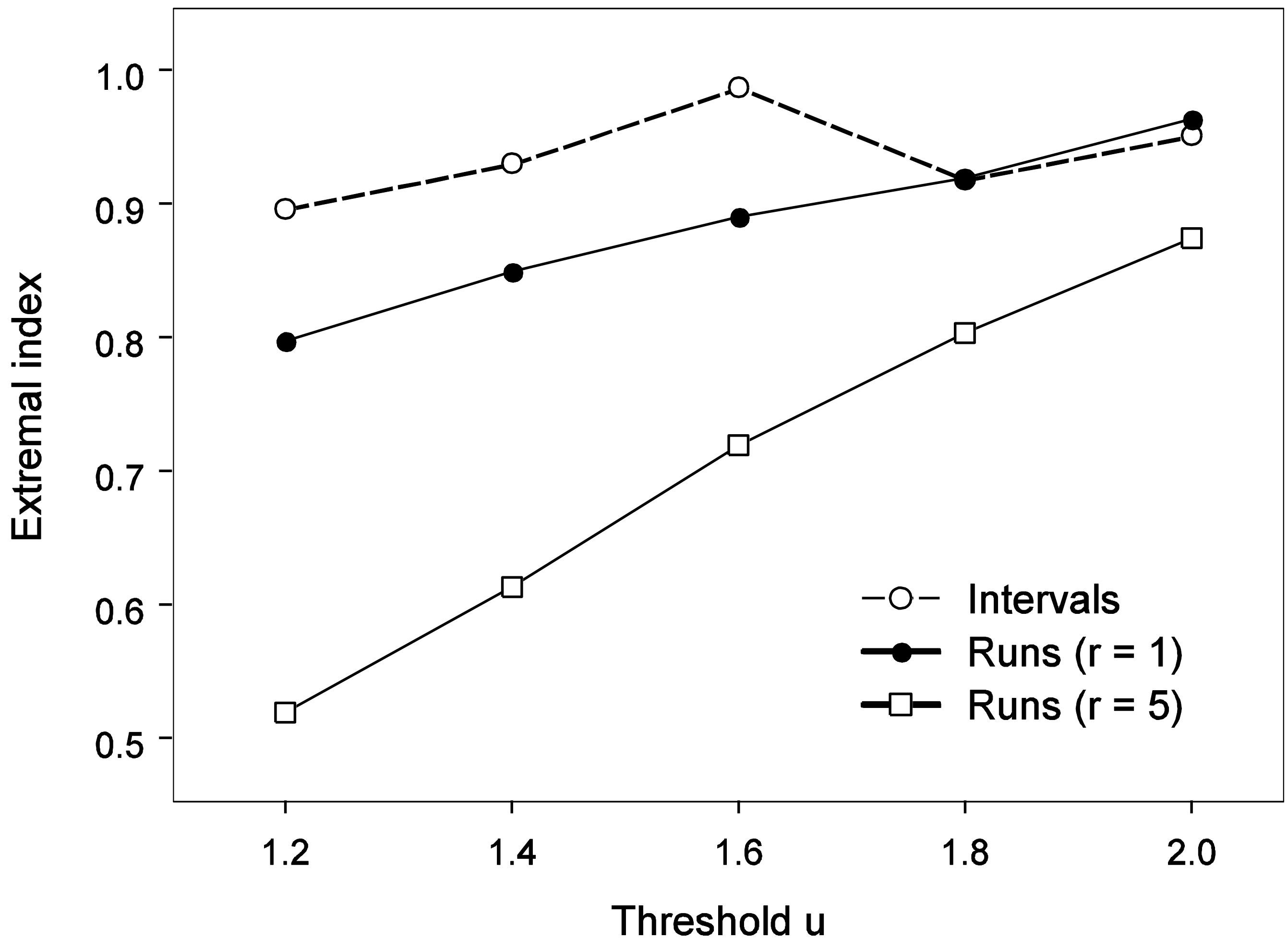
-- Joint distribution

$\{X_t , X_{t+k}\}$ ~ **Bivariate normal with correlation coefficient** $\rho = \phi^k$

