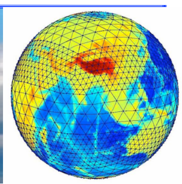
Design and Development of of a Unified Model on Icosahedral-Hexagonal Grids



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Abstract

There are continued attempts towards unifying the general circulation and cloud-resolving models. For designing high-resolution general circulation models, it is necessary to formulate a set of equations for the nonhydrostatic system such that when the nonhydrostatic pressure is neglected, the system of equations reduces to a quasi-hydrostatic compressible model. Following the Arakawa and Konor (2009) approach, the governing equations of a unified model on icosahedral-hexagonal grid are formulated in the hybrid vertical coordinate. Further to this formulation, the flow dependent variables are represented in the basic equations into two parts - grid-resolved and a subgrid part to formulate a system of equations that is capable of simulating the variability of unresolved processes. The advantage of splitting is obvious because the system of equations for grid-resolved variables are indeed those of a quasi-hydrostatic compressible model of the atmosphere. The discrete formulations of divergence, vorticity and gradient are then used to solve the shallow water model on the icosahedral-hexagonal grid as an example of the first stage development of a comprehensive unified model.

Introduction:

The high-resolution operational models of weather forecasting must run with grid of a kilometer on newer computing platforms (GPUs or APUs). Governing equations do not represent the stochastic part in weather and climate that becomes important in atmospheric flows resolved on the scale of a kilometer. Arakawa and Konor (2009) have presented a system of equations that unites the nonhydrostatic anelastic system and quasi-hydrostatic system to derive the governing equations that form the foundation of cloud-resolving models. This is the starting point of unifying the Reynolds stress equations with anelastic, quasi-hydrostatic system of equations. The key assumption here is that even in the unified system, subgrid scale motions of the atmosphere may not have been fully represented the stochastic nature of atmospheric motions. Palmer (1989) showed that adding the stochastic perturbations to the governing equations improves the forecasting skill of the numerical weather prediction (NWP) model.. The multimodel superensemble technique, developed by Krishnamurti and coworkers (Krishnamurti et al. 1999, 2000) has been demonstrated as a powerful post-processing tool for weather forecast parameters. This is another way to incorporate the stochastic nature of the meteorological flows in to weather and climate forecasting. Here we derive the set of equations that becomes an integral part of the anelastic, quasi-hydrostatic system because the stochastic perturbations in the governing equations may be defined by solutions of these equations. An attempt has been made in this paper to include the effect of second order correlation terms in the governing equations. A systematic formulation has been presented which takes the advantage of the advances already made in this direction especially in turbulence modelling, large eddy simulations (LES).

Governing Equations:

We have followed the philosophy of Arakawa and Konor (2009): "One of the main points of the Unified System is that it reduces to a quasi-hydrostatic model when the nonhydrostatic pressure is neglected. In this way the system maintains a close tie with the existing primitive equation models." Thus, the system of equations formulated here using the Reynolds averaging, form an integral part of the governing equations, which reduces to an anelastic, quasi-hydrostatic system if the subgrid scale terms representing stochastic perturbations are neglected in the equations. The solution of the anelastic and quasi-hydrostatic system is assumed in this formulation as the "grid-resolved part" relative to which the perturbation fields are derived. Thus,

$$u=\tilde{u}+u',\quad v=\tilde{v}+v',\quad w=\tilde{w}+w',\quad \omega=\tilde{\omega}+\omega',$$
$$\theta=\tilde{\theta}+\theta',\quad q=\tilde{q}+q',\quad T=\tilde{T}+T'.$$

Operators: $\dfrac{1}{a\cos\varphi}\dfrac{\partial}{\partial\lambda}(.)\equiv\dfrac{\partial}{\partial X}(.)$; $\dfrac{1}{a\cos\varphi}\dfrac{\partial}{\partial\varphi}\cos\varphi(.)\equiv\dfrac{\partial}{\partial Y}(.)$

