# RELATIONSHIP OF GRAUPEL SHAPE TO DIFFERENTIAL REFLECTIVITY: THEORY AND OBSERVATIONS

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## 1. INTRODUCTION

Graupel are the most important ice phase radar targets in cumulonimbus clouds, but are seldom observed at the ground, melting before they get to the surface in summer conditions (Rasmussen and Heymsfield, 1987). The formation of graupel requires the presence of a moderate updraft and supercooled water, so that riming can occur. These conditions are also favorable for icing, which is a threat for aviation. The ability to identify graupel with dual polarization radar would help detect potential icing regions inside a storm.

Graupel comes in a wide variety of shapes, density and sizes, which makes it very difficult to identify with polarimetric radar. The particular case of conical graupel may produce a negative differential reflectivity ($Z_{dr}$), depending on the geometry and fall modes, a condition easily identified since most hydrometeors, both liquid and solid, have positive $Z_{dr}$ (Straka *et al*., 2000; Dolan and Rutledge, 2009). Conditions favorable to the formation of conical graupel are scattered, unorganized convection and locally barotropic conditions that create untilted updrafts.

The frequently observed large areas of slight negative $Z_{dr}$ above the melting layer might indicate that graupel is more prevalent than previously believed in non-winter conditions.

One of the mechanisms proposed for the origin of conical graupel is described in Knight and Knight (1973). They propose a mechanism where planar snow crystals start accreting supercooled drops on the edges of the downward facing side. As the riming continues they start to form cone-like shapes, attached to the crystal. Eventually the crystal breaks. The rimed cones may continue to grow after breakage. Hallett (1965) made detailed observations of conical hailstones that grow from a conical graupel core. The particle structure described is similar to those observed in Knight and Knight (1973), further supporting their theory for conical graupel origin.

In-situ growth conditions play an important role in the geometric shape of the conical graupel, with the cone axis stably aligned with gravity, as shown in the wind tunnel experiments from Cober and List (1993). In particular, the apex angle of the cone ($\alpha$), depends on the temperature, droplet sizes, and updraft vs. fall velocity strength.

The aim of this study is to understand how we can derive the environmental conditions favorable for the growth of conical graupel and/or characterize the supercooled liquid water from dual polarization radar observations.

## 2. METEOROLOGICAL CONDITIONS FOR TWO DISTINCT GRAUPEL EVENTS

Two events where conical graupel was observed on the ground are documented in this paper. The first event occurred on November 10, 2011 in Chesterton, IN, and was observed by the Valparaiso University dual polarization C-band Doppler radar, located about 15 km to the south. The second event took place on April 12, 2012 in Lexington, MA, and was observed by the NEXRAD KBOX dual


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polarization S-band Doppler radar. It is noteworthy that both events occurred in transition seasons between summer and winter, when the height of the melting layer is substantially lower than typical summertime conditions affording graupel the opportunity to reach the ground.

### 2.1 November 10, 2011, near Valparaiso, IN

A shower of conical graupel occurred on the 10$^{th}$ of November in Chesterton IN, around 2330 UTC. Figure 1 shows a picture of a small sample of the observed graupel taken immediately after arrival at the ground. The cone apex angles were measured from this picture by drawing long lines parallel with the edges, and then using a protractor to measure the angles. The results were:

- 1x 40°
- 1x 52°
- 1x 60°
- 2x 50°
- 1x 56°
- 1x 70°.

The surface analysis (figure 2) shows low pressure centered over James Bay, Canada, and a trough axis extending from the center of the depression to the south of Lake Michigan. The low is also visible in the 500 mb analysis (figure 3), as well as the trough. Scattered convection was occurring in Northern Indiana, with mixed precipitation and snow observed at the surface. As shown in figure 4, the surface temperature was just above freezing, and the freezing level was very low (about 600 m ASL, according to the sounding), which allowed for the survival of graupel from its location of growth to the surface.

Figure1. Picture of the conical graupel observed at Chesterton, IN on November 10, 2011.

