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Design and Development of a Unified Model on Icosahedral-Hexagonal Grids

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Abstract: There are continued attempts towards unifying the general circulation and cloud-resolving models. For designing high-resolution general circulation models, it is necessary to formulate a set of equations for the nonhydrostatic system such that when the nonhydrostatic pressure is neglected, the system of equations reduces to a quasi-hydrostatic compressible model. Following the Arakawa and Konor (2009) approach, the governing equations of a unified model on icosahedral-hexagonal grid are formulated in the hybrid vertical coordinate. Further to this formulation, the flow dependent variables are represented in the basic equations into two parts - grid-resolved and a subgrid part to formulate a system of equations that is capable of simulating the variability of unresolved processes. The advantage of splitting is obvious because the system of equations for grid-resolved variables are indeed those of a quasi-hydrostatic compressible model of the atmosphere. The discrete formulations of divergence, vorticity and gradient are then used to solve the shallow water model on the icosahedral-hexagonal grid as an example of the first stage development of a comprehensive unified model.

1. Introduction

The high-resolution operational models of weather forecasting are required to run at a grid size of the order of a kilometer, which can be accomplished as the computing power of newer platforms (GPUs or APUs) is available and affordable. However, the governing equations with traditional approximation do not represent the stochastic part in weather and climate that becomes important in atmospheric flows resolved on the scale of a kilometer and the traditional approximation breaks down. Arakawa and Konor (2009) have presented a system of equations that unites the nonhydrostatic anelastic system and quasi-hydrostatic system to derive the governing equations that form the foundation of cloud-resolving models. This study forms the starting point of unifying the Reynolds stress equations with anelastic, quasi-hydrostatic system of equations. The key assumption here is that even in the unified system, subgrid scale motions of the atmosphere may not have been fully represented by the parameterizations of dynamical and thermodynamical processes. Moreover, Palmer (1989) showed that adding the stochastic perturbations to the governing equations improves the forecasting skill of the numerical weather prediction (NWP) model. The NWP model is also sensitive to initial conditions due to the chaotic behaviour of the atmosphere, but this problem has been dealt effectively with the ensemble approach to numerical weather prediction that is followed at the major forecast centres. The multimodel superensemble technique, developed by Krishnamurti and coworkers (Krishnamurti et al. 1999, 2000a, 2000b, Yun et al. 2005) in several research papers, has been demonstrated as a powerful post-processing tool for weather forecast parameters from different global models with reduced direct model output errors in comparison to individual model forecasts. The superensemble methodology employs a large number of forecasts from different models and the past datasets that collectively represent the training phase to compute weights using least square minimization of the differences between forecast and observed meteorological parameters. The technique successfully extends the limit of predictability even for the precipitation forecast. This is another way to incorporate the stochastic nature of the meteorological flows in to weather and climate forecasting.

The physics of the model also neglects the stochastic nature of various atmospheric processes by parameterizing the physical phenomena in terms of the resolved variables. In the context of parameterization of convection, the computational grid volume is assumed either entirely saturated or entirely unsaturated, which would be a severe limitation until subgrid scale condensation is included in the governing system

(Sommeria 1976, Sommeria and Deardorff 1977). There are several processes that require better representation in a numerical model, which may in turn improve the predictability of the atmospheric flows. Thus, it would have a direct bearing on accuracy and range of the useful forecast from a numerical model.

Here we derive the set of equations that becomes an integral part of the anelastic, quasi-hydrostatic system because the stochastic perturbations in the governing equations may be defined by solutions of these equations. An attempt has been made in this paper to include the effect of second order correlation terms in the governing equations. A systematic formulation has been presented which takes the advantage of the advances already made in this direction especially in turbulence modelling, large eddy simulations (LES) more than three decades ago, and the recent attempts in formulating the cloud resolving models of the atmosphere.