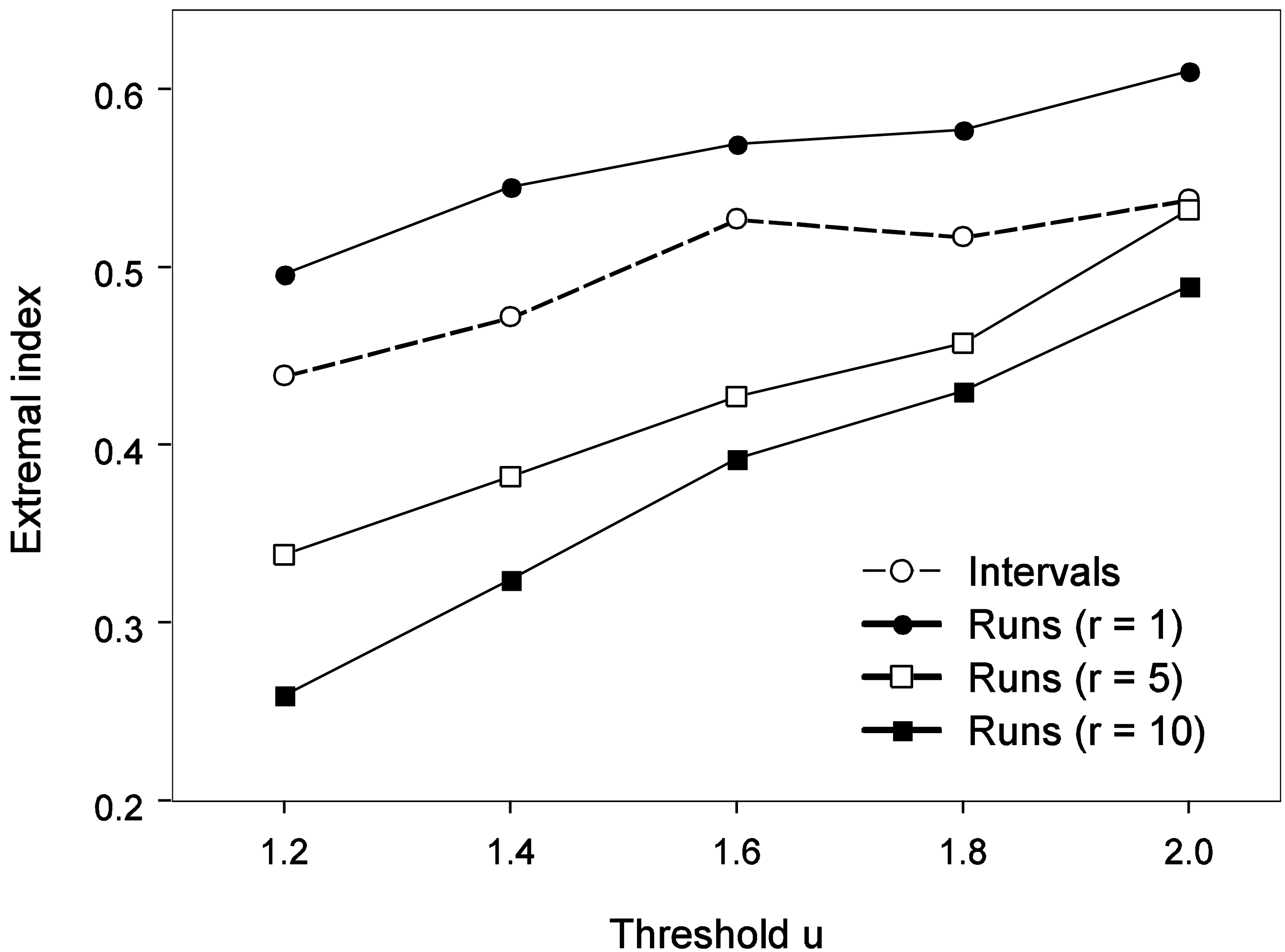
-- Straightforward to show that Extremal Index $\theta = 1$