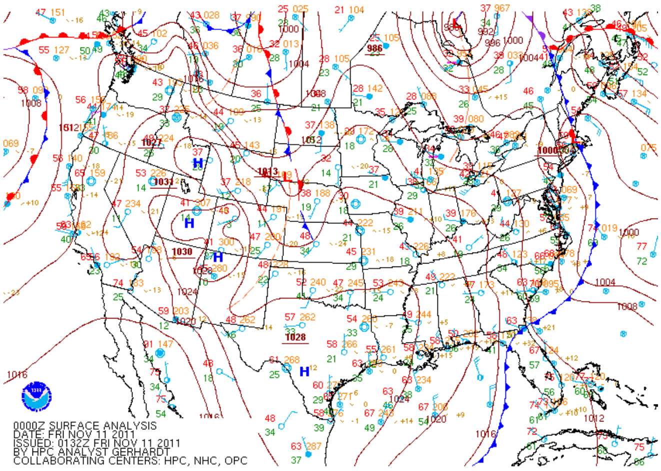


Figure 2. Surface analysis on November 11, 2011 at 0000 UTC.

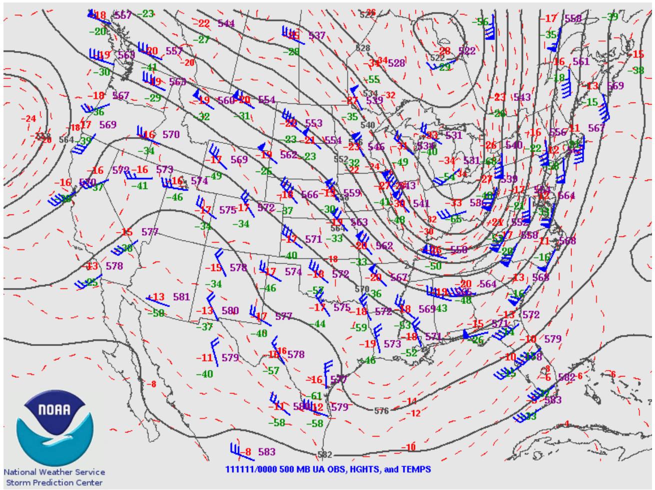


Figure 3. 500 mb analysis on November 11, 2011 at 0000 UTC.

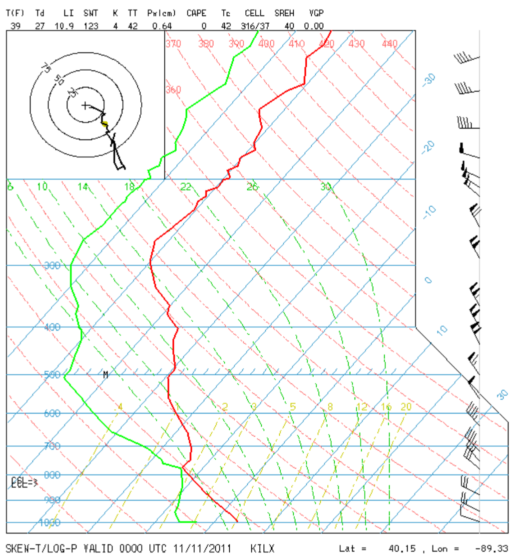


Figure 4. KILX sounding on November 11, 2011 0000 UTC.

## 2.2 April 12, 2012, Lexington, MA

The first of two conical graupel showers observed in Lexington, MA occurred shortly before local noon on April 12, 2012. Figure 5 shows a collection of the graupel gathered rapidly just after falling. The graupel were of a nominal 8 mm diameter equivalent. A review of the surface analysis (figure 6) indicates a low pressure system exiting the northeast into the Atlantic with a surface trough extending back toward the eastern Great Lakes. At 500 mb (figure 7), the attendant upper level low is centered at the Atlantic coast in the northeast.

The resultant weather in Lexington, MA that day was similar to that observed further to the west in the prior two days as this surface and 500 mb system moved east. That is, the cold-core, cut-off low pressure system triggered low-topped convective cells once sufficient surface heating occurred. Figure 8 is the sounding from Chatham, MA for that morning. Note a surface temperature above freezing with a melting level of around 1200 m ASL.

Figure 9 is a visible satellite image of the northeast that represents the type of convective cells that developed on April 12 as well as the days prior when the system was to the west. All three days included reports of small hail from some subset of these cells. The reports do not distinguish the shape of the small hail but it is suggested here that conical graupel was in fact the small hail.

Figure 5. Picture of the conical graupel collected at Lexington, MA on April 12, 2012.