2. Governing Equations

We have followed the philosophy of Arakawa and Konor (2009): "One of the main points of the Unified System is that it reduces to a quasi-hydrostatic model when the nonhydrostatic pressure is neglected. In this way the system maintains a close tie with the existing primitive equation models." Thus, the system of equations formulated here using the Reynolds averaging, form an integral part of the governing equations, which reduces to an anelastic, quasi-hydrostatic system if the subgrid scale terms representing stochastic perturbations are neglected in the equations. For deriving the relevant set of equations, the momentum, thermodynamic and moisture continuity equations are taken in the following form in the pressure coordinate system. For the sake of brevity, we write the Coriolis parameter and its derivative as they appear in the equations,

$$\mathrm{f}=2\Omega\sin\varphi \qquad \mathrm{f}'=2\Omega\cos\varphi \tag{2.1}$$

The choice of vertical coordinate is very important as the continuity equation is used in writing the momentum equations in the flux form. This facilitates the separation of the system of equation for resolved part and the derivation of disturbance equation which are used to derive the equations for the Reynolds stresses Arakawa and Konor (2009) have derived the system of equations of unified system in quasi-hydrostatics pressure coordinate (p) which was introduced by René Laprise (1992) as an independent variable. All exiting primitive equation models use pressure or its variants as a vertical coordinate, and use of pressure as a vertical coordinate in unified system implies that unified system has the same vertical structure as the conventional large scale models. Arakawa and Konor (2009) also showed that the quasi-hydrostatics pressure coordinate could be used in nonhydrostatic atmospheric models based on fully compressible equations. More interestingly, the unified system explicitly deals with quasi-hydrostatic values of thermodynamic state variables (greater merit of Laprise approach). There is, however, a clear advantage of using these equations as their solutions could be taken as a reference state for deriving more comprehensive and complete set of equations for this study.

In order to derive the disturbance equations, a reference state is required. The solution of the anelastic and quasi-hydrostatic system is assumed in this formulation as the "grid-resolved part" relative to which the perturbation fields are derived. It is advantageous to deal with perturbations of the dynamical and thermodynamical equations that are defined with respect to the reference state, which is the solution of a numerical weather prediction system with filtering of sound waves from the system. Like wise, one may also take the solution of hydrostatic primitive equations as a reference state, but it is preferable to take the solutions of anelastic, quasi-hydrostatic system as the reference state, and all the perturbed quantities have been defined with reference to this state as

$$\mathrm{u} = \tilde{\mathrm{u}} + \mathrm{u}', \quad \mathrm{v} = \tilde{\mathrm{v}} + \mathrm{v}', \quad \mathrm{w} = \tilde{\mathrm{w}} + \mathrm{w}', \quad \omega = \tilde{\omega} + \omega',$$
$$\theta = \tilde{\theta} + \theta', \quad q = \tilde{q} + q', \quad T = \tilde{T} + T'. \tag{2.2}$$

The dependent variables are separated into " a grid-resolved part" represented by "tilde" and an unresolved "subgrid part" denoted by "prime" of a dependent variable. The variables with tildes are the solutions of the anelastic, quasi-hydrostatic system over which the perturbations (primed variables) are superimposed. To derive the Reynolds averaged equations with subgrid scale forcing, it is necessary to first write the disturbance equations for the perturbation variables.

Since the algebra involved is tedious, the differential operators are represented as,

$$\frac{1}{a\cos\varphi}\frac{\partial}{\partial\lambda}(.) \equiv \frac{\partial}{\partial X}(.) \;; \qquad \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}\cos\varphi(.) \equiv \frac{\partial}{\partial Y}(.) \tag{2.3}$$

X and Y are working coordinates so that the governing equations may be written in a better form. We shall now develop an appropriate set of equations for the "subgrid part", which becomes considerably simpler, if the equations are written in the following form in the quasi-hydrostatic pressure coordinate system.

Governing Equations in Quasi-hydrostatic Pressure (*p*) Coordinate

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial X}(u^2) + \frac{\partial}{\partial Y}(uv) + \frac{\partial}{\partial p}(u\omega) - fv - \frac{uv\tan\varphi}{a} + w(f' + \frac{u}{a}) + \frac{\partial\Phi}{\partial X} = \mathrm{F}_{ul} + \mathrm{F}^{r}_{us} \tag{2.4}$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial X}(uv) + \frac{\partial}{\partial Y}(v^2) + \frac{\partial}{\partial p}(v\omega) + fu + \frac{u^2\tan\varphi}{a} + \frac{wv}{a} + \frac{1}{a}\frac{\partial\Phi}{\partial\varphi} = \mathrm{F}_{vl} + \mathrm{F}^{r}_{vs} \tag{2.5}$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial X}(uw) + \frac{\partial}{\partial Y}(vw) + \frac{\partial}{\partial p}(w\omega) - \frac{u^2 + v^2}{a} - f'u - g\frac{\partial(\delta p)}{\partial p} + \frac{g\delta\rho}{\rho_{ref}} = \mathrm{F}_{wl} + \mathrm{F}^{r}_{ws} \tag{2.6}$$

