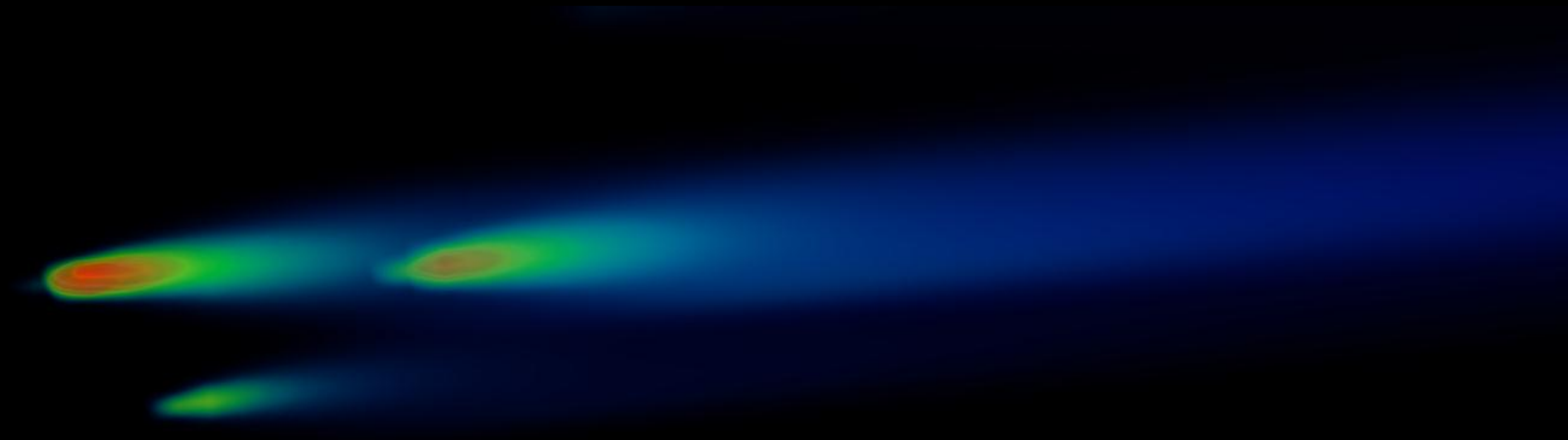

Multi-Source Atmospheric Dispersion Event Reconstruction Using Bayesian Inference and Composite Model Ranking

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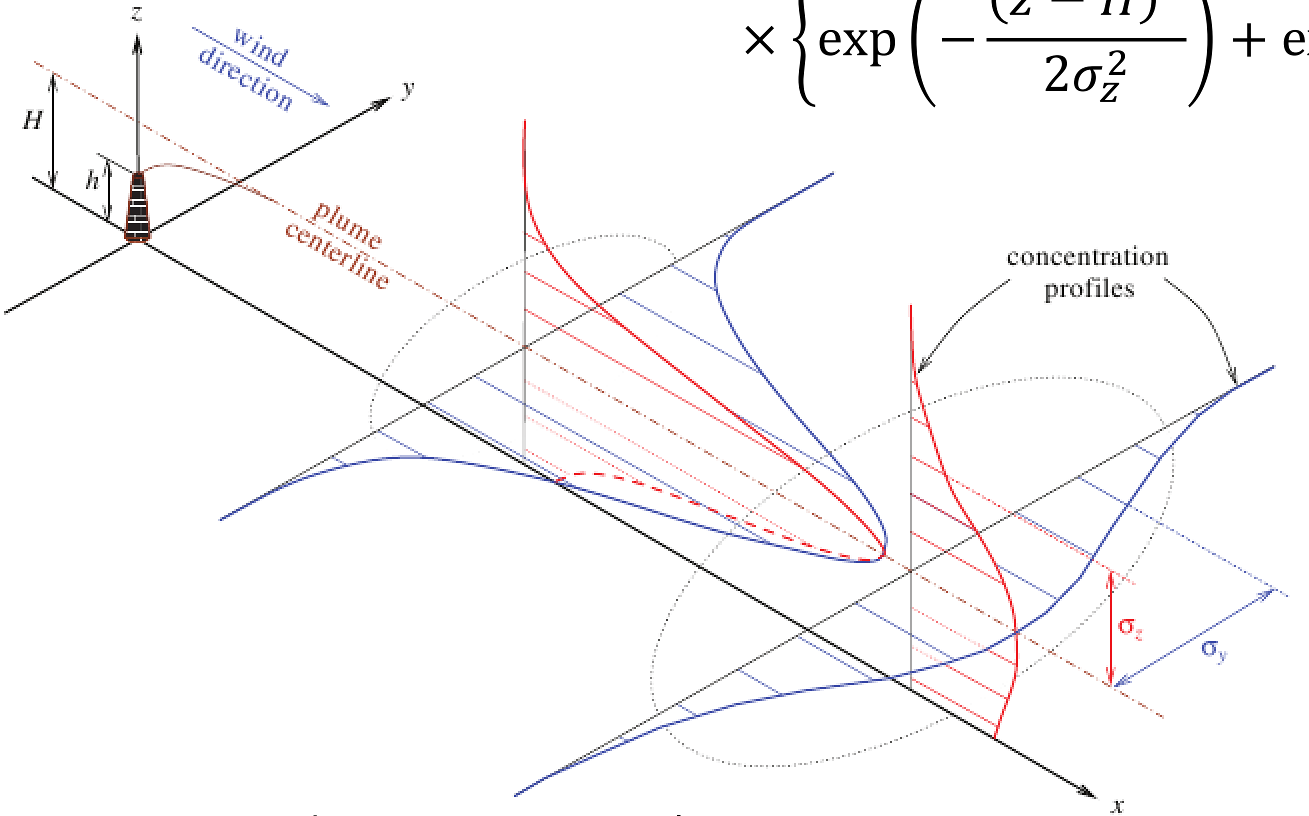
Motivation: Source term estimation of multi-source releases

(source: Red River Radio (NPR))