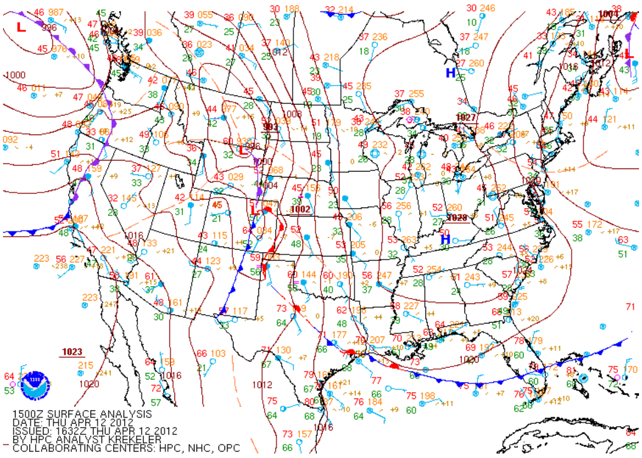


Figure 6. Surface analysis on April 12, 2012 at 1500 UTC.

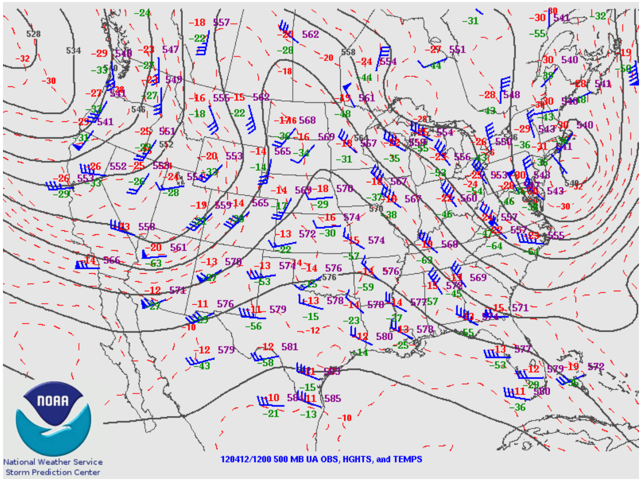


Figure 7. 500 mb analysis on April 12, 2012 at 1200 UTC.

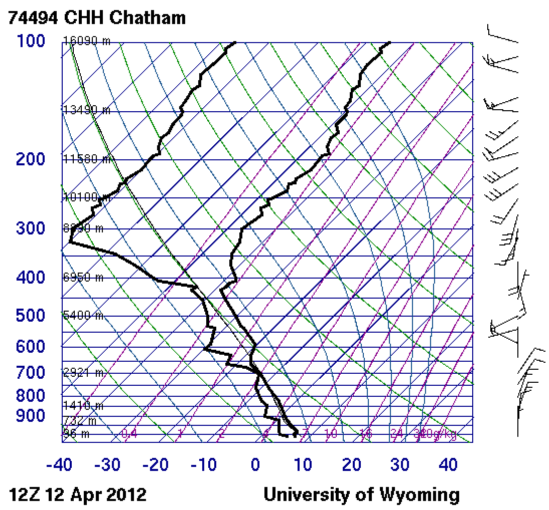


Figure 8. CHH sounding on April 12, 2012, 1200 UTC.

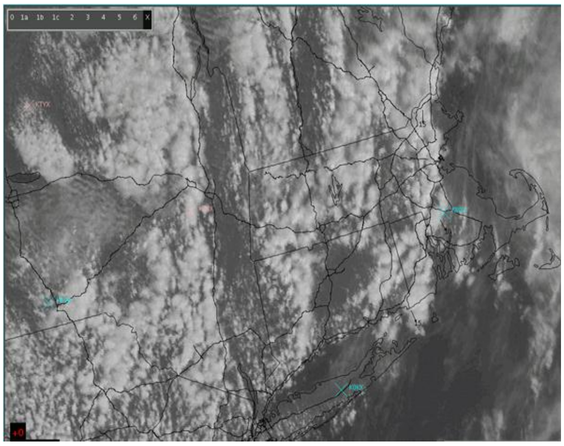

Figure 9. Low topped convective cells producing graupel in Lexington, MA on April 12, 2012.

## 3. $Z_{dr}$ CALCULATIONS FOR IDEALIZED SHAPES

In this section $Z_{dr}$ calculations at 3 GHz are shown for 3 different idealized shapes for conical graupel (0.5 cm overall height): 1) a cone with flat base, shown in Figure 10b, 2) a cone with spherical surface whose center is the cone apex, figure 11b and 3) a cone with hemispherical base, figure 12b. These calculations are intended to explore improving resultant $Z_{dr}$ estimates over earlier approximate backscatter models from Aydin and Seliga (1984a) and Aydin and Seliga (1984b).