$$\frac{\partial\theta}{\partial t} + \frac{\partial}{\partial X}(u\theta) + \frac{\partial}{\partial Y}(v\theta) + \frac{\partial}{\partial p}(\theta\omega) = \frac{Q}{\pi} + F_{\theta l} + F^{r}_{\theta s} \tag{2.7}$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial X}(uq) + \frac{\partial}{\partial Y}(vq) + \frac{\partial}{\partial p}(q\omega) = F_{ql} + F^{r}_{qs} \tag{2.8}$$

$$\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} + \frac{\partial\omega}{\partial p} = 0 \tag{2.9}$$

$$-F^{r}_{us} = \frac{\partial}{\partial X}(\overline{u'^2}) + \frac{\partial}{\partial Y}(\overline{u'v'}) + \frac{\partial}{\partial p}(\overline{u'\omega'}) - \frac{\overline{u'v'}\tan\varphi}{a} + \frac{\overline{u'w'}}{a} \tag{2.10}$$

$$-F^{r}_{vs} = \frac{\partial}{\partial X}(\overline{u'v'}) + \frac{\partial}{\partial Y}(\overline{v'^2}) + \frac{\partial}{\partial p}(\overline{v'\omega'}) + \frac{\overline{u'^2}\tan\varphi}{a} + \frac{\overline{v'w'}}{a} \tag{2.11}$$

$$-F^{r}_{ws} = \frac{\partial}{\partial X}(\overline{u'w'}) + \frac{\partial}{\partial Y}(\overline{v'w'}) + \frac{\partial}{\partial p}(\overline{w'\omega'}) - \frac{\overline{u'^2} + \overline{v'^2}}{a} \tag{2.12}$$

$$-F^{r}_{\theta s} = \frac{\partial}{\partial X}(\overline{u'\theta'}) + \frac{\partial}{\partial Y}(\overline{v'\theta'}) + \frac{\partial}{\partial p}(\overline{\theta'\omega'}) \qquad 2.13)$$

$$-F^{r}_{qs} = \frac{\partial}{\partial X}(\overline{u'q'}) + \frac{\partial}{\partial Y}(\overline{v'q'}) + \frac{\partial}{\partial p}(\overline{q'\omega'}) \tag{2.14}$$

The above set of governing equations include the subgrid scale terms (F_{xl}) which can be explicitly parameterized in terms of the "resolved" large scale variables and the subgrid scale terms (F^{r}_{xs}) which are obtained by solving a set of prognostic equations that are arrived at by using the second-order closure assumption. This is perhaps the best way to include the conventional parameterizations that have made NWP a successful endeavour, and the advances in atmospheric sciences made by large-eddy simulations. We define the total derivative as,

$$\frac{D}{Dt}(.) \equiv \left(\frac{\partial}{\partial t} + \tilde{u}\frac{\partial}{\partial X} + \tilde{v}\frac{\partial}{\partial Y} + \tilde{\omega}\frac{\partial}{\partial p} \right)(.)$$

so that the second-moment equations can be written in a more convenient form as follows.

Equation for $\overline{\mathrm{u}'^2}$

$$\frac{D}{Dt}\left(\frac{\overline{u'^2}}{2}\right) + \left(\frac{\partial \tilde{u}}{\partial X} - \frac{\tilde{v}\tan\varphi}{a} + \frac{w'}{a}\right)\overline{u'^2} + \frac{1}{2}\left(\frac{\partial}{\partial X}(\overline{u'u'u'}) + \frac{\partial}{\partial Y}(\overline{u'u'v'}) + \frac{\partial}{\partial p}(\overline{u'u'\omega'})\right)$$
$$-\frac{\overline{u'u'v'}}{a}\tan\varphi + \frac{\overline{u'u'w'}}{a} + \overline{u'v'}\left(\frac{\partial \tilde{u}}{\partial Y} - \frac{\tilde{u}\tan\varphi}{a} - f\right) + \overline{u'w'}\frac{\partial \tilde{u}}{\partial p} + \frac{\overline{u'w'}}{a}(\tilde{u} + af') = 0 \tag{2.15}$$

