Trajectory-Based Integration of Aircraft Conflict Detection and Weather Avoidance Fields

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ABSTRACT
In anticipation of NextGen requirements for reducing the impact of weather on aviation, we have progressed our research into trajectory-based methods of detecting and resolving aircraft to weather conflicts within automation decision support tools in the en route, terminal, oceanic, and traffic flow management domains. We are building these tools to use gridded weather avoidance fields, as may be available from the NextGen Net Enabled Weather (NNEW)/Single Authoritative Source (SAS) and NextGen Weather Processor (NWP). This year's (2011) work builds on the prototype Lockheed Martin developed in 2010, which displayed conflicts between an aircraft trajectory and gridded weather fields. We have extended the tool set to include user risk preferences, as well adding weather and aircraft conflict resolution options. The components of our work include 1) generation of 4-D prototype weather avoidance field (WAF), 2) development of a WAF conflict detection service, 3) development of weather re-route resolutions that consider other aircraft and 4) visualization of the interaction of weather products, the resulting weather avoidance field and aircraft trajectories. This paper describes the results of the generation, integration and visualization of weather reroute resolution options based on the probing of gridded weather avoidance fields, user risk preferences and the trajectories of other aircraft.

KEY WORDS
NextGen, weather avoidance, integration, trajectory based, NNEW, NWP

1. Introduction

In the summer of 2010, in collaboration with ENSCO Inc., a prototype WAF –to- aircraft detection service was developed [1]. This work utilized a prototype WAF used for the purpose of integrating it with a trajectory-based approach in classifying aircraft conflicts or impacts with WAF according to the estimated hazard severity of the WAF. Additionally, the WAF and the aircraft trajectories were translated to a grid-based construct to implement the service. [2]

The purpose for our 2011 weather integration research was to evolve the research prototype to include a weather conflict resolution service for aircraft whose trajectories are projected to be impacted by WAFs. The WAFs are being evaluated for the purpose of tactical (0-1 hr) avoidance of hazardous weather. To generate tactical routing resolutions around weather, we needed to include the trajectories of proximate aircraft so that the avoidance routing would not reroute aircraft into other aircraft or compromise FAA criteria for aircraft to aircraft separation.

As a last point to the introduction, we wish to allay any concerns by the weather research community regarding our use and application of weather avoidance fields (WAF). We view the WAF as a maturing product, not yet ready for operational use by automation system tactical decision support tools. This is especially true when one considers that a WAF common reference scale is yet to be developed for NextGen users such as controllers and pilots. It is our hope that our work influences the evolution of tactical WAF products so that such data are delivered to automation systems in way that can be implemented using trajectory-based methods and can ultimately represent avoidable hazardous weather beyond convective storms such as icing, turbulence, visibility, ash and so on.

Although the work described in this paper describes gridded WAF data and conflict grids in the context of tactical weather avoidance, it is relatively easy to see its application to strategic flow management as well. A weather integration framework based on a common grid reference that is applicable to weather processing systems (e.g., NWP), terminal systems (e.g., Common ARTS and STARS), en route systems (e.g., ERAM), oceanic (e.g., ATOP) and traffic flow management (e.g., TFMS, TMA) decision support tools that could support both strategic and tactical weather avoidance would seem a very powerful and efficient architecture. Trajectory and conflict information can be exchanged through the use of the Flight Object.

2. Approach

The approach to this year's research included the following components:

1) Use the Denver area prototype 4-D WAF data from Summer 2010
2) Use Trajectory Prediction Service to generate Trajectories
3) Create a single conflict grid containing both WAFs and aircraft trajectories
4) Expand the weather conflict detection service prototype by adding aircraft-to-aircraft conflict detection
5) Develop the Weather Conflict Resolution Service
6) Research and evaluate 4-D path finding algorithms to provide trajectory resolutions

2.1 Four Dimensional (4-D) WAF Data

The concept of a WAF has been previously investigated and published by MIT Lincoln Lab in references [3], [4] and [5]. As described in the cited references, the WAF is based on a convective weather avoidance model (CWAM) which translates convective weather data from CIWS VIL and ET into a pilot deviation probability as a function of time, location and flight altitude. The CWAM was developed using extensive data collection of pilot weather deviations at various en route altitudes operating at various ARTCCs. We recognize the application of the CWAM WAF as primarily an effective strategic traffic flow management, but realized, in its present form, was not directly usable for tactical weather avoidance. Thus, we needed a short-term (0-1 hr) WAF that could be used in the tactical time frame.