The results of the backscatter properties of dielectric cones (dielectric constant 2.0) shown below (Figures 10a, 11a, and 12a) were made using the electromagnetic simulation software FEKO (details in http://www.feko.info/).

The plots in figures 10, 11, and 12 show the $Z_{dr}$ dependence on the cone apex angle for each of the models. It is seen that $Z_{dr}$ is strongly dependent on the angle, exhibiting negative $Z_{dr}$ at small angles, changing sign at a critical angle depending on the shape model, and positive for larger angles. The direction of the incident electromagnetic field is assumed perpendicular to the vertical axis of the cone in all cases. For model 1, the flat base cone, the $Z_{dr}$ sign change occurs at an angle of approximately 49° (figure 10). For the model with curved base, the change is around 50° (figure 11), and for the model with hemispherical base, 74° (figure 12).

## 4. OBSERVED $Z_h$ AND $Z_{dr}$

In this section the radar data are analyzed around the areas where the graupel observations took place, focusing on horizontal and differential reflectivities, $Z_h$ and $Z_{dr}$.

### 4.1 November 10, 2011, Chesterton, IN

Figure 13 shows the location of the graupel observation relative to the radar. The distance is about 15 km. The radar images in figures 14 and 15, are showing PPI's of $Z_h$ and $Z_{dr}$ at a 1.5° elevation. In order to better see the details, the images are zoomed. The radar and the graupel observation are marked with an X and a + respectively for reference.

Reflectivity shows scattered convection around the location where the graupel was observed. $Z_h$ reaches relatively high values, between 20 and 30 dBZ, consistent with the presence of graupel. Negative $Z_{dr}$ values (between -1 and 0 dB) are seen roughly at the same location.

To get a better idea of the actual radar values, plots in the $Z_h$-$Z_{dr}$ space were created of the data from the radar range bins vertically and azimuthally surrounding the location of the graupel. This allows the visualization of both variables simultaneously for each gate.

The points used are shown in figure 16. We use the first 6 tilts (figure 16a), and 9 points per tilt (figure 16b), for a total of 54 points. The result is shown in figure 17. The contours represent the number of points in a specific $Z_h$-$Z_{dr}$ range. $Z_h$ is concentrated between 10 and 32 dBZ, while $Z_{dr}$ varies between -1 and 1 dB. There is a distinct maximum with reflectivity centered at 24 dBZ and differential reflectivity at -0.3 dB. The observed range of $Z_h$ and $Z_{dr}$ values are consistent with the presence of graupel, as indicated in previous studies like Straka *et al*. (2000) or Dolan and Rutledge (2009).

## Shape Model 1: Flat base

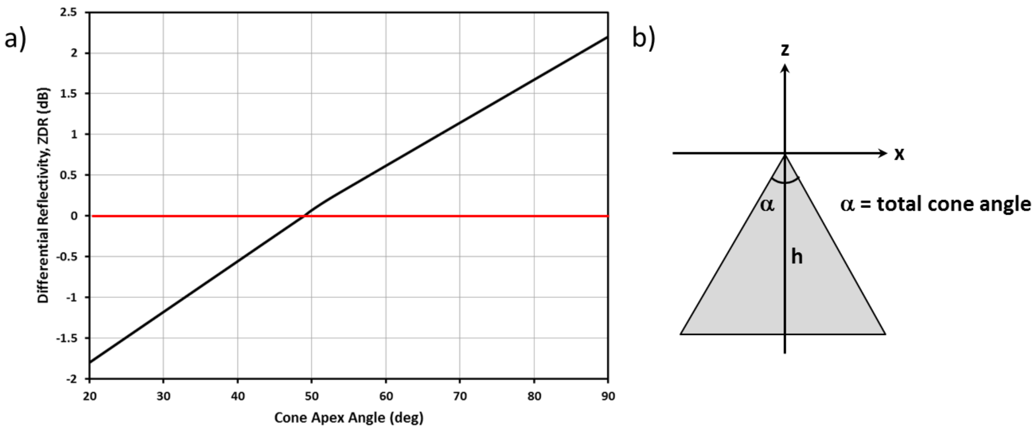


Figure 10. a) $Z_{dr}$ as a function of the apex angle for the flat base model. Calculations for 3 GHz electromagnetic frequency and a dielectric constant of 2 b) Schematic shape of the conical graupel with flat base.