- Fukushima Nuclear Reactor Meltdown
- Reactor unreachable
- Radioactive material released from **multiple locations**

Gaussian Plume Forward Model

$$C_m(x,y,z) = \frac{Q}{2\pi U\sigma_y\sigma_z}\exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$
$$\times\left\{\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right)+\exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right\}$$



Dispersion of passive contaminant over flat terrain with steady wind conditions

(Picture Source: Stockie, SIAM Review, vol 53, No. 2, pp. 349-372. 2010)

Inverse problem

$$\boldsymbol{m} \approx F^{-1}(\boldsymbol{d})$$

where,

- F is the forward model, the Gaussian plume model in this case (can be other)
- $\boldsymbol{d}$ is a vector of observed concentration values (i.e. sensor readings)
- $\boldsymbol{m}$ is a set of forward model parameters (i.e. source location, emission rate, etc.)
- Probabilistic approach

Bayesian Formulation

$$P(\boldsymbol{m}|\boldsymbol{d}) \propto L(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})$$

where,

- $P(\boldsymbol{m}|\boldsymbol{d})$ is the probability of the forward model parameters, given the observed concentrations
- $L(\boldsymbol{d}|\boldsymbol{m})$ is the Likelihood of the observations given the forward model parameters
- $P(\boldsymbol{m})$ is a set of prior probabilities for each forward model parameter

Conditional Likelihood Formulation

$$L(d_i|\boldsymbol{m}) = \begin{cases} \exp\left(-\alpha \cdot \widehat{C}_i\right), & d_i = 0 \\ \dfrac{1-\exp\left(-\alpha \cdot \widehat{C}_i\right)}{\sqrt{2\pi}\sigma d_i} \exp\left(\dfrac{1}{2\sigma^2}\left(\ln(d_i) - \ln\left(\widehat{C}_i\right)\right)^2\right), & d_i > 0 \end{cases}$$

(Senocak et al., 2008)

- $\alpha = \dfrac{1}{C_{th}} \ln(2)$, due to ideal sensor detection with probability of ½.
- Given model parameters, $\boldsymbol{m}$, an ideal sensor, ξ_i, has a lognormal distribution with density:

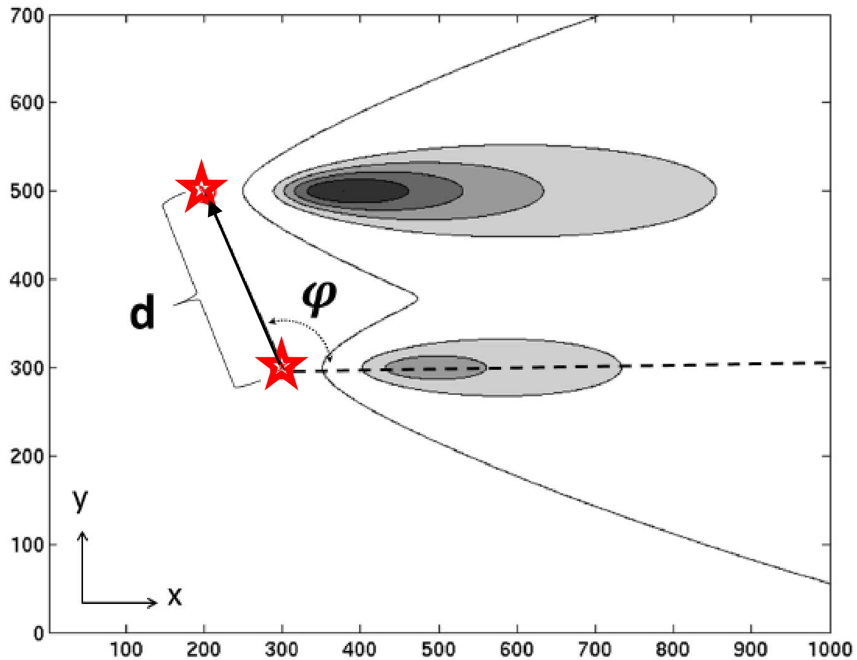
$$p(\xi_i|\boldsymbol{m}) = \frac{1}{\sqrt{2\pi}\sigma\xi_i} \exp\left(\frac{1}{2\sigma^2}\left(\ln(\xi_i) - \ln\left(\widehat{C}_i\right)\right)^2\right)$$

(Senocak et al., 2008)

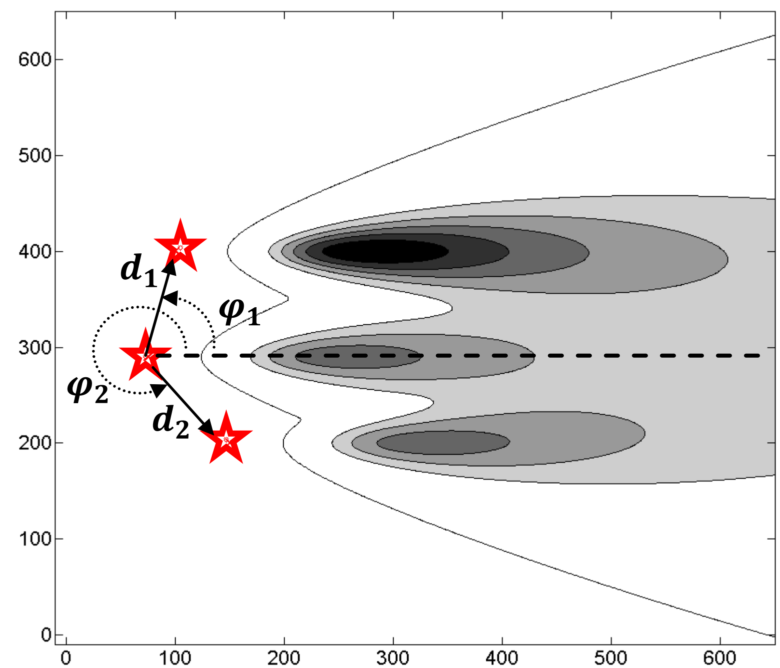
Extending it to Multi-source

$$\boldsymbol{m} = \left[x_{s1}, y_{s1}, \left(\frac{Q}{U}\right)_1, \theta, \zeta_1, \zeta_2, \sigma^2, d, \varphi, \left(\frac{Q}{U}\right)_2\right]$$