Our prototype WAF, developed by ENSCO, is not a CWAM-based WAF. For the purpose of developing an automation framework for integration of tactical weather avoidance, we needed a WAF product that could be used for any altitude, whose severity was indexed solely by atmospheric measurement. In the CWAM-based WAF, the convective weather deviation probability by the pilot is derived from empirical data. However, given the NextGen TBO conops, we viewed the WAF from a different perspective. Our approach to the WAF is to base the WAF solely on atmospheric data such as CIWS VIL and ET and let the aircraft derived trajectory (i.e., intent) or other flight management system (FMS) provided parameter dictate pilot preferred avoidance strategy. In other words, it is the aircraft trajectory that contains aircraft type and equipment, pilot or airline risk preferences for weather avoidance. Trajectory exchanges between the ground system and the aircraft can be synchronized and negotiated for common resolution by both controller and pilot [6].

The concept of a WAF extends beyond convective weather to other forms of potentially hazardous weather such as turbulence, icing, ash, or ceiling/visibility. A WAF derived from measurement of the atmosphere would then provide a basis for creating a WAF based on any hazardous weather phenomena. Likewise, the risk preferences to be used in avoiding whatever the hazardous weather would be contained in aircraft-supplied data as provided by an FMS. Given the idea that a WAF could eventually represent multiple forms of hazardous weather, how would one assign a single probability of deviation to a WAF that could represent varying forms of hazardous weather (e.g., turbulence, icing, hail, etc.) as evaluated by pilots of varying experience flying aircraft of varying capabilities and missions operating in various ATC domains (e.g., enroute, terminal, etc.) and subject to individual or company operating rules?

In order to modify our conflict probes from a trajectory-polyhedron based evaluation to a grid-based trajectory-WAF evaluation, we needed a gridded 4-D WAF product. In this paper, we will summarize the WAF data and refer the reader to References [1] and [2] for additional details.

Examples of VIL and ET products are shown in Figures 1 and 2. These are examples and not the specific data used in our project.

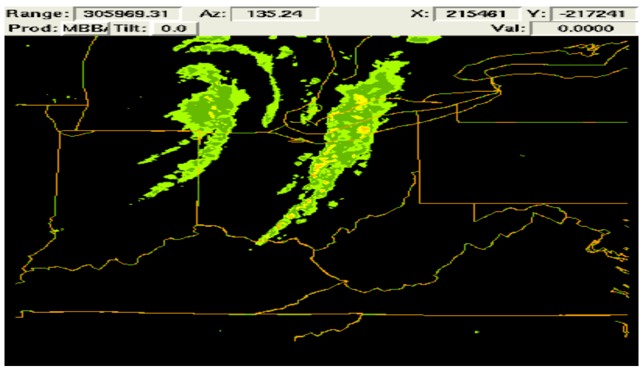

Figure 1. CIWS 2-D VIL Example (Courtesy: MIT Lincoln Labs) [1]

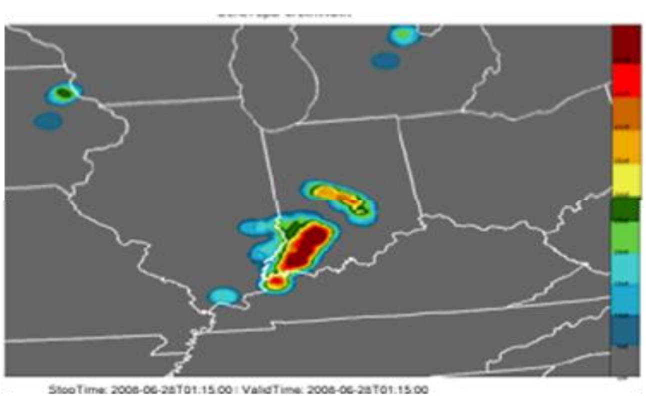

Figure 2. CIWS 2-D ET Example (Courtesy: MIT Lincoln Labs) [1]

We needed CIWS data for a sample day in the Denver region that had experienced convective activity. The Denver area was the area of choice as our prototype trajectory service included adaptation data configured for the Denver Air Route Traffic Control Center (ARTCC) area.

