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Approximated Equations of Urban Climate by Scale Analysis

Ali Gholizadeh Touchaei

Building, Civil and Environmental Engineering Department, Concordia University, Montreal, QC, Canada

Introduction

Traditionally, first and the most important step for developing a solution of meteorological problem was scale analysis, used in many references (Roger A. Pielke, 2002; Holton & Tennekes, 1979; Houghton, 2002; Haltiner & Williams, 1980). Nowadays, while reasonable and accurate numerical simulation methods are proposed, it is needed to provide a scale analysis for large eddy scale motion of regional climate to reduce unimportant terms in order to decrease computation cost and improve calculation efficiency. In this research an approximation for climate equations of urban areas is presented.

Theory

Like other fluid problems basic equations are based on conservation of mass, momentum and heat.

Conservation of Mass: $\frac{D\rho}{Dt}=\frac{\partial\rho}{\partial t}+\nabla.\rho\vec{V}=0$

Conservation of Water: $\frac{Dq_n}{Dt}=\frac{\partial q_n}{\partial t}+\vec{V}.\nabla q_n=S_{q_n}$ $(n=1,2,3)$ n_1, n_2, n_3 are solid, liquid and vapor phases.

Conservation of other Gaseous Materials: $\frac{D\chi_m}{Dt}=\frac{\partial\chi_m}{\partial t}+\vec{V}.\nabla\chi_m=S_{\chi_m}$ $(m=1,\dots,M)$, S_{χ_m} is source or sink.

Conservation of Momentum: $\frac{D\vec{V}}{Dt}=\frac{\partial\vec{V}}{\partial t}+\vec{V}.(\nabla\vec{V})=-\frac{1}{\rho}\nabla P-g\vec{k}-2\vec{\Omega}\times\vec{V}$

Conservation of Heat: $\frac{D\theta}{Dt}=\frac{\partial\theta}{\partial t}+\vec{V}.\nabla\theta=S_\theta$, S_θ is source or sink.

In all equations the interest is to keep velocity so all terms were compared with horizontal term of velocity $\left|\vartheta_0\frac{\partial u}{\partial x}\right|=\frac{\vartheta_0 U}{Lx}$ because our interest is to retain horizontal velocities in equations.

Results

Conservation of mass: $\left|\frac{\partial\vartheta'}{\partial t}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{L_x\vartheta'}{Ut_\vartheta\vartheta_0}$; Available data indicated that the characteristic length over horizontal velocity is changing rapidly compare to time needed for density (or specific volume) to change so this term is negligible even without considering specific volumes ratio. $\left|u.\frac{\partial\vartheta'}{\partial x}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{\vartheta'}{\vartheta_0}$; Specific volumes ratio is small so this term is also negligible. $\left|v.\frac{\partial\vartheta'}{\partial y}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{VL_x\vartheta'}{UL_y\vartheta_0}$; Both length and wind velocity in different direction in horizontal surface have the same order but specific volumes ratio is still small. $\left|w.\frac{\partial\vartheta'}{\partial z}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{WL_x\vartheta'}{UL_z\vartheta_0}$; Although *W* is much smaller than *U* but L_x is larger than L_z with the same order by multiplying to specific volumes ratio it becomes small. $\left|\vartheta_0\frac{\partial v}{\partial y}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{VL_x}{UL_y}$; Both length and wind velocity in different direction in horizontal surface have the same order so this term will remain. $\left|\vartheta_0\frac{\partial w}{\partial z}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{WL_x}{UL_z}$; Although *W* is much smaller than *U* but L_x is larger than L_z with the same order so this term has the same order as horizontal velocity order. $\left|w.\frac{\partial\vartheta_0}{\partial z}\right|/\left|\vartheta_0\frac{\partial u}{\partial x}\right|\sim\frac{WL_x}{H_\vartheta U}$; L_x and H have same order but it's not the same for W and U. Finally continuity equation became: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$.

Conservation of momentum: Time required by x- component of horizontal velocity t_u and its equal to $\frac{L_x}{U}$ and it's the same for y- and z-component of velocity; $\frac{U}{t_u}\sim\frac{U^2}{L_x}\sim\frac{UV}{L_y}\sim\frac{UW}{L_z}$ and $\frac{V}{t_v}\sim\frac{UV}{L_x}\sim\frac{V^2}{L_y}\sim\frac{VW}{L_z}$ also for vertical component $\frac{W}{t_w}\sim\frac{WU}{L_x}\sim\frac{WV}{L_y}\sim\frac{W^2}{L_z}$. In addition, with respect to provided data, it can be concluded that pressure is almost constant and neglecting pressure terms in x- and y-component equations are acceptable; $\left|\frac{1}{\rho}\frac{\partial P}{\partial y}\right|\sim\left|\frac{1}{\rho}\frac{\partial P}{\partial x}\right|\sim 0$ but for z-component $\left|\frac{1}{\rho}\frac{\partial P}{\partial z}\right|\sim\frac{\Delta P}{\rho L_z}$. The Coriolis Effect is also negligible in all three momentum equations ($\Omega=2.77\times10^{-4}rev/s$). At last, three components of momentum conservation equation become: $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=0$, $\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=0$, $\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}-g$.