## Shape Model 2: Curved base

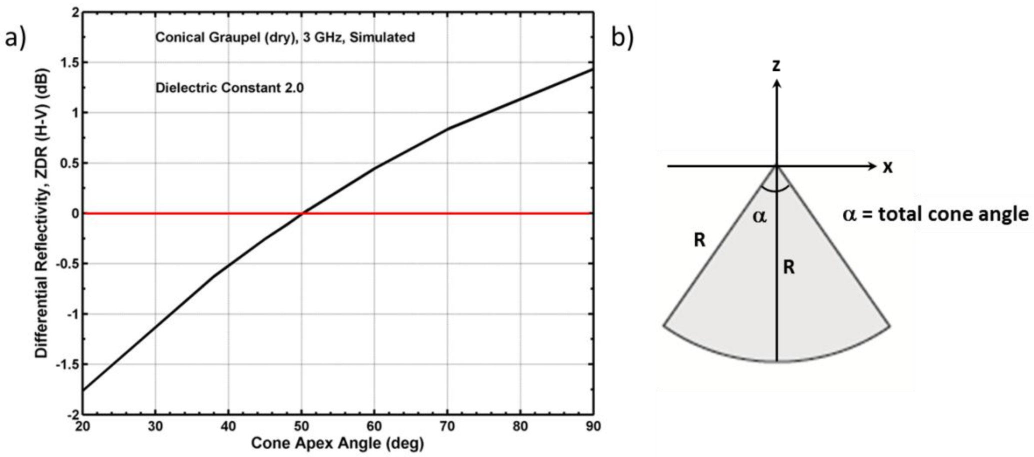


Figure 11. a) $Z_{dr}$ as a function of the apex angle for the curved base model. Calculation details are the same as figure 10. b) Schematic shape of the conical graupel with curved base.

## Shape Model 3: Hemispherical base

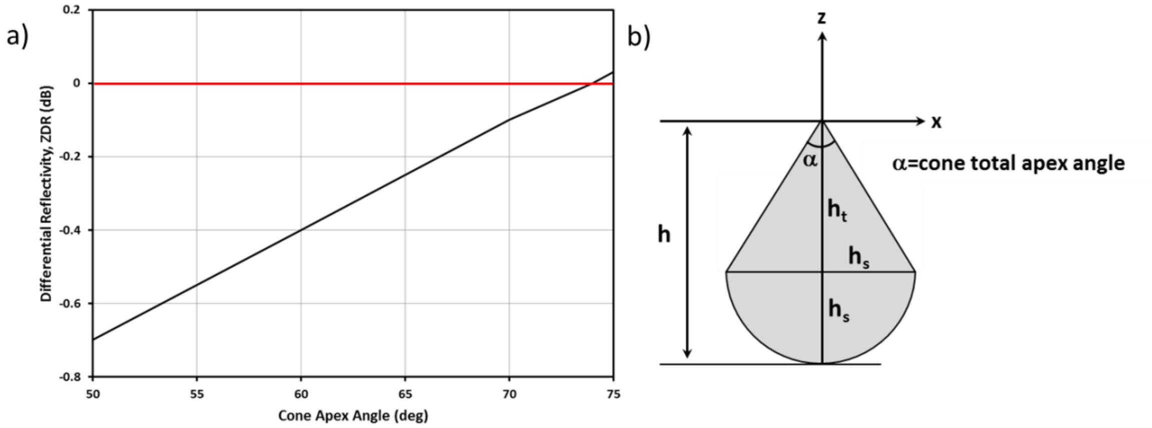


Figure 12. a) $Z_{dr}$ as a function of the apex angle for the hemispherical base model. Calculation details are the same as figure 10. b) Schematic shape of the conical graupel with hemispherical base.

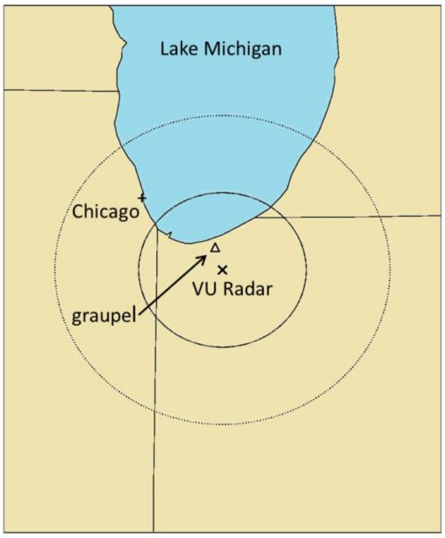


Figure 13. Map showing the location of the VU radar (X) in Valparaiso, IN, and the place where graupel was observed on the ground (triangle). The circles represent the 50 km and 100 km range. The distance between the two locations is about 15 km.