Because the focus of our project was on tactical weather avoidance, we limited the conflict look-ahead time to 30 minutes since beyond that time weather forecast uncertainties would need to be modeled. Thus, our WAF data set includes the current time data plus six additional forecast data sets where each data set corresponds to a 5 minutes future forecast. Since the CIWS VIL and ET products were 1 km x 1 km resolution, the resulting WAF data would also be 1 km x 1 km resolution. The sample data set provided by MIT Lincoln Laboratory, shown in Figure 3, included gridded VIL and ET in netCDF4 format. An example of the combined WAF, ET data set is depicted in Figure 4. This is a ¼ scale portable network graphics (PNG) image of the entire CIWS region with the hazard value plotted in subset area with a rectangular orange border that roughly corresponds to the area covered by the Denver ARTCC airspace. The Figure 4 map projection is different from the one in Figure 3. The red areas have both high VIL and ET and are thus most likely to be avoided by all aircraft. The blue areas have lower VIL and ET values that some aircraft may choose to penetrate. The gray areas have much lower VIL and ET values and thus are less likely to present an obstacle to air traffic.

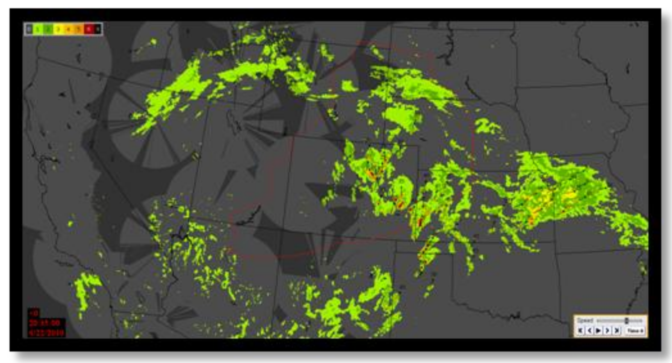

Figure 3. CIWS VIL Data Set from 2035z, April 22, 2010 (Courtesy: MIT Lincoln Labs) [1]

From the 2-D ENSCO combined WAF-ET data set, LM created a 3-D WAF data set by extending the WAF value at a given grid point downward to the surface. We did this only for a convective WAF field to ensure that aircraft would not be routed under the convective weather. We would anticipate that such 3-D WAF data would be eventually be published by the 4-D Weather Cube/SAS for a requested area of concern.

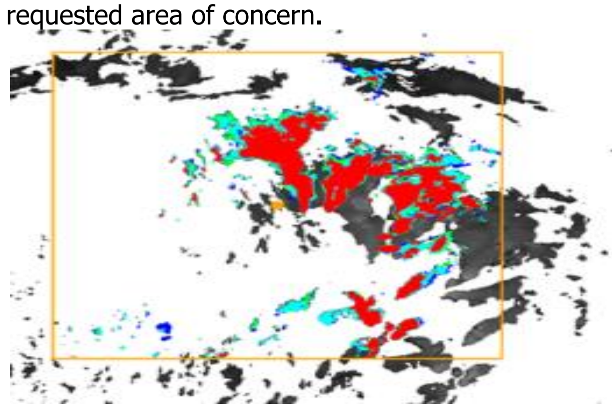

Figure 4. Combined 2-D WAF-ET Data Set [1]

The process used to create the 3-D WAF was to consider a given grid cell and to extend the grid cell's WAF value from the surface up to and including the altitude of the grid's ET value. This array is stored in netCDF4 format. Figure 5 depicts a Google Earth™ visualization of the 3-D WAF data from reference [1].

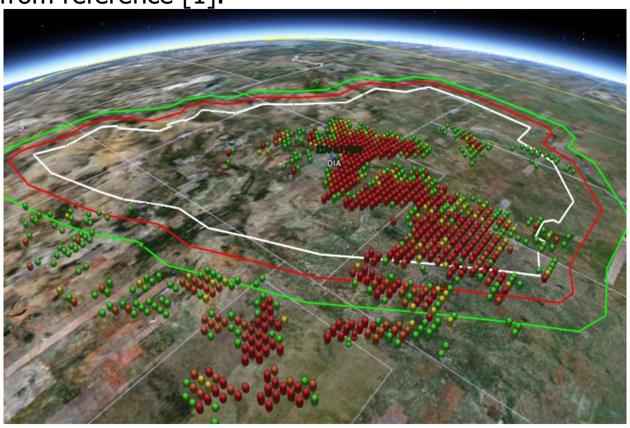

Figure 5. Visualization of 3-D WAF Array(10k x 10k)

2.2 Flight Plan and Trajectory Prediction Service

We used our prototype Common Trajectory Prediction Service to accept either existing flight plans or new flight plans and produce a 4-D trajectory using algorithms similar to those used in the current en route automation system. See Figure 6.