Equation For $\overline{v'^2}$

$$\frac{D}{Dt}\left(\frac{\overline{v'^2}}{2}\right) + \overline{v'^2}\left(\frac{\partial \tilde{v}}{\partial Y} + \frac{w'}{a}\right) + \overline{u'v'}\left(\frac{\partial \tilde{v}}{\partial X} + \frac{2\tilde{u}\tan\varphi}{a} + f\right) + \overline{v'w'}\frac{\partial \tilde{v}}{\partial p} + \frac{\overline{v'w'}}{a}\tilde{v}$$
$$+\frac{1}{2}\left(\frac{\partial}{\partial X}(\overline{v'v'u'}) + \frac{\partial}{\partial Y}(\overline{v'v'v'}) + \frac{\partial}{\partial p}(\overline{v'v'\omega'})\right) + \frac{\overline{u'u'v'}}{a}\tan\varphi + \frac{\overline{v'v'w'}}{a} = 0 \tag{2.16}$$

Equation for $\overline{w'^2}$

$$\frac{D}{Dt}\left(\frac{w'^2}{2}\right) + \overline{u'w'}\left(\frac{\partial \tilde{w}}{\partial X} - \frac{2\tilde{u}}{a} - f'\right) + \overline{v'w'}\left(\frac{\partial \tilde{w}}{\partial Y} - \frac{2\tilde{v}}{a}\right) + \overline{\omega'w'}\frac{\partial \tilde{w}}{\partial p} - \left(\frac{\overline{u'u'w'} + \overline{u'v'w'}}{2}\right)$$
$$+\frac{1}{2}\left[\frac{\partial}{\partial X}\left(\overline{u'w'w'}\right) + \frac{\partial}{\partial Y}\left(\overline{v'w'w'}\right) + \frac{\partial}{\partial p}\left(\overline{\omega'w'w'}\right)\right] = 0 \tag{2.17}$$

Equation for $\overline{u'v'}$

$$\frac{D(\overline{u'v'})}{Dt} + \overline{u'v'}\left(\frac{\partial \tilde{u}}{\partial X} + \frac{\partial \tilde{v}}{\partial Y} - \frac{\tilde{v}\tan\varphi}{a} + \frac{2\tilde{w}}{a} + f'\right) + \overline{u'w'}\frac{\partial \tilde{v}}{\partial p} + \overline{v'w'}\frac{\partial \tilde{u}}{\partial p} + \overline{u'w'}\frac{\tilde{v}}{a} + \overline{v'w'}\frac{\tilde{u}}{a} +$$
$$\overline{u'^2}\left(\frac{\partial \tilde{v}}{\partial X} + \frac{2\tilde{u}\tan\varphi}{a} + f\right) + \overline{v'^2}\left(\frac{\partial \tilde{u}}{\partial p} - \frac{\tilde{u}\tan\varphi}{a} - f\right) - \frac{\tan\varphi}{a}(\overline{u'u'u'} - \overline{u'u'v'}) + \frac{2u'v'w'}{a} +$$
$$\left(\frac{\partial}{\partial X}(\overline{u'u'v'}) + \frac{\partial}{\partial Y}(\overline{v'u'v'}) + \frac{\partial}{\partial p}(\overline{\omega'u'v'})\right) = 0 \tag{2.18}$$

Added to the above set of equations are 16 more equations for other second order correlations terms of similar type, which have been omitted here. In each of the equation, the triple correction terms are to be parameterized, which is the closure problem. We believe that the Reynolds Averaged Navier-Stokes (RANS) approach appears a practical option, which would allow sufficiently fine resolution of the order of a kilometer with the basic "resolved" state that is also evolving in time as model integrations advance. An important simplification has been introduced in deriving the second-moment equations: correlations of pressure with dynamical, thermodynamical and humidity variables have been neglected in the second-moment equations. Most importantly, the pressure strain waves (Hanjalić and Launder, 2011, p28; Daly and Harlow 1970) or the disturbances propagating with the speed of sound have been removed from the system through this artifact. It is to be noted that resolved component of the atmospheric flows (represented by quantities with ~) would be obtained by solving the system of equations formulated following the theory of Arakawa and Konor (2009).

