3A.3: Spatial Extreme Value Analysis for Large-Scale Severe Weather Indicators

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Scale of Extreme Weather Events

2006 European Heat Wave (Fig. from KNMI)

F5 Tornado in Elie Manitoba on 22\textsuperscript{nd} June 2007

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Large-scale indicators (CAPE and Shear)

W_{\text{max}} = \sqrt{2 \text{CAPE}} \text{ (m/s)}

<table>
<thead>
<tr>
<th>Non-severe</th>
<th>Hail &lt; 1.9 cm Winds &lt; 55kts. No tornado</th>
</tr>
</thead>
<tbody>
<tr>
<td>Severe</td>
<td>Hail ≥ 1.9 cm 55 ≤ Winds &lt; 65 Or tornado</td>
</tr>
<tr>
<td>Significant Non-tornadic</td>
<td>Hail ≥5.07 cm Winds ≥ 65 kts.</td>
</tr>
<tr>
<td>Significant tornadic</td>
<td>Same as Significant non-tornadic, but with ≥ F2 tornado</td>
</tr>
</tbody>
</table>
Extreme Value Analysis

• Rare Events
• Only one Maximum in a Dataset
• Very few points above high threshold
• Theory suggests appropriate family of distributions for analyzing extremes
  – Generalized Extreme Value (GEV) df
  – Generalized Pareto (GP) df
  – Point Process characterization
Extreme Value Analysis

• GEV (For large $n$)

$$
\Pr\left\{ \max( X_1, \ldots, X_n ) \leq x \right\} = F(x)
$$

• GP (For large $u$)

$$
\Pr\left\{ X \leq u + x \mid X > u \right\} = F(x)
$$
Spatial Extremes:
Different Choices for Different Goals

• Interpolate Extremes to Unobserved Locations
• Statistical Inference in the Face of Spatial Dependence
• Identify Sources of Variability in Space
• Analyze Extremes Jointly Over Space
• Smooth Data Over Space?
Spatial Extremes: Methods

- Univariate Extremes with Spatial Covariates
- Multivariate Extremes
- Max-Stable Processes
- Copulas
- Bayesian Hierarchical Modeling (BHM)
- BHM + Max-Stable Processes
- Conditional Extremes
Spatial Extremes: Methods

\[ \text{Pr}[T_A < 1, T_B < 1] = \Phi_2(\Phi^{-1}(F_A(1)), \Phi^{-1}(F_B(1)), \gamma) \]

Recipe for Disaster: The formula that Killed Wall Street

Wired Magazine, 2/23/2009, by Feliz Salmon
Conditional Extremes

\[ X \mid Y = y, \text{ for } y \text{ large} \]

\( X,Y \) Follow marginal standard EVDs

\( X \) may or may not be extreme.
Conditional Extremes

\[ X \mid Y = y, \text{ for } y \text{ large} \]

\[ X, Y \text{ follow marginal standard EVDs.} \]

If positively associated, then

\[ [X \mid Y = y] = \alpha y + y^\beta Z \]

Conditional Extremes

$\alpha = 0.3, \beta = 0.7$

$\alpha = 0.8, \beta = 0.1$

Keef et al. (2009) Fig. 3
J. Hydrology, 240 - 252
Conditional Extremes

\[ [X_1, \ldots, X_n \mid Y = y] = (\alpha_1, \ldots, \alpha_n) y + y^{(\beta_1, \ldots, \beta_n)}(Z_1, \ldots, Z_n) \]

Dependence is determined by the parameters alpha and beta and the distribution function G(z).

\( \alpha \) in \([0, 1)\) describes the strength of dependence, with \( \alpha = 1 \) perfect dependence.

\( \beta \) in \((-\infty, 1]\) describes the scale/dispersion of dependence.

Unknown what G is or should be.
Choose Conditioning Variable

- Something to measure the energy in the field at a given time.
- \( q75 = \text{Upper quartile of } W_{\text{max}} \times \text{Shear (WmSh)} \) over space.
- Univariate quantity over time. Condition on its being large.
Spring WmSh (m/s)^2

1958 - 1978

1979 - 1992

1993 - 1999
1958 - 1978

1979 - 1992

Summer WmSh (m/s)^2

1993 - 1999
1958 - 1978

1979 - 1992

Fall WmSh (m/s)^2

1993 - 1999

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Current (1979 – 2004) RCM3

Mean Simulated WmSh from Conditional Model

Mean Simulated WmSh from Conditional Model

5th percentile of Simulated WmSh from Conditional Model

95th percentile of Simulated WmSh from Conditional Model
Summary

• Univariate EVA well studied
• Spatial Extremes is an active area of research
• Current spatial extremes methods require strong assumptions
• Conditional approach alleviates problems with assumptions
• Estimation for conditional approach is tricky, and is an active area of research
• Conditional approach shows a lot of promise for making statistical inferences in the face of spatial dependence
• Challenge in determining how to incorporate future climate model output
Thank you for your attention

Watch for extRemes version 2.0-0

http://www.assessment.ucar.edu/toolkit