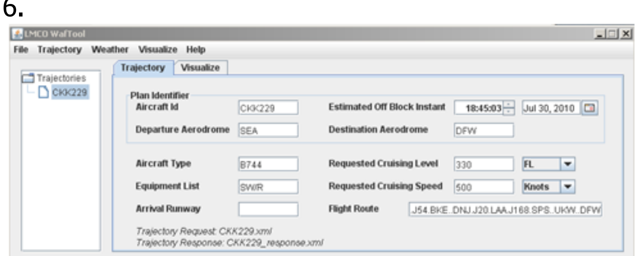

Figure 6. Flight Plan/Trajectory Service Panel [1]

2.3 Developing the Conflict Grid

In 2010, , we used a single conflict grid, with a 1 km x 1 km x 1000 ft cell resolution, to determine aircraft–to-WAF conflicts. The detail of how the weather conflicts were evaluated is in reference [1]. The new single conflict grid will be a section of a digitized volume of the Denver ARTCC area of responsibility (AOR) and will contain both aircraft trajectories and WAF data. However, the 1 km x 1 km grid resolution will not be sufficient to ensure the required separation between aircraft. For example, suppose we wanted to ensure that aircraft are separated laterally and vertically by 1000 ft. (See figure 7). For this we would need a grid resolution of 500 ft.

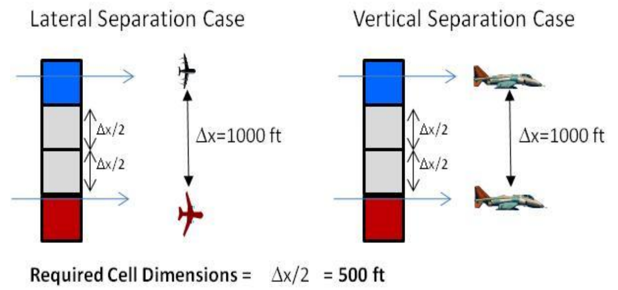

Figure 7 Determining Grid Cell Dimensions

Note that the in-trail separation is variable depending on a number of factors, primarily flight domain (e.g., en route, oceanic, terminal, etc.). For example, a nominal en route separation distance is 5 nm. See Figure 8.

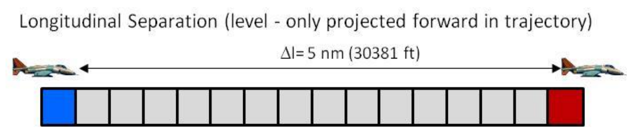

Figure 8 In trail separation

While the technique taken thus far for aircraft separation is known as grid cell buffering, the single grid construct lends itself to stochastic representation as described in [7].

With the conflict grid established, next is the task of mapping the aircraft trajectories and WAF onto the conflict grid. The mapping of trajectories to grids cells is accomplished by explicitly following the trajectory through the grid cells, finding the intersection of the 3D path with each edge of the cell and hence determining what the "next" cell will be. Once the intersection points are known, the time span in the cell can be calculated from the 4D path. The incoming WAF data are on 1 km x 1 km x 1000 ft cell dimensions and must be translated to the finer resolution conflict grid. If the WAF cell overlaps any part of a 500 x 500 ft cell, then that cell is assigned that level of WAF.

2.4 Weather Conflict Detection (WCDS) Service

The WCDS service and its interaction with other services used on the project is depicted in Figure 9.

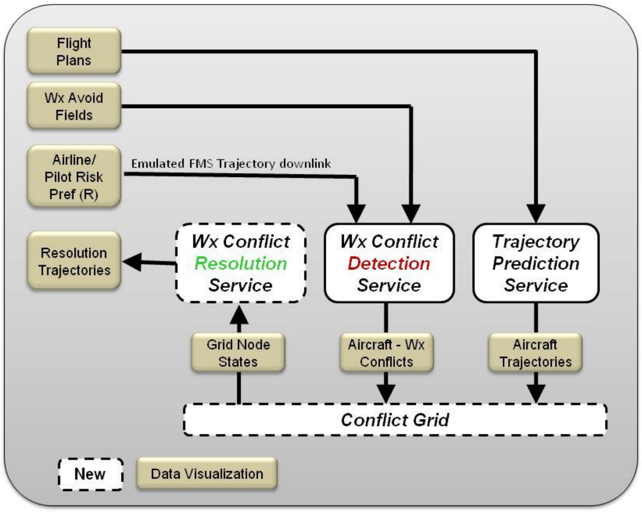

Figure 9. Weather Conflict Detection and Resolution Services Block Diagram

The WCDS uses the following generalized approach to detection of aircraft trajectory–to-WAF conflicts:

1) Test for WAF Existence: Because we specified the sample data to include convective activity within the Denver ARTCC AOR, as a simplification, we did not need to perform a test for WAF existence.