3. Filtering of Sound Waves

As pointed out by Daly and Harlow (1970) and, Hanjalić and Launder (2011) the correlations of velocity fluctuations with pressure fluctuations will propagate with the speed of sound, which will immediately mask the meteorologically important waves. Thus in our design the acoustic waves have been removed at each stage:

(i) Sound waves have been eliminated from the system representing the resolved part of motion following the theory of Arakawa and Konor (2009);

(ii) Sound waves have been eliminated from "subgrid part" of the motion by neglecting the correlations of pressure and velocity fluctuations.

4. Second-Moment Closures:

The set of equations given is Sec. 2 involves triple correlation terms such as $\overline{u_i'u_j'u_k'}$, $\overline{\theta' u_j'u_k'}$, $\overline{q' u_j'u_k'}$ etc. are to parameterized. These terms are modelled following the "generalized gradient diffusion hypothesis (GGDH)" of Daly and Harlow (1970) in the manner as proposed by Hanjalić and Launder (1972, 2011). Thus a model for $\overline{u_i'u_j'u_k'}$ consists of three terms:

$$-\overline{u_i'u_j'u_k'} = c_s \frac{\hat{k}}{\varepsilon}\left[\overline{u_i'u_l'}\frac{\partial}{\partial X_l}(\overline{u_j'u_k'}) + \overline{u_j'u_l'}\frac{\partial}{\partial X_l}(\overline{u_i'u_k'}) + \overline{u_k'u_l'}\frac{\partial}{\partial X_l}(\overline{u_i'u_j'})\right] \tag{4.1}$$

$$c_s = 0.1\,; \quad \hat{k} = \frac{\left[\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_2'^2}\right]}{2} = \frac{\overline{u_i'^2}}{2} = \frac{\left[\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right]}{2} \tag{4.2}$$

Here ε is the turbulence dissipation parameter. We can easily formulate the equation for calculating the turbulence kinetic energy $\hat{k}$, but it remains to provide a means of determining the dissipation rate of turbulence energy, ε . For atmospheric flow, it appears that the turbulence energy dissipation rate ε may be related the mean square of the fluctuations part of the enstrophy; that is $\varepsilon = \mathrm{D}\varsigma$;

$$D = c_l \frac{(\Delta x)^2}{\Delta t} \tag{4.3}$$

Δx is the grid size and Δt is the time step in the model. Note that parameter c_l has to be chosen. An appropriate value should be such that, $0 < c_l < 1$.

Expression for Triple Correlation Terms

Following (4.1) we write the expression for Triple correlations:

$$-\overline{u'u'u'} = \frac{c_s 3\hat{k}}{\varepsilon}\left[\overline{u'^2}\frac{\partial \overline{u'^2}}{\partial X} + \overline{u'v'}\frac{\partial \overline{u'^2}}{\partial Y} + \overline{u'\omega'}\frac{\partial \overline{u'^2}}{\partial p}\right] = \frac{c_s 3\hat{k}}{\varepsilon}\left[\overline{u'u'_l}\frac{\partial \overline{u'^2}}{\partial X_l}\right] \tag{4.3}$$

$$-\overline{u'u'v'} = \frac{c_s 2\hat{k}}{\varepsilon}\left[\overline{u'u'_l}\frac{\partial}{\partial X_l}\left(\overline{u'v'}\right)\right] + \frac{c_s \hat{k}}{\varepsilon}\left[\overline{v'u'_l}\frac{\partial \overline{u'^2}}{\partial X_l}\right] \tag{4.4}$$

Here $u'_1 = u',\ u'_2 = v',\ u'_3 = w',\ \mathrm{X}_1 = X,\ \mathrm{X}_2 = Y,\ \mathrm{X}_3 = p$; repeated index represent summation.

$$-\overline{u'u'w'} = \frac{c_s \hat{k}}{\varepsilon}\left[2\overline{u'u'_l}\frac{\partial}{\partial X_l}\left(\overline{u'\omega'}\right) + \overline{\omega'u'_l}\frac{\partial \overline{u'^2}}{\partial X_l}\right] \tag{4.5}$$

$$-\overline{u'u'w'} = \overline{u'u'\frac{\left(-\rho_{qs} g w'\right)}{\rho_{qs} g}} = \frac{1}{\rho_{qs} g}\overline{u'u'\omega'} \qquad (\omega = -\rho_{qs} g w \text{ for large scale flows}) \tag{4.6}$$

There are other 35 expressions for the other triple correlation terms.