2 source



3 source



MCMC Sampling

- Markov Chain Monte Carlo (MCMC) via the Metropolis Algorithm (Metropolis et al. 1953) simulates samples from the posterior distributions
- Candidate state $\boldsymbol{m}^*$ is sampled from Gaussian distribution centered on previous state, $\boldsymbol{m}$, and accepted with probability

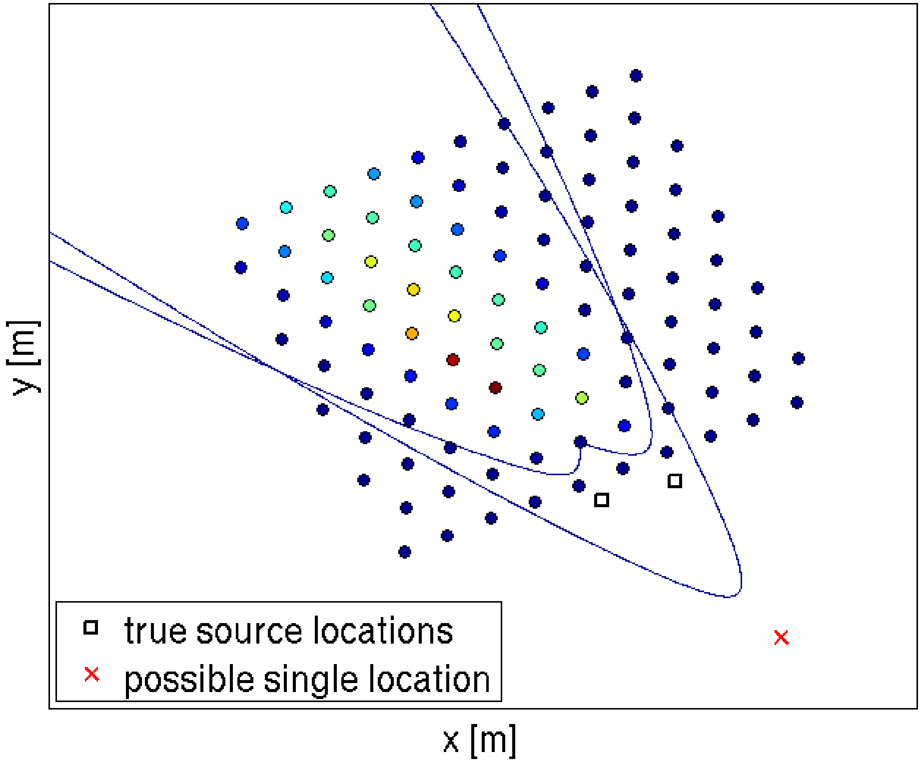
$$p(\boldsymbol{m}, \boldsymbol{m}^*) = \min\left(\frac{\pi(\boldsymbol{m}^*)}{\pi(\boldsymbol{m})}, 1\right)$$

Where target distribution is

$$\pi(\boldsymbol{m}) = L(\boldsymbol{d}|\boldsymbol{m}) \cdot p(\boldsymbol{m})$$

How many sources are there?

- Possible to get a reasonable result with incorrect number of sources
- Proposed a **composite model ranking** system to quantitatively choose the correct number of sources



Ranking Metrics

Ranking Formula

- Overall Ranking Range: 0 (worst) – 3 (best)

$$RANK = \left(1 - \|\boldsymbol{x}\|_{2,scaled}\right) + \left(1 - \frac{|FB|}{2}\right) + R^2$$

- Fractional Bias
  - Quantifies Bias (Over/Underestimation)

$$FB = 2\left(\frac{\bar{C} - \bar{\hat{C}}}{\bar{C} + \bar{\hat{C}}}\right)$$

- l_2 norm of relative error
  - Quantifies Error

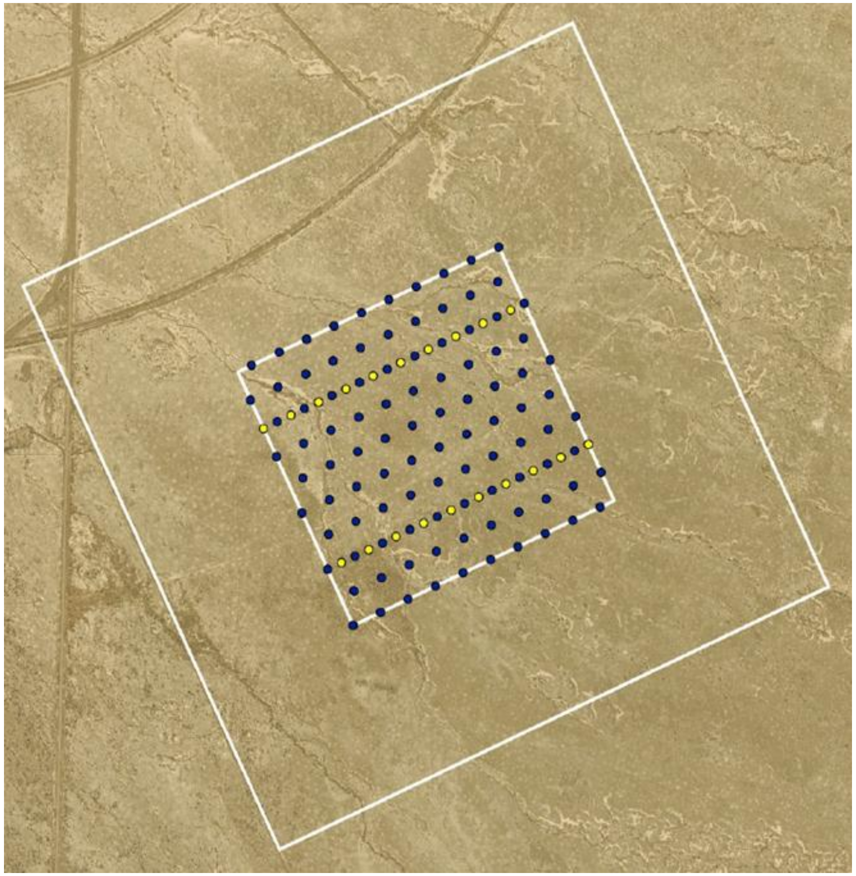
$$\|\boldsymbol{x}\|_2 = \sqrt{\sum_{i=1}^{n}\left(\frac{|C_i - \hat{C}_i|}{C_i}\right)^2}$$