2) Bounding Volume Intersection Test (BVIT) The BVIT is a first order test to determine if there is an intersection between an aircraft trajectory including its 3-D WAF buffer margins bounding volume (BV) and the WAF 3-D BV. The BVs would be created for each time increment.

3) Determine WAF Avoidance Margin Search Domain To determine the WAF avoidance margin search domain, we need to establish the number of buffer cells surrounding the aircraft trajectory cell of interest at any time increment, that is, a cell occupied by an aircraft according to its trajectory at some time. The lateral search domain can be thought of as a single 2-D rectangular stereographic surface level plane consisting of three nested rectangular regions. The three nested avoidance regions can be visualized by considering a Russian wooden Matroshka doll analogy, with the doll figures represented as rectangular regions at 5 km, 15 km and 20 km buffer margins latitudinally and longitudinally. The vertical search domain consists of nested vertical regions starting at the aircraft's current altitude extending above and below by 1000 ft, 3000 ft and 5000 ft margins.

The buffer margins referenced above were for research purposes only and not suggested to be used operationally. The actual buffer margins used operationally by general aviation pilots or commercial airline pilots will be in accordance with personal weather minimums or airline company policy, respectively. The Aeronautical Information Manual [8] suggests pilots avoid severe thunderstorms by at least 20 nm (i.e., ~ 37 km) laterally and by at least 1000 ft vertically for each 10 knots of wind speed at the cloud top. The margins described above were intended to be used as starting points in defining a pilot/company provided risk preference.

4) Search Algorithm to Determine AC-WAF Conflicts Once the first order BV test passes, the detailed search algorithm begins using the WAF avoidance margin search domain. Each grid cell that the trajectory passes through is compared against the grid for the appropriate forecast time interval (there are grids at 5 minute intervals that represent time up to 30 minutes into the future). We merge all the WAFS together in time if appropriate (e.g., if weather was slow moving and did not vary appreciably in the 5 min intervals). So if weather of a certain intensity was present for 25 minutes straight, there would be one check against that 25 minute time period to see if it overlapped with the time period that the aircraft would be in the cell. Any grid cell found to be within the WAF avoidance margin of the trajectory cell will cause a conflict to be generated.

The search algorithm has been modified for this year to include a single parameter, R, which represents the pilot's risk preference as supplied by the aircraft trajectory. The parameter can be used to increase the search buffer by a value corresponding to the degree of risk. For example, R can be one of three values: Low Risk where the search margins are the default values, medium risk where the search margins are multiplied by a factor of 1.5 and high risk, where the search margins are multiplied by a factor of 2. This single parameter is notional and provides a simple construct to illustrate one possible implementation of a risk index supplied by the aircraft. The R parameter allows different aircraft types or airline preferences to have different avoidance margins; for example, a cargo flight may wish to assume a higher risk and fly closer to a given level WAF than a passenger flight.

When the search algorithm finds a WAF within the aircraft trajectory WAF buffer region, the trajectory segment for the corresponding trajectory segment at the time increment under evaluation is highlighted in a color corresponding to the trajectory buffer margin penetration and of the proximate WAF cell value. The output of the conflict service returns the subset of aircraft trajectory segments with a conflict including the value of the conflict (WAF 1-4). See Figure 10. [1]

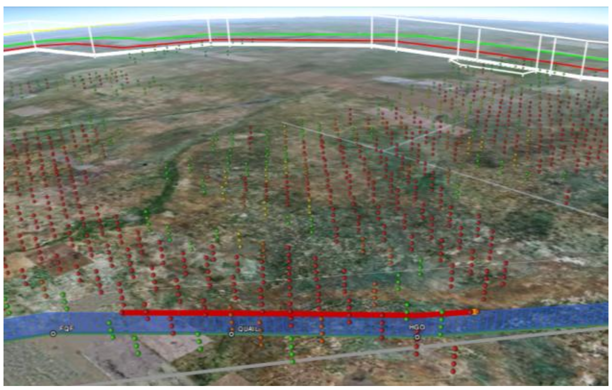

Figure 10. Aircraft - WAF Conflicts for SEA-DFW flight

The output of the WCDS service produces a conflict as the starting and ending time on the trajectory. The starting time is when the trajectory enters the first cell in conflict; the ending time is the time when the trajectory exits the last cell in conflict. The conflict is annotated according to the corresponding WAF severity level of the proximate WAF cells. The WCDS correspondingly marks the trajectory grid cells in conflict. At this point, the cells in the conflict grid are marked and time indexed according to whether they are occupied by an aircraft trajectory, the aircraft trajectory buffer, the WAF data trajectory conflicts with WAFs.