Governing Equations in Quasi-hydrostatic Pressure (p) Coordinate

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial X}(u^2)+\frac{\partial}{\partial Y}(uv)+\frac{\partial}{\partial p}(u\omega)-fv-\frac{uv\tan\varphi}{a}+w(f'+\frac{u}{a})+\frac{\partial\Phi}{\partial X}=\mathrm{F}_{ul}+\mathrm{F}^r_{us}$$

$$\frac{\partial v}{\partial t}+\frac{\partial}{\partial X}(uv)+\frac{\partial}{\partial Y}(v^2)+\frac{\partial}{\partial p}(v\omega)+fu+\frac{u^2\tan\varphi}{a}+\frac{wv}{a}+\frac{1}{a}\frac{\partial\Phi}{\partial\varphi}=\mathrm{F}_{vl}+\mathrm{F}^r_{vs}$$

$$\frac{\partial w}{\partial t}+\frac{\partial}{\partial X}(uw)+\frac{\partial}{\partial Y}(vw)+\frac{\partial}{\partial p}(w\omega)-\frac{u^2+v^2}{a}-f'u-g\frac{\partial(\delta p)}{\partial p}+\frac{g\delta\rho}{\rho_{ref}}=\mathrm{F}_{wl}+\mathrm{F}^r_{ws}$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial}{\partial X}(u\theta)+\frac{\partial}{\partial Y}(v\theta)+\frac{\partial}{\partial p}(\theta\omega)=\frac{Q}{\pi}+F_{\theta l}+F^r_{\theta s}$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial X}(uq)+\frac{\partial}{\partial Y}(vq)+\frac{\partial}{\partial p}(q\omega)=F_{ql}+F^r_{qs}$$

$$\frac{\partial u}{\partial X}+\frac{\partial v}{\partial Y}+\frac{\partial\omega}{\partial p}=0$$

$$-F^r_{us}=\frac{\partial}{\partial X}(\overline{u'^2})+\frac{\partial}{\partial Y}(\overline{u'v'})+\frac{\partial}{\partial p}(\overline{u'\omega'})-\frac{\overline{u'v'}\tan\varphi}{a}+\frac{\overline{u'w'}}{a}$$

$$-F^r_{vs}=\frac{\partial}{\partial X}(\overline{u'v'})+\frac{\partial}{\partial Y}(\overline{v'^2})+\frac{\partial}{\partial p}(\overline{v'\omega'})+\frac{\overline{u'^2}\tan\varphi}{a}+\frac{\overline{v'w'}}{a}$$

$$-F^r_{ws}=\frac{\partial}{\partial X}(\overline{u'w'})+\frac{\partial}{\partial Y}(\overline{v'w'})+\frac{\partial}{\partial p}(\overline{w'\omega'})-\frac{\overline{u'^2}+\overline{v'^2}}{a}$$

$$-F^r_{\theta s}=\frac{\partial}{\partial X}(\overline{u'\theta'})+\frac{\partial}{\partial Y}(\overline{v'\theta'})+\frac{\partial}{\partial p}(\overline{\theta'\omega'})$$

$$-F^r_{qs}=\frac{\partial}{\partial X}(\overline{u'q'})+\frac{\partial}{\partial Y}(\overline{v'q'})+\frac{\partial}{\partial p}(\overline{q'\omega'})$$

The above set of governing equations include the subgrid scale terms.

$$\frac{D}{Dt}(.)\equiv\left(\frac{\partial}{\partial t}+\tilde{u}\frac{\partial}{\partial X}+\tilde{v}\frac{\partial}{\partial Y}+\tilde{\omega}\frac{\partial}{\partial p}\right)(.)$$

Equation for $\overline{u'^2}$

$$\frac{D}{Dt}\left(\frac{\overline{u'^2}}{2}\right)+\left(\frac{\partial\tilde{u}}{\partial X}-\frac{\tilde{v}\tan\varphi}{a}+\frac{w'}{a}\right)\overline{u'^2}+\frac{1}{2}\left(\frac{\partial}{\partial X}(\overline{u'u'u'})+\frac{\partial}{\partial Y}(\overline{u'u'v'})+\frac{\partial}{\partial p}(\overline{u'u'\omega'})\right)$$
$$-\frac{\overline{u'u'v'}}{a}\tan\varphi+\frac{\overline{u'u'w'}}{a}+\overline{u'v'}\left(\frac{\partial\tilde{u}}{\partial Y}-\frac{\tilde{u}\tan\varphi}{a}-f\right)+\overline{u'w'}\frac{\partial\tilde{u}}{\partial p}+\frac{\overline{u'w'}}{a}(\tilde{u}+af')=0$$