5. Governing equations of the reference state

The reference state is the solution of anelastic, quasi-hydrostatic equation system of equations with the quasi-hydrostatic pressure (p) as the vertical coordinate. This system is obtained by setting the terms F_{xl} and F^r_{xs} to zero in equations (2.4)-(2.8) and the continuity equation (2.9). The nonlinear terms have been split in to a gradient term and the vorticity (Sadourny and Laval 1984, Hourdin et al. 2006). The derivation follows the procedure suggested by Arakawa and Konor (2009) and we obtain the following set of equations for calculating the reference state. The momentum, continuity, thermodynamic and other equations are as follows. Note that $p \equiv p_{qs}$ in the following equations.

$$\frac{\partial V}{\partial t} = J_H - \frac{1}{\rho_{qs}}\nabla_H(\delta p) - \nabla_H \Phi \frac{\partial}{\partial p}(\delta p) \tag{5.1}$$

$$\frac{\partial w}{\partial t} = J_z + g\frac{\partial}{\partial p}(\delta p) \tag{5.2}$$

$$J_H = -\left[\nabla_H\left(K + \Phi\right) + Zk \times V + wG + \omega\frac{\partial V}{\partial p}\right] \tag{5.3}$$

$$J_z = -\left[V \cdot \nabla_H w + \omega\frac{\partial w}{\partial p} - \frac{u^2 + v^2}{a} - f'u + \frac{g(\delta p)}{\rho_{qs}}\right] \tag{5.4}$$

$$Z = \mathrm{f} + \frac{u \tan\varphi}{a} + \zeta \tag{5.5}$$

$$\zeta = k.\nabla_H \times V = rot\, V \tag{5.6}$$

$$\rho_{qs} = \frac{p}{R\pi_{qs}\theta}\ , \qquad \frac{\delta\pi}{\pi_{qs}} \approx \kappa \frac{\delta p}{p} \tag{5.7}$$

$$\frac{\partial\Phi}{\partial p} = -\frac{R\pi_{qs}}{p}\theta \tag{5.8}$$

$$\Phi = \Phi_B + \int_p^{p_B} R\theta \frac{\pi_{qs}}{p} dp \tag{5.9}$$

$$\frac{\partial\Phi}{\partial t} = \left[R\theta \frac{\pi_{qs}}{p} \right]_B \left(\frac{\partial p_B}{\partial t} \right) + \int_p^{p_B} R \frac{\pi_{qs}}{p} \left(\frac{\partial\theta}{\partial t} \right) dp \tag{5.10}$$

$$\nabla_H \cdot V + \frac{\partial\omega}{\partial p} = 0 \tag{5.11}$$

$$\frac{\partial p_B}{\partial t} = -\int_{p_T}^{p_B} \left(\nabla \cdot V \right) dp \tag{5.12}$$

$$\left(\frac{\partial\theta}{\partial t} \right) + V \cdot \nabla_H (\theta) + \omega \frac{\partial\theta}{\partial p} = \frac{Q}{\pi} \tag{5.13}$$

Here $\omega = \frac{Dp}{Dt}$; and taking $\left(\frac{\partial p}{\partial t} \right)_{top} = 0$ makes the procedure simpler.