2.5 Weather Conflict Resolution Service (WCRS)

Once aircraft to WAF conflicts are evaluated, cells marking the beginning and end of trajectory segments impacted by the WAF, are passed to the conflict resolution service. Based on the WAF and trajectories of aircraft in the vicinity and times of the conflict space, the conflict grid marks grid cells as nodes that are either open, blocked, path-start or path-end. The path-start and path-end nodes were marked by the WCDS according to the start and end points of a trajectory segment that is in conflict with weather and requires resolution. The node states are based on the value of the grid cell. For example, nodes marked as blocked could be due to a grid cell corresponding to a WAF value of 4 (most severe) or to a cell marked as either an aircraft trajectory cell or trajectory buffer cell. Nodes marked as open could be due to no aircraft or aircraft trajectory buffer or have no weather or WAF values below 1.

With the various node states identified, the WCRS treats the conflict grid as a directed, weighted 3-D graph in which to apply 3-D path finding algorithms to the nodes marked as the path-start and path-end. This resolution path takes into account WAFs, other aircraft and the user's risk preference in finding candidate paths. In order to evaluate the path finding algorithms as well as the necessary rule sets, we needed a tool that would be flexible enough to allow us to easily modify the attributes of not only the conflict grid, but also the path finding algorithm. Additionally, it would be helpful if this tool was easily integrated with a visualization tool. Under consideration were Matlab® and LuciadMap (see http://luciad.com). Last year, Google Earth™ was our visualization tool because it was quick and easy to use. However, due to the increased cell resolution needed for this year's project, we chose LuciadMap and its real-time extension for visualization of the WAF and conflict data. Since we chose LuciadMap for visualization, we decided to use Luciad Network Planner to construct the conflict grid as well as evaluating various path finding algorithms. The Luciad Network planner defines both an interface and implementation for network resolution algorithms; this allows us to substitute our own implementation if we so choose.

Another challenging task performed by the WCRS, was the reconstruction of resolution trajectory segments based on discrete movement in 3-D. To do this, a 3-D least squares fit algorithm is used. This segment is then combined with the remaining segments to describe the full trajectory. While our initial approach will involve a least squares fit, other techniques may be called upon depending on the complexity of the segments involved.

2.6 Research and Evaluation of 3-D path finding algorithms

The problem of path finding through a network is found in several domains including network planning, artificial intelligence, robotics, and computer gaming. Some path finding algorithms are shortest path algorithms, such as Dikstra and A*, and are applicable to 2-D domains (e.g., network optimization, route finding) using directed graphs. A directed graph is a graph with arrows on the edges indicating direction. [9] Each edge represents an ordered pair of nodes with the order of the nodes indicating directionality. The trajectory segment in conflict that needs re-routing will include start and finish nodes, which are exactly what the A* algorithm was designed for – to find the shortest path from start node to finish node. However, because trajectory resolution movements are not restricted to 2-D grids, we need a version of A* that works on 3-D grids.

There are many 3-D path finding algorithms available. It is important that we find an algorithm that can use any angle to progress to the next grid cell and not be constrained to grid edges. Because the conflict grid node structure will be of uniform dimensions, the method of finding a route from start to finish point will use node traversal rather than edge traversal. Of the several we investigated so far, D* [10], Theta* [11] and Phi* [12] were the most notable.

Theta* is a variant of A* where information is propagated along edges without constraining the paths to edges. [11]. Phi*, is an improvement on Theta* where it is an incremental version of Basic Theta* that has been shown to speed up Theta* by an order of magnitude [11]. For this initial research, we used a fairly simple form of the Theta* algorithm. The Luciad Network Planner provides the platform for evaluating the different algorithms. However, further experimentation and evaluation of path finding algorithms will be the subject of future research.

3.0 Results

To define the problem space for the WCRS, we created three flights, obtained from actual flight routes, operating through the Denver ARTCC. One flight is en route from Seattle (SEA) to Dallas-Ft. Worth (DFW), the second is en route from DFW to Salt Lake City, and the third is a short flight from Denver to Albuquerque. The WCDS returned WAF conflicts for the SEA-DFW as shown in Figure 11. The presence of the other two flights, while not in conflict with each other or the SEA-DFW flight, pose a challenge to the path finding algorithm such that the WCRS needs to find a resolution path for the SEA-DFW trajectory segments that are in conflict with weather. Figure 11 shows the extent of all three trajectories. This effectively represents the trajectories input to the WCRS.