Equation For $\overline{v'^2}$

$$\frac{D}{Dt}\left(\frac{\overline{v'^2}}{2}\right)+\overline{v'^2}\left(\frac{\partial\tilde{v}}{\partial Y}+\frac{w'}{a}\right)+\overline{u'v'}\left(\frac{\partial\tilde{v}}{\partial X}+\frac{2\tilde{u}\tan\varphi}{a}+f\right)+\overline{v'w'}\frac{\partial\tilde{v}}{\partial p}+\frac{\overline{v'w'}}{a}\tilde{v}$$
$$+\frac{1}{2}\left(\frac{\partial}{\partial X}(\overline{v'v'u'})+\frac{\partial}{\partial Y}(\overline{v'v'v'})+\frac{\partial}{\partial p}(\overline{v'v'\omega'})\right)+\frac{\overline{u'u'v'}}{a}\tan\varphi+\frac{\overline{v'v'w'}}{a}=0$$

Equation for $\overline{w'^2}$

$$\frac{D}{Dt}\left(\frac{\overline{w'^2}}{2}\right)+\overline{u'w'}\left(\frac{\partial\tilde{w}}{\partial X}-\frac{2\tilde{u}}{a}-f'\right)+\overline{v'w'}\left(\frac{\partial\tilde{w}}{\partial Y}-\frac{2\tilde{v}}{a}\right)+\overline{\omega'w'}\frac{\partial\tilde{w}}{\partial p}-\left(\frac{\overline{u'u'w'}+\overline{u'v'w'}}{2}\right)$$
$$+\frac{1}{2}\left[\frac{\partial}{\partial X}\left(\overline{u'w'w'}\right)+\frac{\partial}{\partial Y}\left(\overline{v'w'w'}\right)+\frac{\partial}{\partial p}\left(\overline{\omega'w'w'}\right)\right]=0$$

Equation for $\overline{u'v'}$

$$\frac{D(\overline{u'v'})}{Dt}+\overline{u'v'}\left(\frac{\partial\tilde{u}}{\partial X}+\frac{\partial\tilde{v}}{\partial Y}-\frac{\tilde{v}\tan\varphi}{a}+\frac{2\tilde{w}}{a}+f'\right)+\overline{u'w'}\frac{\partial\tilde{v}}{\partial p}+\overline{v'w'}\frac{\partial\tilde{u}}{\partial p}+\overline{u'w'}\frac{\tilde{v}}{a}+\overline{v'w'}\frac{\tilde{u}}{a}+$$
$$\overline{u'^2}\left(\frac{\partial\tilde{v}}{\partial X}+\frac{2\tilde{u}\tan\varphi}{a}+f\right)+\overline{v'^2}\left(\frac{\partial\tilde{u}}{\partial p}-\frac{\tilde{u}\tan\varphi}{a}-f\right)-\frac{\tan\varphi}{a}(\overline{u'u'u'}-\overline{u'u'v'})+\frac{2\overline{u'v'w'}}{a}+$$
$$\left(\frac{\partial}{\partial X}(\overline{u'u'v'})+\frac{\partial}{\partial Y}(\overline{v'u'v'})+\frac{\partial}{\partial p}(\overline{\omega'u'v'})\right)=0$$

Added to these equations are 16 more equations for other second order correlations terms of similar type, which have been omitted here. In each of the equation, the triple correction terms are to be parameterized, which is the closure problem.

Filtering of Sound Waves

As pointed out by Daly and Harlow (1970) and, Hanjalić and Launder (2011) the correlations of velocity fluctuations with pressure fluctuations will propagate with the speed of sound, which will immediately mask the meteorologically important waves. Thus in our design the acoustic waves have been removed at each stage:

(i) Sound waves have been eliminated from the system representing the resolved part of motion following the theory of Arakawa and Konor (2009);

(ii) Sound waves have been eliminated from "subgrid part" of the motion by neglecting the correlations of pressure and velocity fluctuations.

Second-Moment Closures:

The set of equations given is Sec. 2 involves triple correlation terms such as , , etc. are to parameterized. These terms are modelled following the "generalized gradient diffusion hypothesis (GGDH)" of Daly and Harlow (1970) in the manner as proposed by Hanjalić and Launder (1972. 2011). Thus a model for consists of three

$$-\overline{u_i'u_j'u_k'}=c_s\frac{\hat{k}}{\varepsilon}\left[\overline{u_i'u_l'}\frac{\partial}{\partial X_l}(\overline{u_j'u_k'})+\overline{u_j'u_l'}\frac{\partial}{\partial X_l}(\overline{u_i'u_k'})+\overline{u_k'u_l'}\frac{\partial}{\partial X_l}(\overline{u_i'u_j'})\right]$$

E is the dissipation parameter and $\hat{k}$ is the kinetic energy of velocity perturbations.