- Pearson's correlation coefficient
  - Quantifies Correlation

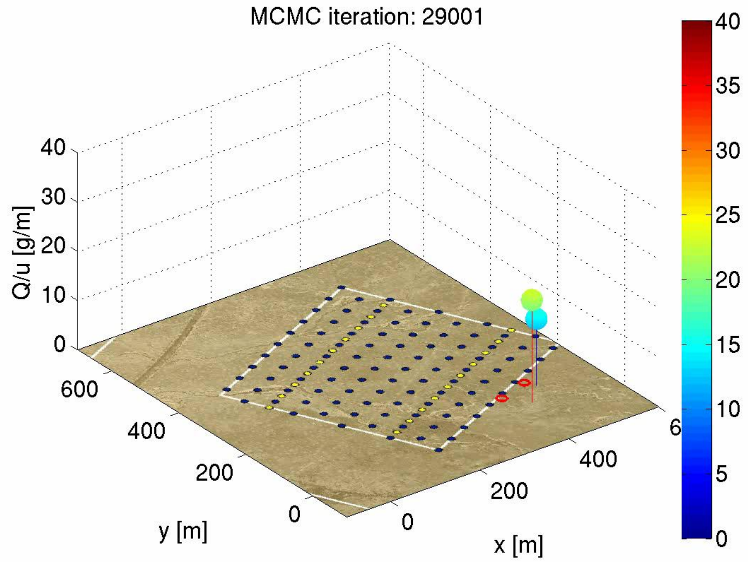
$$R = \frac{\sum_i \left[(C_i - \bar{C})\left(\hat{C}_i - \bar{\hat{C}}\right)\right]}{\left[\sqrt{\sum_i (C_i - \bar{C})^2}\right]\left[\sqrt{\sum_i \left(\hat{C}_i - \bar{\hat{C}}\right)^2}\right]}$$

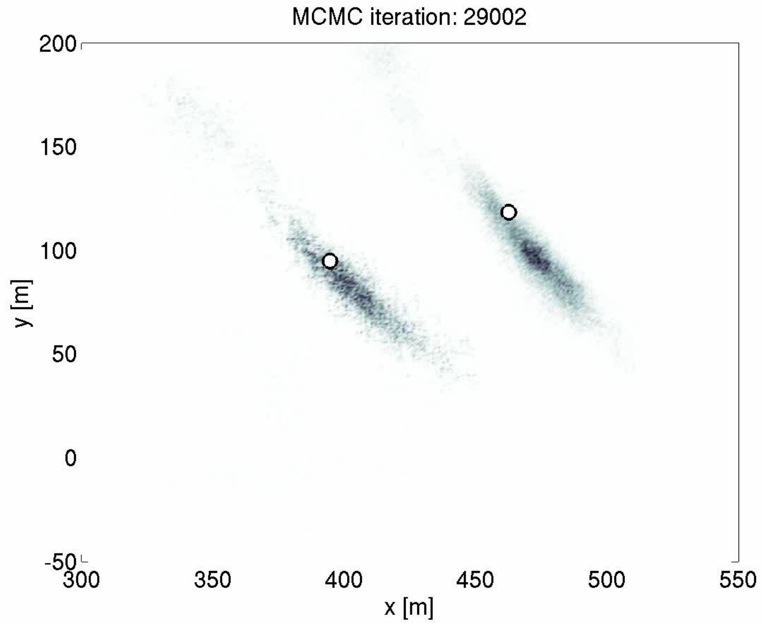
Application to FUSION Field Trials 2007 (FFT-07) and Synthetic Trials

- Real trials conducted by U.S. Army at Dugway Proving Grounds, Utah
- 100 sensors placed 50m apart in test space 1km x 1km
- Single and multiple source releases of Propylene Gas in continuous and puff trials as part of FFT-07 data set.
- **Goal**: based on sensor data, reconstruct the source and all associated parameters and correctly determine number of sources involved in release