Finally the equation for nonhydrostatic pressure δp is as follows,

$$\nabla \cdot \left(\frac{1}{\rho_{qs}} \nabla + \nabla\Phi \frac{\partial}{\partial p} \right) \delta p + g^2 \frac{\partial}{\partial p} \left(\rho_{qs} \frac{\partial(\delta p)}{\partial p} \right) = \nabla \cdot J_H - g \frac{\partial}{\partial p} (\rho_{qs} J_z) + g \frac{\partial}{\partial t} \left(\frac{\partial\omega}{\partial p} \right) \tag{5.14}$$

6. Computational Procedure and Discretization on icosahedral-hexagonal grid

After adding the second-moment terms to the anelastic, quasi-hydrostatic equations with Boussinesq approximation, the governing set may be regarded more comprehensive and complete, which is devoid of sound waves. The icosahedral hexagonal grids, first introduced by Sadourny et al. (1968) are shown in Fig. 1, which shows the discretization on the sphere at Level-6 and Level-10; that is, discretization remains uniform as one moves to finer and finer resolutions. Hence, this grid system is very suitable to design high-resolution models, and there is a surge of paper with this grid system in the last decade (Heikes and Randall (1995), Giraldo 1997, Thuburn 1997 Majeskwi et al. 2002, Tomita and Satoh 2004, Giraldo 2006, Mittal et al. 2007, many more studies in recent years).

362 gridpoints 1002 gridpoints

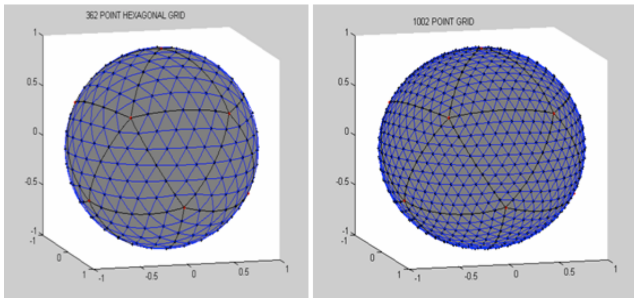

Figure 1: Icosahedral-hexagonal grids: Level-6 and Level-10

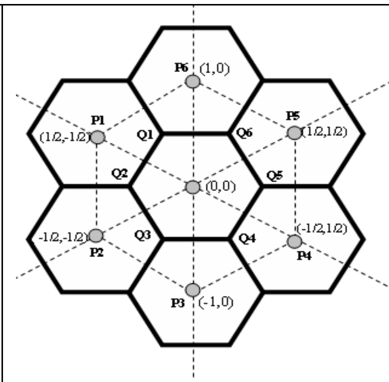

Figure 2: Vornoi-Delaunay associations

The computational geometry of the grid system has been described by Mittal (2008) which includes grid generation, node numbering and node search algorithms, and grid refinement through Vornoi-Delaunay associations, data structure and storage. The discrete operators on this grid system are as follows:

Gradient: $$\left\{\nabla\psi\right\}_k Area(H_k) = -\frac{1}{2}\left[\psi\cdot\nabla\phi Area(T)\right]_{H_k} \qquad (6.1)$$

Divergence: $$\left\{\nabla\cdot\vec{Q}\right\}_k Area(H_k) = -\frac{1}{2}\left[(p,q)\cdot\nabla\phi Area(T)\right]_{H_k} \qquad (6.2)$$

Curl: $$\left\{k\cdot\nabla\times\vec{Q}\right\}_k Area(H_k) = -\frac{1}{2}\left[(p,-q)\cdot\nabla\phi Area(T)\right]_{H_k} \qquad (6.3)$$

In the above expressions for *grad, div, and curl*, ψ is a scalar and *(p,q)* are the components of the vector $\vec{Q}$. Laplacian of a scalar φ will be obtained using the sequence *div grad* φ, so first the *grad* φ will be computed and then its divergence will be computed on the icosahedral-hexagonal grid. T represents a triangular element of the hexagonal/pentagonal element H_k.

We now present the flow of computations for entire model dynamics. First and foremost requirement is to prepare the initial data. This would be done by directly analyzing the meteorological observations on the icosahedral-hexagonal grid using suitable radial functions at a resolution of 50 km. Another requirement is the climatology for each day in a year, which is being prepared from the 12-year ECMWF data at native grid at 75 km and 25 km. Since our reference model is LMDZ, so the governing equations will be transformed in the hybrid coordinate system.

Computation of reference state: Equations (5.1)-(5.14) will be solved in the hybrid coordinate system using the icosahedral-hexagonal grids. To achieve this, we compute the nonhydrostatic part of the pressure (δp) from equation (5.14) with prescribed boundary conditions, as it is an equation of elliptic type. On realizing this step, the equations (5.1)-(5.13) are solved to obtain the horizontal wind field and the vertical velocities surface pressure temperature and other necessary variables.