Conservation of heat: Temperature of urban areas is changing in vertical direction (Bornstein, 1968). Monthly and yearly data provided for temperature horizontal change and its standard deviation on ground indicated that temperature is varying rapidly with time or in another word t_θ is small in the order of a change in horizontal wind speed. When pressure change is insignificant variation of θ becomes similar to variation of temperature so $\frac{\theta}{t_\theta}\sim\frac{U\theta}{L_x}\sim\frac{V\theta}{L_y}\sim\frac{W\theta}{L_z}\gg|S_\theta|$. As a result potential temperature conserves following: $\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=0$.

Conservation of other contaminant in the atmosphere: Rate of production and generation of atmospheric aerosols and contaminants and even water is much less than rate of their dispersion by wind so their conservation equations simplified in the same manner as conservation of heat. $\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial x}+v\frac{\partial q}{\partial y}+w\frac{\partial q}{\partial z}=0$ Because $\frac{q}{t_q}\sim\frac{Uq}{L_x}\sim\frac{Vq}{L_y}\sim\frac{Wq}{L_z}\gg|S_q|$.

Summary and Conclusion

Simplification of basic fluid equations is the first step to provide a sensible boundary for an urban climate simulation problem. In this paper, we tried to provide a suitable framework for ongoing researches in this field. Current research is a proposition for climate simulation models of urban areas, which have large growth in size and remarkable population increase, and results can be applied for other purposes like avionic science as well. Basic conservation equations were converted to characteristic form, and scale analysis was performed based on desired data to calculate their values. Desired terms to be retained in the conservation of mass, momentum, were horizontal velocities terms, and in the conservation equations of heat and mass of other atmospheric contaminants was the rate of change of potential temperature and the mass of air contaminant, respectively. In order to evaluate the results of simplified equations, observational data for 14 different cities (Atlanta, Chicago, Dallas, Houston, Miami, New Orleans, New York, Los Angeles, Philadelphia, Phoenix, Washington DC, Toronto, Vancouver and Montreal), which are selected based on a parent project, were gathered. Observational data are provided by National Renewable Energy Laboratory (NREL) named Typical Meteorological Year (TMY3) for US cities and Environment Canada named Canadian Weather for Energy Calculations (CWEC) for Canadian cities on hourly basis. Data are analyzed statistically on monthly and yearly basis to find out mean, maximum, minimum, mode and standard variation of important parameters in governing equations; wind speed, temperature and pressure. Data showed that pressure and density changes are negligible for different urban areas and different time of a year, they both have maximum standard deviation occurred in Montreal with the values of no more than 0.06 kg/m3 and 8.66 mbar which with respect to mean value of 1.25 kg/m3 and 1010 mbar have less than 5% and 1% change, respectively. Wind and temperature were highly variable as expected by sunlight direction during days and its absence during nights. Montreal had the most unstable weather condition between those urban areas and it had maximum error for this simplification. Vancouver and New Orleans had lowest wind speed mode which made them more stable than other selected urban areas and less generated error. The most important result of this paper is negligible variation of pressure in horizontal surface. We verified it by simulation of Montreal, as the most unstable city, using WRF (version 3.3). Simulation also verified that the maximum variation of the hydrostatic pressure in the modeling domain is less than 1%. Although there is no preferred geometric shape exists for urban areas, scale of interest was assumed to be less than a square with 30km×30km sides which applies for most of metropolitan around the world. Resulted equations are simplest form that should be treated for parameterization before using numerical methods for climate simulation. Final resulted equations are as follows:

Continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

X-momentum: $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=0$

Y-momentum: $\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=0$

Z-momentum: $\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}-g$

Heat: $\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=0$

Continuity of contaminants: $\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial x}+v\frac{\partial q}{\partial y}+w\frac{\partial q}{\partial z}=0$

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Concordia University

Ali.gholizade@gmail.com

Http://user.encs.concordia.ca/~al_gho