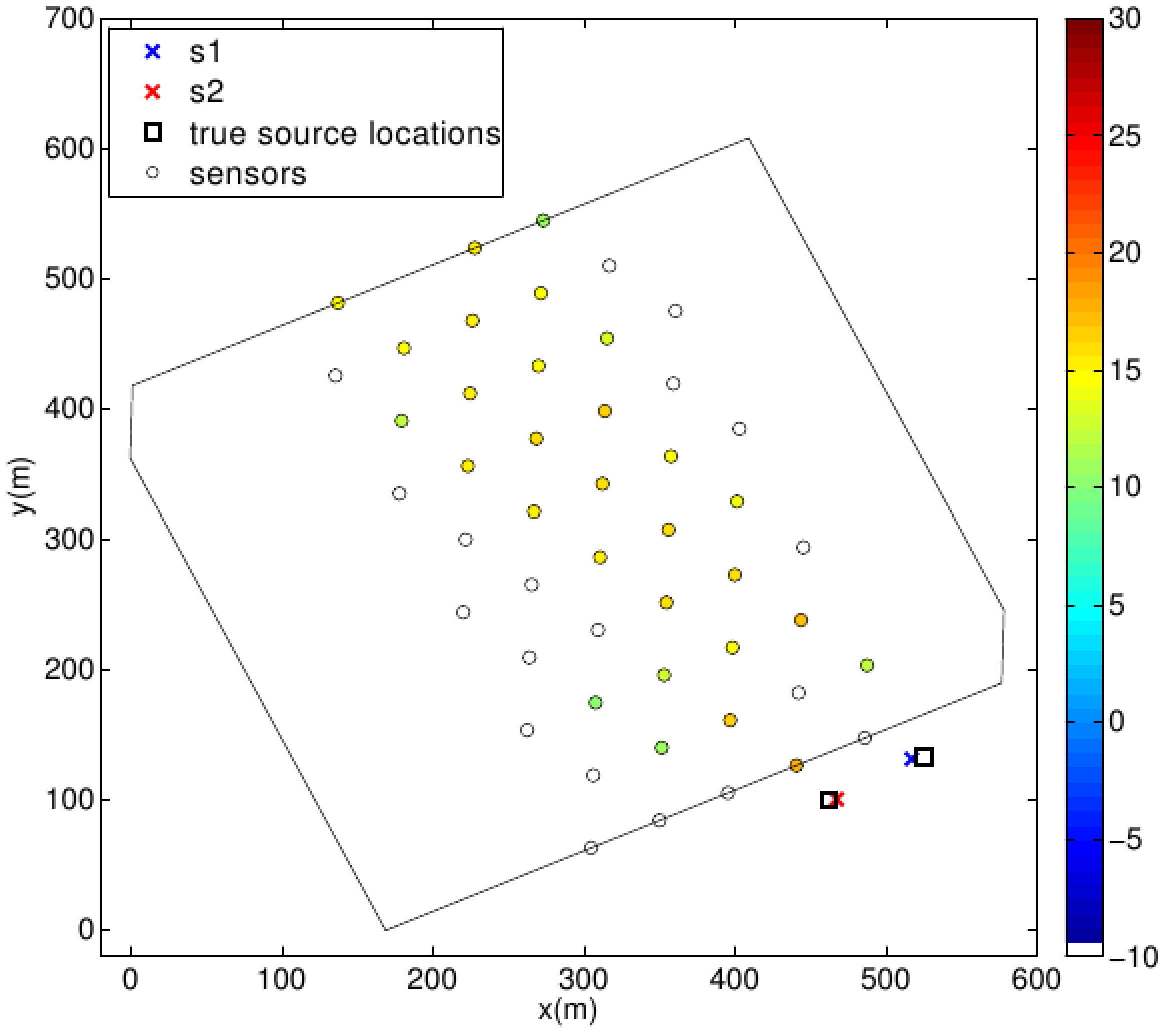
Synthetic Trial MCMC movie





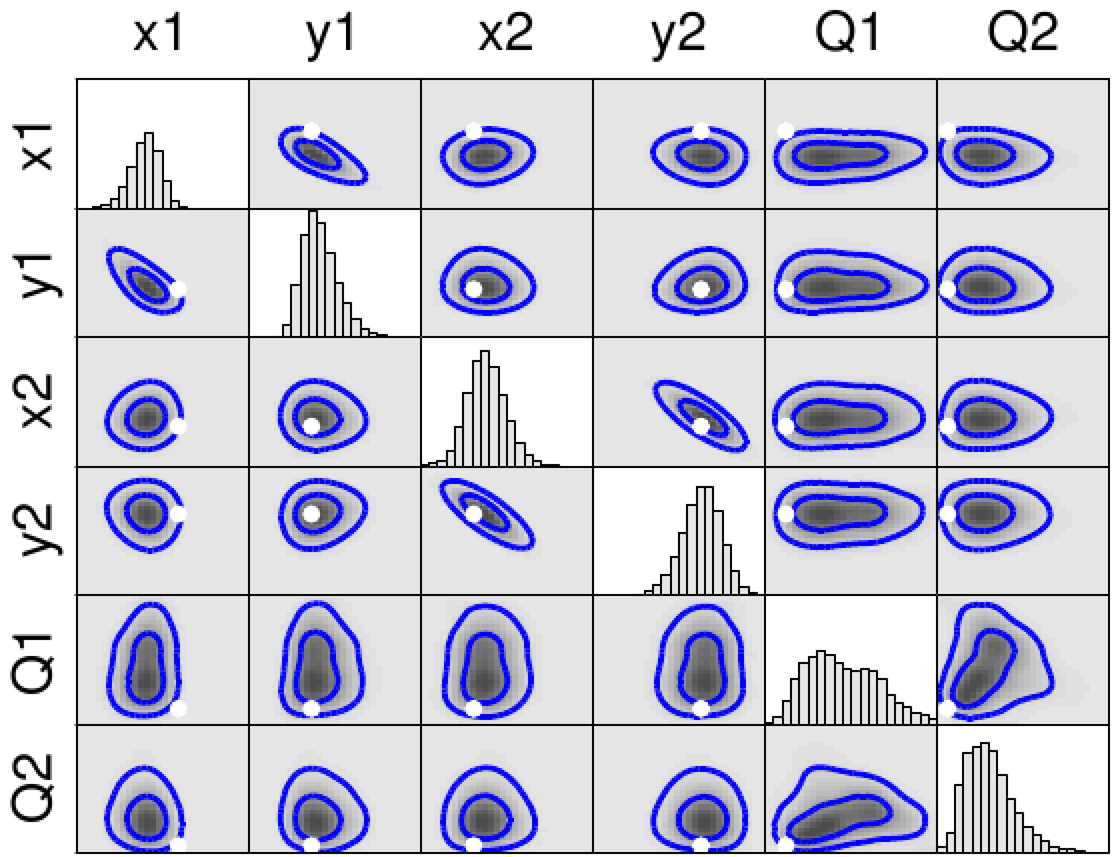
Trial 40 from FFT-07 Data Set

- 48 sensors used
- Error of 8m and 6m for source 1 and source 2, respectively