Bivariate Normal Distribution: $\Pr\{X_{t+k} > u \mid X_t > u\}$



AR(1) Process: $\phi = 0.25$



AR(1) Process: $\phi = 0.75$

(4) Application to Daily Maximum Temperature

- **Estimate Extremal Index θ by two different methods**

(i) Intervals Estimator of θ

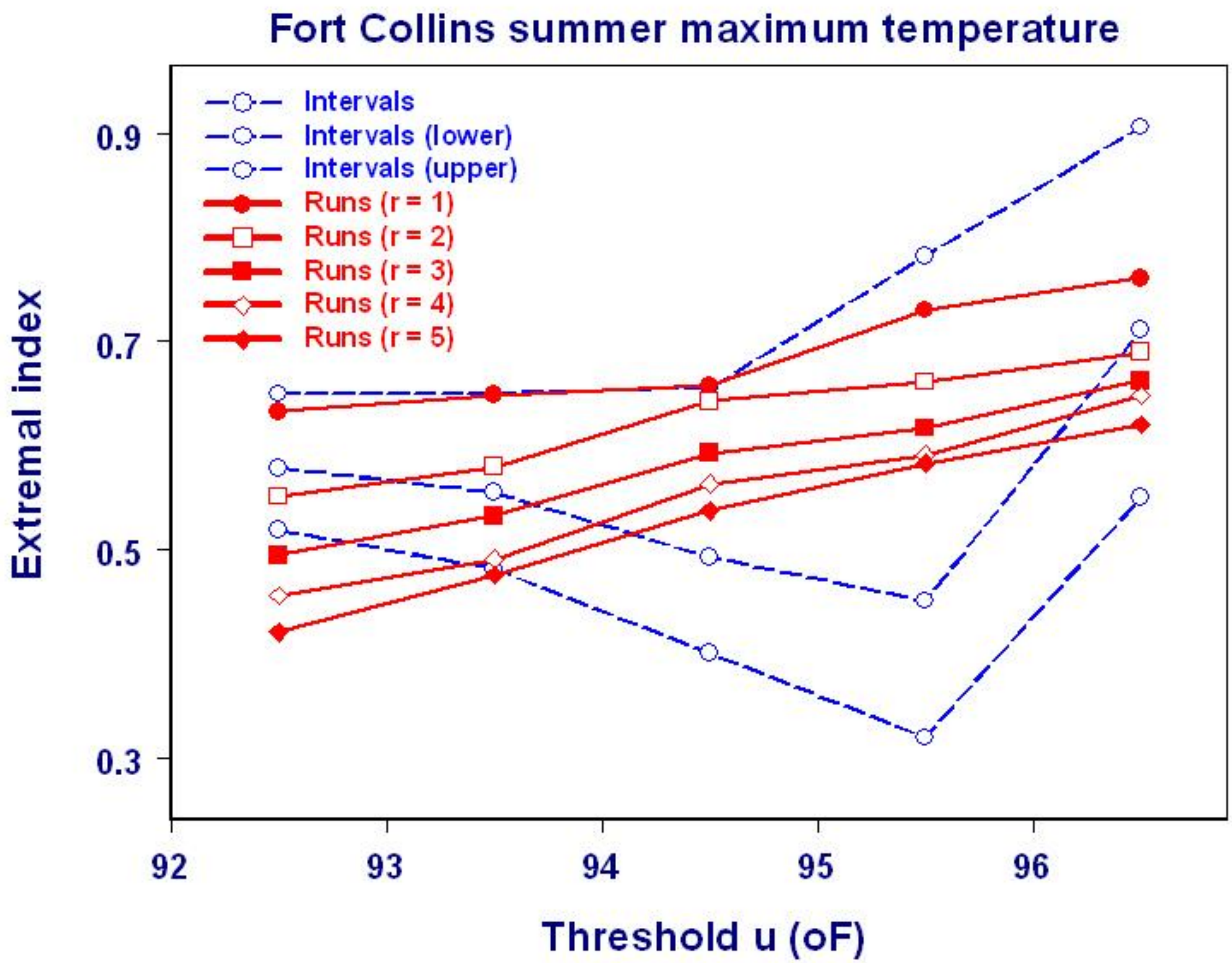
-- Vary threshold *u*

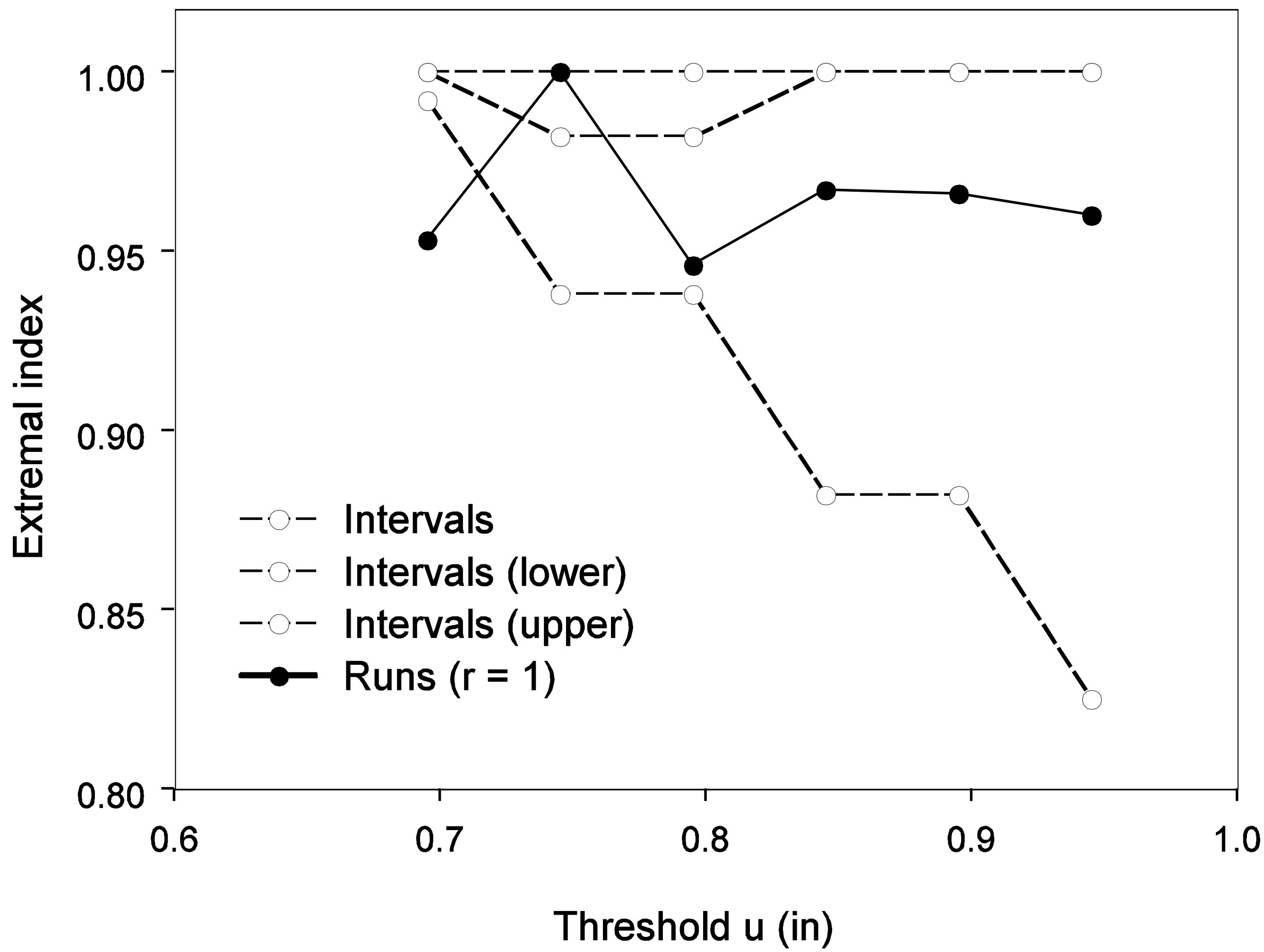
-- Attach confidence interval based on bootstrap

(i) Runs Estimator of θ

-- Runs parameter *r* = 1, 2, 3, 4, 5

-- Vary threshold *u*


Fort Collins summer maximum temperature
Intervals
Intervals (lower)
Intervals (upper)
Runs (r = 1)
Runs (r = 2)
Runs (r = 3)
Runs (r = 4)
Runs (r = 5)
0.9
0.7
0.5
0.3
Extremal index
92
93
94
95
96
Threshold u (oF)



Fort Collins July precipitation

(5) Discussion

- **Dilemma**

(i) *Financial statistics*

Strong evidence that Gaussian copula is inappropriate model for simultaneous occurrence of extremes

(ii) *Climate statistics*

Gaussian models are useful -- Should *not* be viewed as if produce no dependence of extremes

But do Gaussian models treat joint dependence of extremes realistically enough?