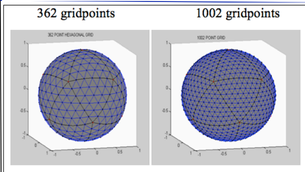

Figure 1: Icosahedral-hexagonal grids: Level-6 and Level-10

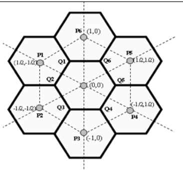

Figure 2: Vornoi-Delaunay associations

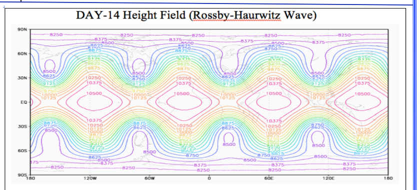

Figure 3: Height field at Day-14 from Shallow-water model integration on ico-hexa grids

Governing equations of the reference state:

The reference state is the solution of anelastic, quasi-hydrostatic equation system of equations with the quasi-hydrostatic pressure (p) as the vertical coordinate.

$$\frac{\partial V}{\partial t}=J_H-\frac{1}{\rho_{qs}}\nabla_H(\delta p)-\nabla_H\Phi\frac{\partial}{\partial p}(\delta p)$$

$$\frac{\partial w}{\partial t}=J_z+g\frac{\partial}{\partial p}(\delta p)$$

$$J_H=-\left[\nabla_H\left(K+\Phi\right)+Zk\times V+wG+\omega\frac{\partial V}{\partial p}\right]$$

$$J_z=-\left[V\cdot\nabla_H w+\omega\frac{\partial w}{\partial p}-\frac{u^2+v^2}{a}-f'u+\frac{g(\delta p)}{\rho_{qs}}\right]$$

Finally the equation for nonhydrostatic pressure δp is as follows,

$$\nabla\cdot\left(\frac{1}{\rho_{qs}}\nabla+\nabla\Phi\frac{\partial}{\partial p}\right)\delta p+g^2\frac{\partial}{\partial p}\left(\rho_{qs}\frac{\partial(\delta p)}{\partial p}\right)=\nabla\cdot J_H-g\frac{\partial}{\partial p}(\rho_{qs}J_z)+g\frac{\partial}{\partial t}\left(\frac{\partial\omega}{\partial p}\right)$$

The computational geometry of the icosahedral-hexagonal grid system has been described by Mittal (2008) which includes grid generation, node numbering and node search algorithms, and grid refinement through Vornoi-Delaunay associations, data structure and storage. The discrete operators on this grid system are as follows:

Gradient: $\{\nabla\psi\}_k\,Area(H_k)=-\frac{1}{2}[\psi\cdot\nabla\phi Area(T)]_{H_k}$

Divergence: $\{\nabla\cdot\vec{Q}\}_k\,Area(H_k)=-\frac{1}{2}[(p,q)\cdot\nabla\phi Area(T)]_{H_k}$

Curl: $\{k\cdot\nabla\times\vec{Q}\}_k\,Area(H_k)=-\frac{1}{2}[(p,-q)\cdot\nabla\phi Area(T)]_{H_k}$

Test Problem: Rossby-Haurwitz wave

The discrete operators (6.1)-(6.3) were verified with several test cases for the shallow-water equations on a sphere (Mittal 2008) using the icosahedral-hexagonal grids. Tests for the Rossby-Haurwitz wave is discussed here. The shallow-water equations were integrated up to day-14 at two different resolutions: one at Level-48 (23,042 gridpoints) and at Level-64 (40,962 gridpoints). Using these forms of the discrete operators for grad, div and curl in the shallow-water equations produced results that match well with other studies (Thuburn, 1997) for this test case. Most importantly, the global invariants (energy and enstrophy) were very maintained. Here we only show in Fig. 3 the results of day-14 for this test case for the height field. The ratios of total energy and enstrophy with their respective day-0 values were RTE = 1.0004908; RTZ = 1.0013307. The ratio of the total mass on day-14 to day-0 and the day-0 total mass was 0.99997794.

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Conclusion

Second-moment equations have been added to the anelastic, quasi-hydrostatic system derived by Arakawa and Konor (2009). The sound waves have been filtered from the second-moment equations by neglecting the pressure-velocity correlations as the nonhydrostatic pressure δp, is a resolved variable in the reference state. The discrete representations of *grad, div and curl* have been tested with the shallow-water equations on icosahedral-hexagonal grids. The dynamical core for the global icosahedral-hexagonal model will be implemented on a GPU Cluster. The work in this direction is progressing in a group involving 14 members from different science and engineering departs at IIT Delhi.

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