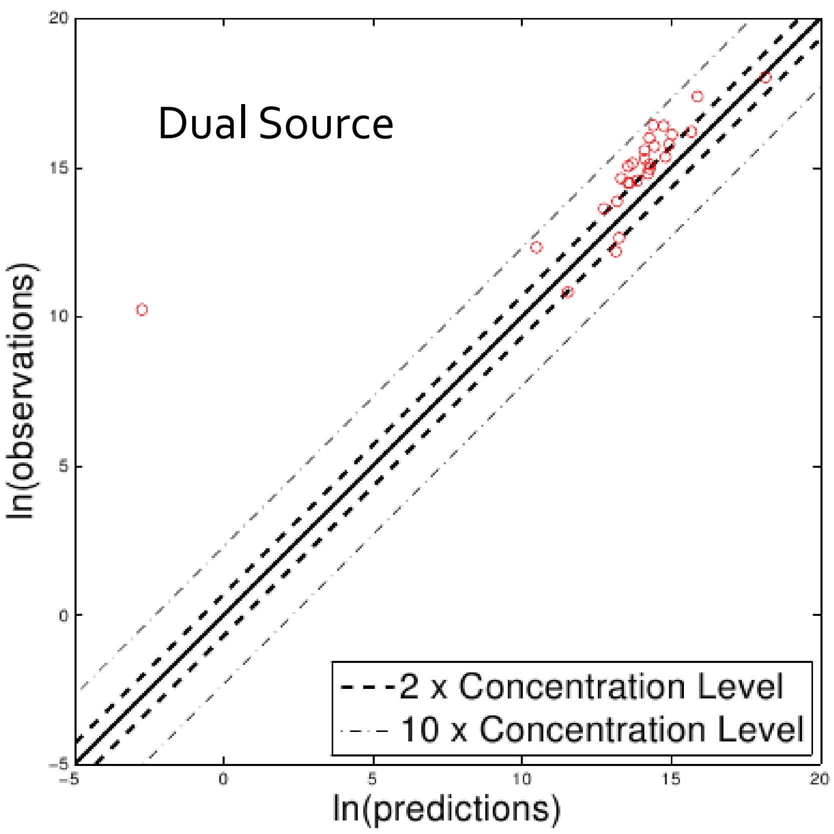
Trial 40 from FFT-07 Data Set

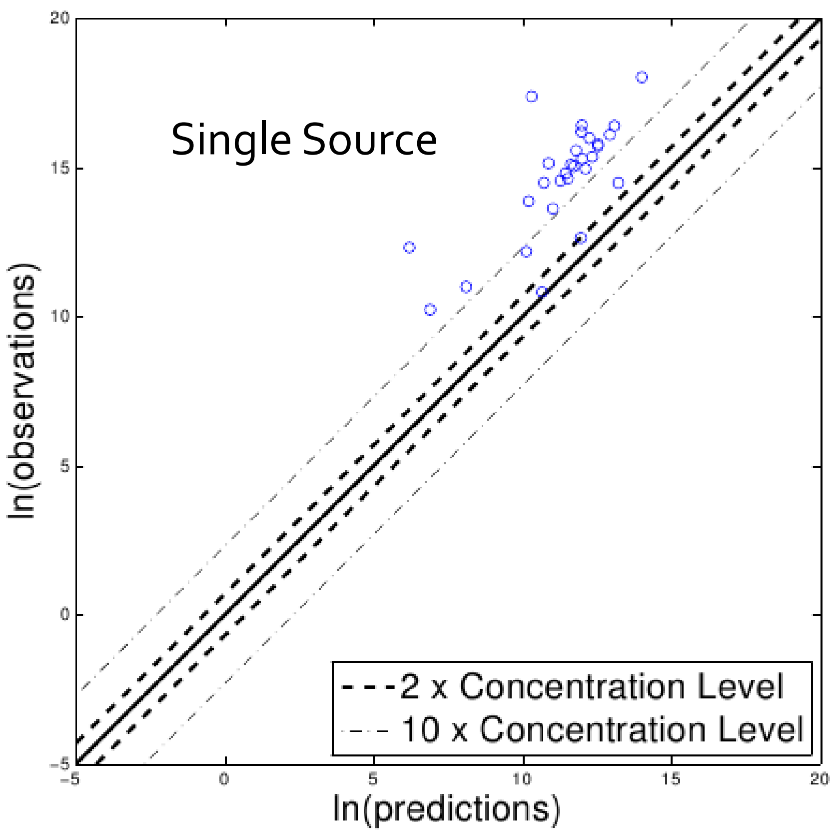
- Bivariate and marginal posterior probabilities for location and release rate, Q.
- Marginal distributions approximately Gaussian



