# Theoretical Analysis of Polarization Characteristics for Planar and Cylindrical Phased Array Radars

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Abstract- In this work, antenna polarization characteristics of the two arrav configurations (i.e., planar and cylindrical structures) are analyzed and compared, and polarimetric parameter (e.g., differential reflectivity) bias correction techniques for the two antennas are presented. The crosspolarization of one polarization in the main beam direction can be calculated and corrected by adjusting the amplitude and phase of the other polarization. In the case of the cylindrical polarimetric phased array radar (CPPAR), the beam is formed at the bisector of the cylindrical sector, and the cross-polarizations caused by opposing elements in azimuth cancel each other, vielding a very low cross-polarization level. Copolar and cross-polar patterns of planar polarimetric phased array radar (PPPAR) and those of CPPAR are compared.

## **1. INTRODUCTION**

Polarimetry and phased array are two advanced radar technologies that have received much attention and are making contributions in the weather community (Doviak et al. 1993, 2000; Zrnić et al. 2007; Smith et al. 2008). It is, however, a big challenge to combine the two technologies into one system for the future Multi-mission Phased Array Radar (MPAR). The challenge comes from the fact that the polarization base changes as radar beam electronically scans off broadside of a planar array antenna. For PPPAR, horizontally (H) and vertically (V) polarized wave fields are not perpendicular when the beam scans away from the broadside. That is, the H-pol and V-pol fields are coupled, which causes high cross-polarization for planar structure. We need to calibrate for this by either applying the correction matrix (or the projection matrix) or solving the radar parameters (Zhang et al. 2009).

To preserve the H, V bases, a cylindrical array configuration has been recently proposed as a candidate for MPAR, which was introduced by Zhang et al. (2011). The H and V polarized wave fields will be orthogonal in all beam directions. The CPPAR would essentially eliminate the beam-to-beam calibration that is required for the PPPAR. In the azimuth, the mainlobe is always at broadside and the scan is achieved by shifting the column of active elements. And very low cross-polarization level is achieved. CPPAR has the characteristics of polarization purity and a scan-invariant beam in the azimuth.

In this paper, bias corrections of planar and cylindrical PPAR are studied in section 2 and 3, respectively. The comparison of them is in section 4.

## 2. PPPAR BIAS CORRECTION

There are two methods of bias calibration. One is to directly apply the correction matrix (or projection matrix), which can be realized in hardware by adjusting the amplitudes of the H and V polarizations. The other is calculating the bias of radar parameters, which can restore the estimated radar parameters to their true values by subtracting the bias. We study only the first bias correction method for PPPAR and CPPAR in this paper.

If an array element is the ideal dipole, the transmitted wave field is only H-polarization when the bean points at broadside. However, when the beam scans away from principle planes, the H-pol dipole generated field will have a V-pol component (i.e.,  $-\vec{\theta}$ ), causing a high cross-pol level. Assuming that the PPPAR transmits H-pol only, but has a V-pol in error, we can adjust the amplitude of the antenna elements to cancel the cross-pol by applying the projection matrix at the boresight (Zhang et al., 2009). The procedure to obtain the amplitudes of H and  $\Delta V$  is shown below.

For ideal dipoles the projection matrix is (Zhang et al., 2009):

$$\mathbf{P} = \begin{bmatrix} \cos\phi & 0\\ -\cos\theta\sin\phi & \sin\theta \end{bmatrix}$$

The correction procedure for the ideal dipole is from eq. (1) to (4)

$$\vec{E}_{i} = \mathbf{P}\vec{E}_{t} = \begin{bmatrix} \cos\phi & 0\\ -\cos\theta\sin\phi & \sin\theta \end{bmatrix} \begin{bmatrix} E_{th}^{(c)}\\ \Delta E_{tv}^{(c)} \end{bmatrix}$$
$$= \begin{bmatrix} \cos\phi E_{th}^{(c)}\\ -\cos\theta\sin\phi E_{th}^{(c)} + \sin\theta\Delta E_{tv}^{(c)} \end{bmatrix} ,$$
(1)

where  $\vec{E}_i$  is the incident field and  $\vec{E}_t$  is the transmitting field. <sup>(c)</sup> means calibration. Subscript h means horizontal polarization; subscript v means vertical polarization.