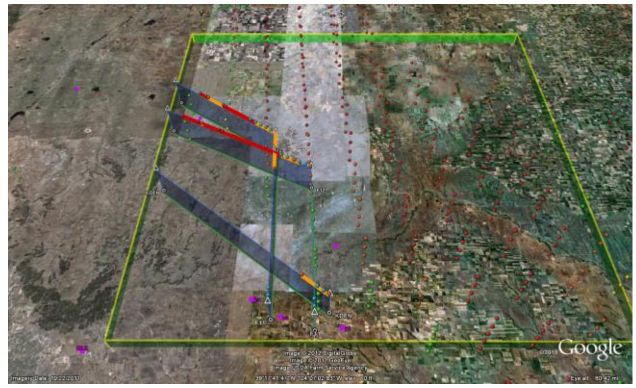

Figure 11. Test Trajectories Input to WCRS

At this time, conflicts and resolutions are visualized using Google Earth ™. However, we will be transitioning the visualization component to LuciadMap with its real-time extensions to gain fine-grained control over the display and improve performance. The output of the WCRS service as represented in Luciad Network Planner is depicted on the conflict grid.

The trajectory segments that are impacted by weather defined by the WAFs are passed into the conflict resolution service. The conflict resolution service then constructs a weather reroute trajectory, that considers a) other hazardous WAF areas, b) other aircraft, and c) route restrictions. As a follow on project, a weather-avoided, aircraft-conflict free trajectory as produced by the ground system can then be exchanged with the FMS derived weather avoidance trajectory or intent for trajectory negotiation. A full implementation of a resolution service would have to ensure that all flights impacted by a weather system were treated fairly; that is, no one airline was consistently given the longest re-calculated routes.

The process of tweaking the path finding algorithm has been incremental as can be seen for the SEA-DFW flight in Figures 12a and 12b. In Figure 12a, early path resolution attempts show multiple discrete “stair step” segments for the resolution path that avoids hazardous weather to the northeast while avoiding a DFW-SLC flight to the southwest. Adjustment of the graph node state rules yielded the smoothed results shown in Figure 12b. In both Figures 12a and 12b, while the end of the resolution path occurs after the conflict, additional node tweaking was required to ensure the start of the resolution starts before the weather conflict. In both figures 12a and 12b, the resolution segment highlighted by the white oval was created to avoid the DFW-SLC flight whose trajectory lies to the southwest of the highlighted resolution segment.