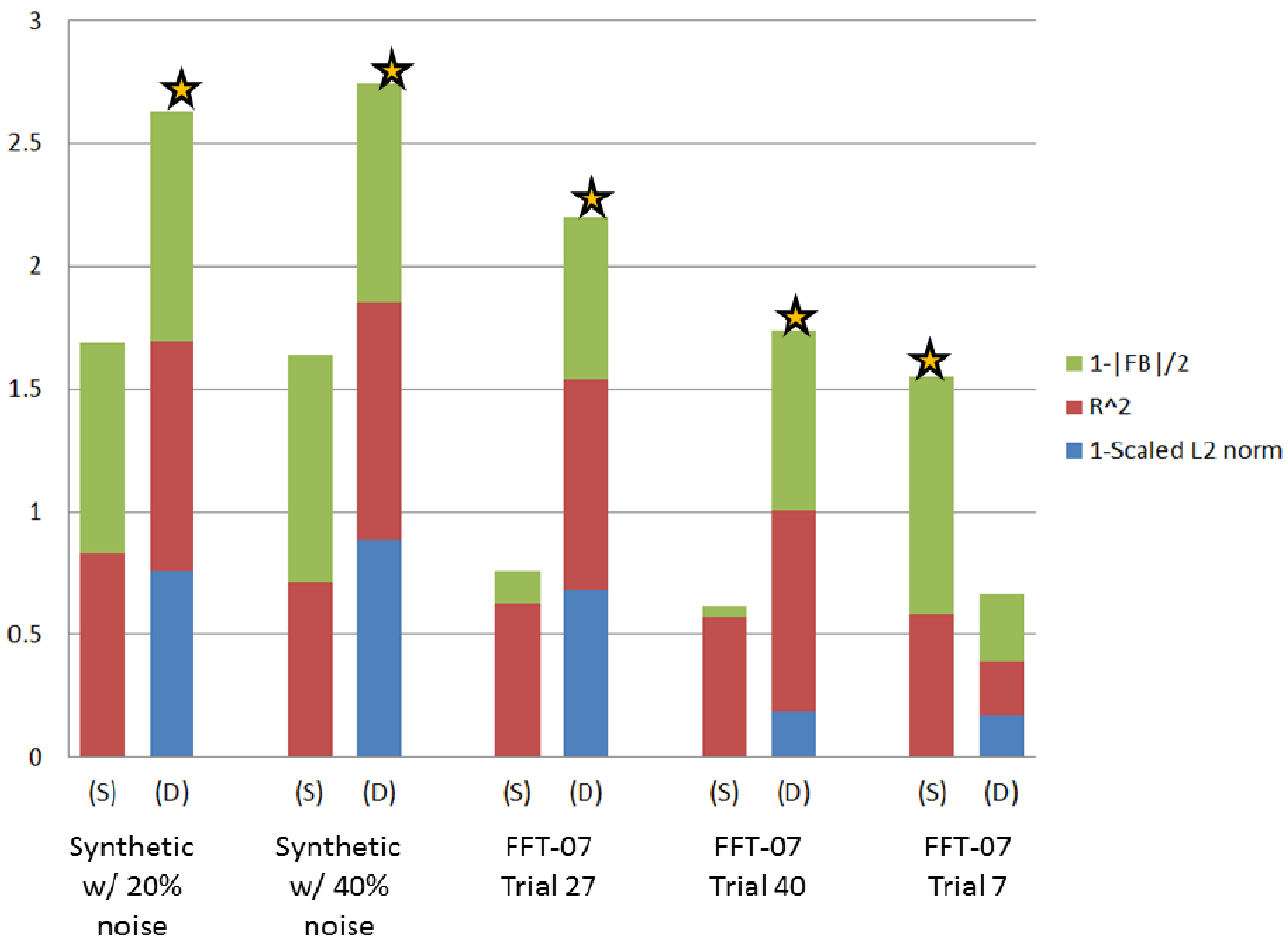
Trial 40 from FFT-07 Data Set

- Observed data vs. calculated (predicted) data





Composite Model Ranking Results



References* and Contact Info

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Acknowledgements:
NSF-ATD Award No. 1043107

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, E., 1953. Equations of state calculations by fast computing machines. Journal of Chemical Physics 224, 560-568.

Senocak, I., Hengartner, N., Short, M., Daniel, W., 2008. Stochastic event reconstruction of atmospheric contaminant dispersion using bayesian inference. Atmospheric Environment 42, 7718-7727.

Stockie, J., 2011. The mathematics of atmospheric dispersion modeling. SIAM Review.

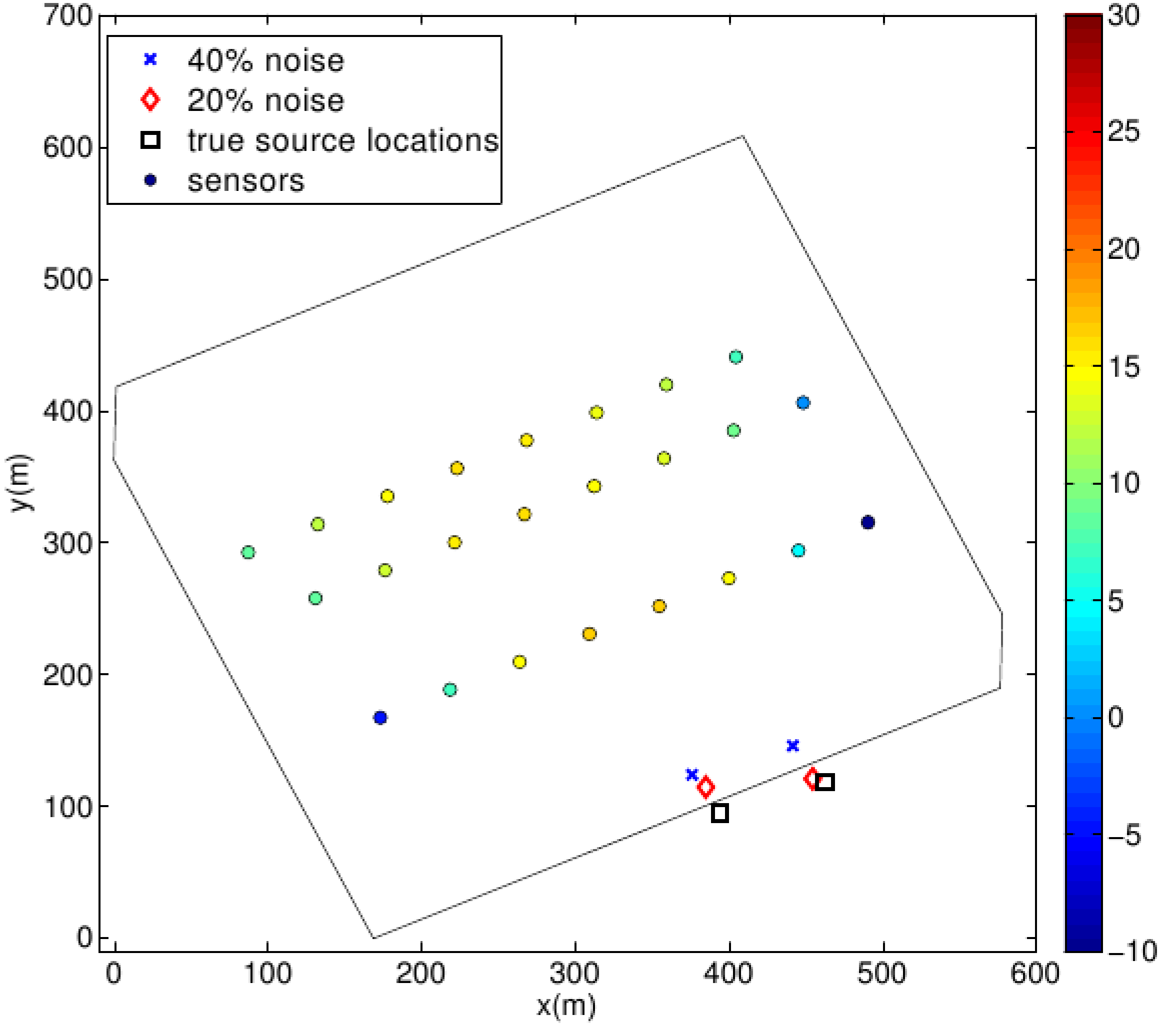
Storwold Jr., D., 2007. Detailed Test Plan for the Fusing Sensor Information from Observing Netwoks (FUSION) Field Trial 2007 (FFT-07). Technical Report WDTC- TP-07-078. U.S. Army.