Solutions of Reynolds stress equations: The system consists of 20 and more equations, for which the resolved fields input to equations (5.1)-(5.14) and the averaged values of quantities such as u'^2, v'^2, $u'v'$

etc., which need to be computed. If the variables are not staggered then one may take the average of a dynamic or thermodynamic variable carried at the points P_1, P_2, P_3, P_4, P_5 and P_6 ; and the average value will be defined at the centre (0,0) as shown in Figure 2. Using this average value, we then can find the deviation by subtracting this average value from the value of the variable in question at point (0,0). In doing so we also assume that the dynamical variable is a random variable such that the time and spatial averages converge to an ensemble average (ergodicity). Thus initial values of all the second-moment terms can be defined to solve the second-moment equations (2.16) etc.

Complete solution: The solution of second-moment equations is then used to compute derivative of the variables in F_{xs}^{r} as they appear in equations (2.9)-(2.14), and are then substituted in equations (2.1)-(2.8). At this stage, the forcing has been modified by the subgrid scale processes in these equations, which will be solved on the icosahedral-hexagonal grids for the complete solution of the modified anelastic, quasi-hydrostatic system with Reynolds stresses.

Test Problem: Rossby-Haurwitz wave

The discrete operators (6.1)-(6.3) were verified with several test cases for the shallow-water equations on a sphere (Mittal 2008) using the icosahedral-hexagonal grids. Tests for the Rossby-Haurwitz wave is discussed here. The shallow-water equations were integrated up to day-14 at two different resolutions: one at Level-48 (23,042 gridpoints) and at Level-64 (40,962 gridpoints). Using these forms of the discrete operators for grad, div and curl in the shallow-water equations produced results that match well with other studies (Thuburn, 1997) for this test case. Most importantly, the global invariants (energy and enstrophy) were very maintained. Here we only show in Fig. 3 the results of day-14 for this test case for the height field. The ratios of total energy and enstrophy with their respective day-0 values were RTE = 1.0004908; RTZ = 1.0013307. The ratio of the total mass on day-14 to day-0 and the day-0 total mass was 0.99997794.

DAY-14 Height Field (Rossby-Haurwitz Wave)

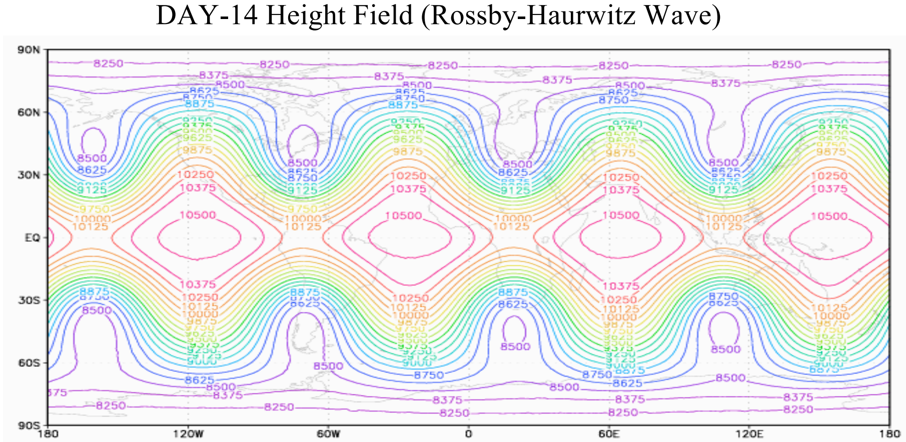

Figure 3: Height field at Day-14 from Shallow-water model integration on ico-hexa grids

Conclusion

Second-moment equations have been added to the anelastic, quasi-hydrostatic system derived by Arakawa and Konor (2009). The sound waves have been filtered from the second-moment equations by neglecting the pressure-velocity correlations as the nonhydrostatic pressure δp , is a resolved variable in the reference state.

The discrete representations of *grad, div and curl* have been tested with the shallow-water equations on icosahedral-hexagonal grids. The dynamical core for the global icosahedral-hexagonal model will be implemented on a GPU Cluster. The work in this direction is progressing in a group involving 14 members from different science and engineering departs at IIT Delhi.

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