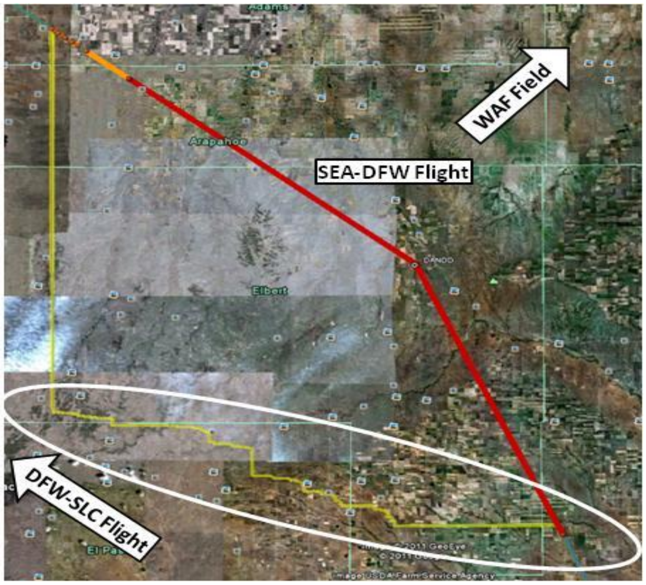

Figure 12a. SEA-DFW Flight with initial resolution attempt

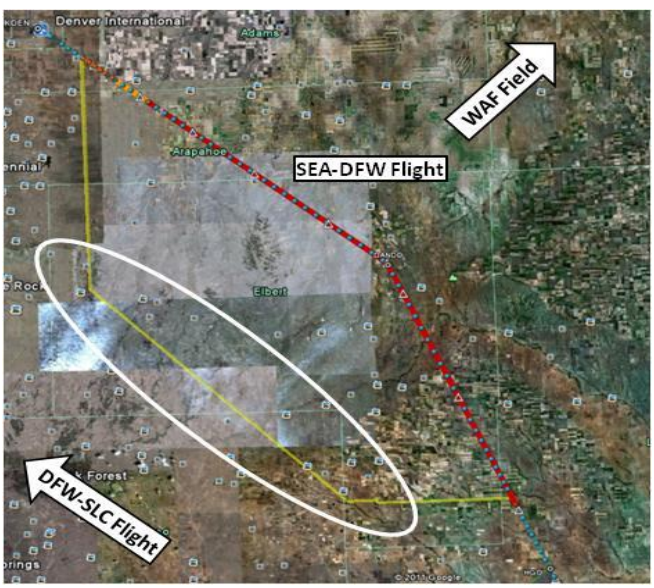

Figure 12b. SEA-DFW Flight with smoothed resolution path

4. Summary

In 2010, we published the results of suggested approaches to the integration of NextGen weather data, specifically WAF in a grid-based architecture using trajectory methods to detect conflicts or impacts of hazardous weather to aircraft trajectories. A prototype weather conflict detection service was developed that produced the specific segments of trajectories impacted by weather. In 2011, we included resolutions to those detected conflicts. In order for those resolutions to be viable, they need to account for other aircraft in the space being evaluated for weather avoidance. In order to accomplish that, we included the aircraft-to-aircraft conflict detection logic as well. All of these integration goals pointed to the more pressing need for an effective integration framework. This framework would ingest weather data, a converged air/ground based trajectory, aeronautical information, and user (pilot/airline) risk preferences such as provided by an FMS. Then, separate services can provide conflict and resolution services by accessing a grid-construct that can handle both deterministic data for use in tactical weather avoidance coordinated with the FMS as well as probabilistic data for use in strategic traffic flow management. The next step for this activity is to continue development of grid-based 4D conflict resolution algorithms to include the synchronization of a flight crew preferred weather avoidance trajectory, as downlinked by an FMS, with the resolution trajectory as determined by the ground-based weather conflict resolution service

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References

[1] R. Avjian, J. Dehn and J. Stobie, "NextGen En Route Weather Integration Strategies", *Proceedings of the ATCA 55th Annual Conference and Exposition,* Washington, DC Oct 2010.

[2] R. Avjian, J. Stobie, Strategies for Air Traffic Management Integration of Hazardous Weather. *Proceedings of the ATCA 54th Annual Conference and Exposition,* Washington, DC October 2009

[3] R. DeLaura, M. Robinson, M. Pawlak, J. Evans, Modeling Convective Weather Avoidance in En Route Airspace. *13th Conference on Aviation, Range and Aerospace Meteorology (ARAM), Amer. Meteor. Soc.,* New Orleans, LA January 2008

[4] R. DeLaura, B. Crowe, R. Ferris, J. Love, and W. Chan, Comparing convective Weather Avoidance Models and Aircraft-based Data. *89th Annual Meeting of the American Meteorological Society: Aviation, Range and Aerospace Meteorology Special Symposium on Weather – Air Traffic Impacts,* Phoenix, AZ, January, 2009 *.*

[5] Matthews, Michael P. and Rich DeLaura: "Evaluation of Enroute Convective Weather Avoidance Models Based on Planned and Observed Flights," *14th Conference on Aviation, Range, and Aerospace Meteorology (ARAM)*, Atlanta, GA, 2010

[6] Klooster, J. Torres, S., Earman, D. Castillo-Effen, M. Subbu, R., Kammer, L., Chan, D., Tomlinson, T, Trajectory Synchronization and Negotiation in Trajectory Based Operations, *29th Digital Avionics Systems Conference (DASC)*, October 2010

[7] Jardin, Matt R, Grid-Based Air Traffic Control Strategic Conflict Detection, *AIAA Guidance, Navigation and Control Conference and Exhibit*, 15-18 August 2005, San Francisco, CA

[8] Federal Aviation Administration, *Aeronautical Information Manual*, February 11, 2010. http://www.faa.gov/air_traffic/publications/ATpubs/AIM/

[9] Tucker, A., 1980: *Applied Combinatorics*, John Wiley & Sons, 385 pp.

[10] Ferguson, D. and Stentz, A., Using interpolation to improve path planning: The Field D* algorithm. *Journal of Field Robotics*, 23(2):79–101, 2006

[11] Nash, A, Daniel, K., Koenig, S, and Felner, A., Theta*: Any-angle path planning on grids. In *Proceedings of the AAAI Conference on Artificial Intelligence*, pages 1177–1183, 2007.

[12] Nash, A, Koenig, S, Likhachev, M, Incremental Phi*: Incremental Any-Angle Path Planning on Grids *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pages 1824-1830, 2009