*Extended Reference List Available

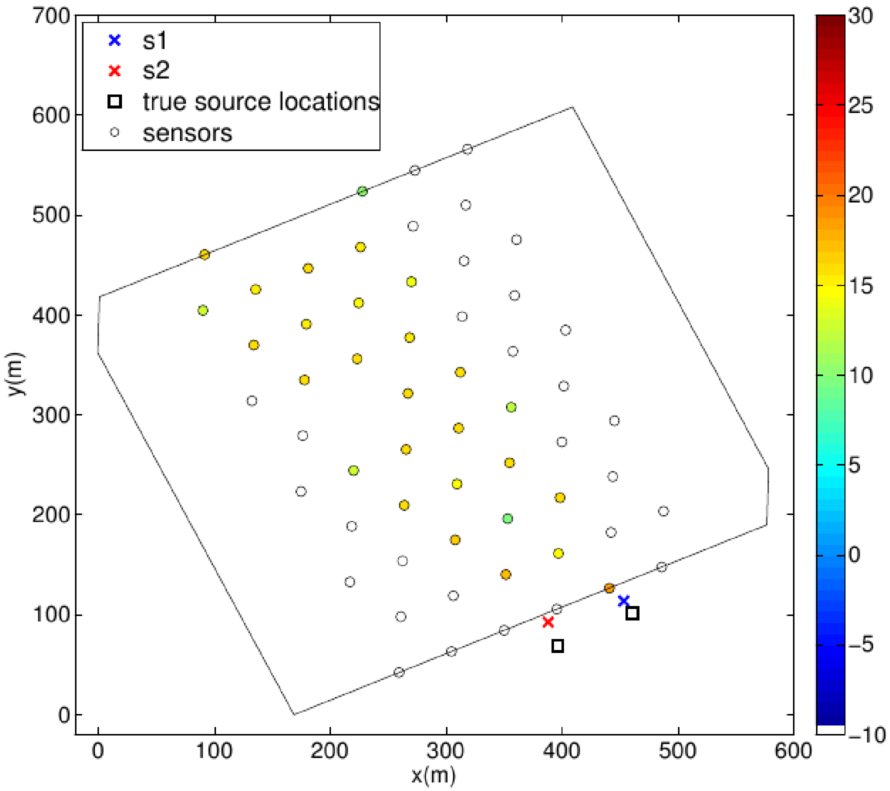
Additional Trial Data

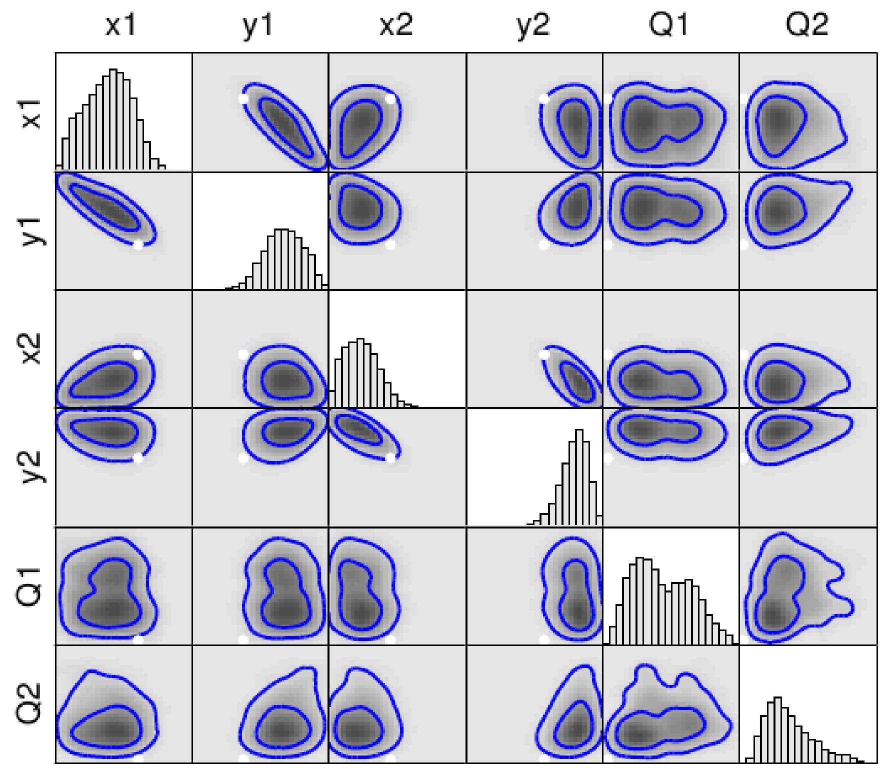
Synthetic Trial Results

- 24 sensors used
- Tested with 20% and 40% Gaussian noise
- Average error of 15.5m and 35.6m, respectively



Trial 27 from FFT-07 Data Set





- Source location Errors of 15m and 25m