At the boresight  $(\theta_0, \phi_0)$ , we assume that there is no V component (i.e.,  $-\vec{\theta}$ ):

$$\vec{E}_{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{2}$$

Combine (1) and (2), we have the new amplitudes at the transmitter for H and  $\Delta V$ :

$$E_{th}^{(c)} = \frac{1}{\cos \phi_0}$$

$$\Delta E_{tv}^{(c)} = \frac{\cos \theta_0 \sin \phi_0}{\sin \theta_0 \cos \phi_0},$$
(3)

Therefore, the amplitudes used in the crosspolarization array pattern (i.e.,  $F = \sum_{n=1}^{N} A(\theta, \phi) \exp(j\varphi_n) \text{ calculations are:}$  $A(\theta, \phi) = -\cos\theta \sin\phi \left(\frac{1}{\cos\phi_o}\right) + \sin\theta \left(\frac{\cos\theta_o \sin\phi_o}{\sin\theta_o \cos\phi_o}\right), \quad (4)$ 

Thus at the boresight direction, there is no cross-pol after correction. We use simulations to verify this (Fig. 1). The parameters in the simulation are: element separation is  $0.5\lambda$ , beam points at  $(\theta, \phi) = (70^\circ, 45^\circ)$ , and the antenna diameter is 8.54m to mimic that of the WSR-88D. The cross-pol patterns are normalized by the co-pol pattern peak. As shown in Fig.1, the cross-pol peak is -9.3dB before correction (Fig.1a) and –Inf dB after correction (Fig.1b) at the boresight. In Fig.1b, the two cross-pol peaks adjacent to the boresight direction are about -43.6 dB.

On the other hand, we transmit V-pol and assume there is no H component (i.e.,  $\vec{f}$ ). A new set of amplitudes for V and  $\Delta$ H can be calculated by the same procedure from (1) to (4). By adjusting the amplitudes at the transmitter, the incident wave always has no couplings. And the amplitudes are a function of boresight direction.

This procedure can also apply to other kinds of antenna elements by simply changing the projection matrix in the Eqs. (1) to (4). Ideal, simulated or measured pattern can be used. The general projection matrix of any kinds of element patterns can be written as

$$\mathbf{P} = \begin{bmatrix} E_{\phi}^{(\mathrm{h})} & E_{\phi}^{(\mathrm{v})} \\ -E_{\theta}^{(\mathrm{h})} & -E_{\theta}^{(\mathrm{v})} \end{bmatrix},$$

where  $E_{\phi}^{(h)}$  is the electrical field in  $\phi$ direction from horizontally polarized antenna element and it is a function of ( $\theta$ ,  $\phi$ ).  $E_{\theta}^{(h)}$  is the electrical field in  $\theta$  direction from horizontally polarized antenna element.  $E_{\phi}^{(v)}$  is the electrical field in  $\phi$  direction from vertically polarized antenna element.  $E_{\theta}^{(v)}$  is the electrical field in  $\theta$  direction from vertically polarized antenna element.



Fig.1 (a) cross-pol pattern of PPPAR before correction (b) cross-pol pattern of PPPAR after correction

#### **3. CPPAR BIAS CORECTION**

Directly applying the correction matrix (or projection matrix) to the CPPAR can also correct the cross-pol pattern at the boresight. Assuming that CPPAR transmit H and a  $\Delta V$  at the transmitter, the incident waves has H polarization only. The antenna element is dipole. By using the projection matrix, we

can calculate the amplitudes of H and  $\Delta V$ . The procedure is described below.

$$\vec{E}_{i} = \mathbf{P}\vec{E}_{t}$$

$$= \begin{bmatrix} \cos(\phi + \Delta\phi_{n}) & 0\\ -\cos\theta\sin(\phi + \Delta\phi_{n}) & \sin\theta \end{bmatrix} \begin{bmatrix} E_{th}^{(c)}\\ \Delta E_{tv}^{(c)} \end{bmatrix} ,$$

$$= \begin{bmatrix} \cos(\phi + \Delta\phi_{n})E_{th}^{(c)}\\ -\cos\theta\sin(\phi + \Delta\phi_{n})E_{th}^{(c)} + \sin\theta\Delta E_{tv}^{(c)} \end{bmatrix}$$
(5)

where  $\Delta \phi_n$  is the angle of nth column antenna element as shown in Fig.2.



Fig.2 Part of a cylindrical array (top view) At the boresight,

$$\vec{E}_{i} = \begin{bmatrix} 1\\0 \end{bmatrix},\tag{6}$$

Combine (5) and (6), we have the new amplitudes:

$$E_{th}^{(c)} = \frac{1}{\cos(\phi_o + \Delta\phi_n)}$$

$$\Delta E_{tv}^{(c)} = \frac{\cos\theta_o \sin(\phi_o + \Delta\phi_n)}{\sin\theta_o \cos(\phi_o + \Delta\phi_n)},$$
(7)

Therefore, the amplitudes used in the crosspolarization array pattern (i.e.  $F = \sum_{n=1}^{N} A(\theta, \phi) \exp(j\varphi_n)$ ) calculations are:

$$A(\theta, \phi) = -\cos\theta \sin(\phi + \Delta\phi_n) \left(\frac{1}{\cos(\phi_o + \Delta\phi_n)}\right) + \sin\theta \left(\frac{\cos\theta_o \sin(\phi_o + \Delta\phi_n)}{\sin\theta_o \cos(\phi_o + \Delta\phi_n)}\right)$$
(8)

We found that the adjusted amplitudes are different from column to column for CPPAR. At the boresight, there is no cross-pol after correction. Simulations are used to verify this. In the simulation, the antenna element separation is  $0.5\lambda$ , the beam points at  $(\theta, \phi) = (70^\circ, 0^\circ)$ , 90° sector of the CPPAR is used, and the projection area of CPPAR is to mimic WSR-88D. The height of CPPAR is 8.54m and the radius is 6m. In Fig.3, we can see the obvious decrease of cross-pol. Before correction, the two cross-pol peaks adjacent to the boresight are -20.1 dB. And after correction the cross-pol peaks adjacent to the boresight are -53.0 dB.



Fig. 3 (a) cross-pol pattern of CPPAR before correction (b) cross-pol pattern of CPPAR after correction

The correction for CPPAR is more complicate than that for PPPAR because the adjusted amplitudes are different from column to column. However, because -20 dB cross-pol peak before correction is good enough for weather measurements using alternative transmission, the correction is not necessary for this array if the elevation angle is less than 20 degree.

#### 4. SMMARY AND DISCUSSIONS

The cross-pol pattern of CPPAR is much lower than the PPPAR. As discussed in section 3, the PPPAR has -9.3dB cross-pol peak and needs corrections to lower the cross-pol level. The CPPAR does not need the bias correction because the cross-pattern peak is -20.1 dB, which satisfies the requirement of most weather applications.

The elements spacing of cylindrical array are studied in terms of ring array (Hansen 2009; Lei et al. 2011). Ring array needs more dense separations than the linear array to avoid grating lobes but has lower grating lobe level than the linear array. The amplitude tapering of CPPAR is studied by Zhang et al. 2011.

The cross-pol level of cylindrical array is much lower than that of planar array. The high cross-pol of planar array can be adjusted to very low level by applying correction matrix. The other way to reduce cross-pol effect is to correct the polarimetric parameters. The cylindrical cross-pol is so low that adjustment is not needed if the elevation angle is less than 20 degree. The CPPAR uses fewer elements for 360° azimuth scan and has preserved beam shape. However, the manufacture of CPPAR is more complicated.

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