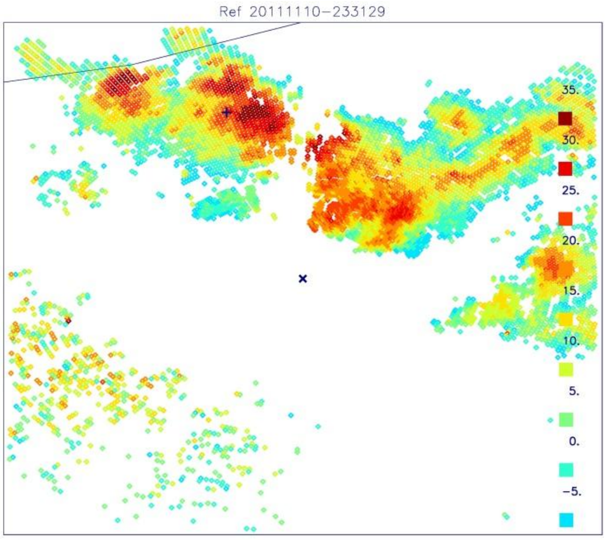


Figure14. Reflectivity $Z_h$ in dBZ on November 10, at 2331 UTC, the time the graupel was observed. The + sign to the northwest depicts the location of the graupel. The radar scan elevation is 1.5°.

From the graupel pictured in figure 1, it seems like the curved base cone model 2 in figure 11b is closer to that observed. As shown with the calculations in section 3, figure 11a, $Z_{dr}$ should change sign from negative to positive around an apex angle of 50°. The observed $Z_{dr}$ is mainly between -0.5 and -0.1 dB, which should correspond to an apex angle between 40° and 50°. However, most of the angles measured are larger than 50°. This could be partially due to melting of the particles occurring preferentially at the apex of the cone resulting in an increase in the apparent angle, and/or due to a chipping of the tip upon the impact when landing. In any case, we believe that the sample measured is too small and may not be representative of the whole set of particles.

$Z_{dr}$ for the Valparaiso University radar is calibrated frequently by pointing vertically in rain events, and this had been performed just a few days prior to this event.

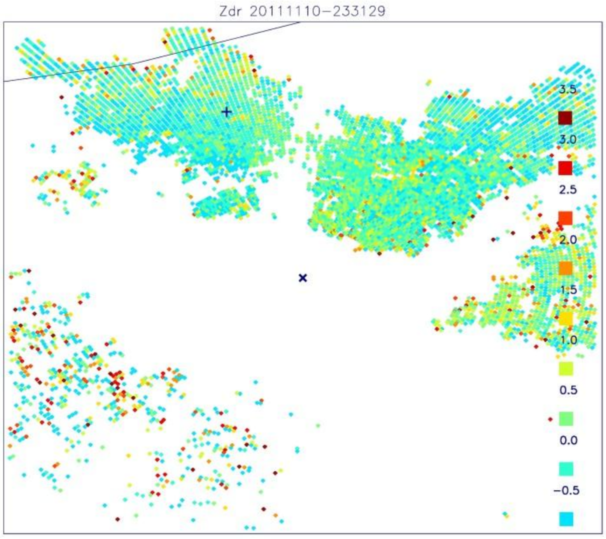


Figure 15. Same as figure 14, but for $Z_{dr}$, in dB.

**4.2 April 12, 2012, Lexington, MA**

The PPI of the KLOT reflectivity at the time of the graupel shower (April 12, 2012, 1557 UTC) is shown in figure 18. The red sector includes the high reflectivity graupel core (observed at Lincoln Laboratory) and areas further north with presumed graupel. Figure 19 shows the same PPI but for $Z_{dr}$. The stronger $Z_h$ values are between 30-40 dBZ, while $Z_{dr}$ varies between -1 and 0.5 dB. As with the Indiana case, these values are consistent with the presence of graupel.

Figure 20 is a scatterplot of Zh vs. Zdr, each of the points representing a radar bin inside the red sector in figures 18 and 19. There is a substantial amount of points with negative Zdr supporting the evidence that conical graupel is responsible for that signature.

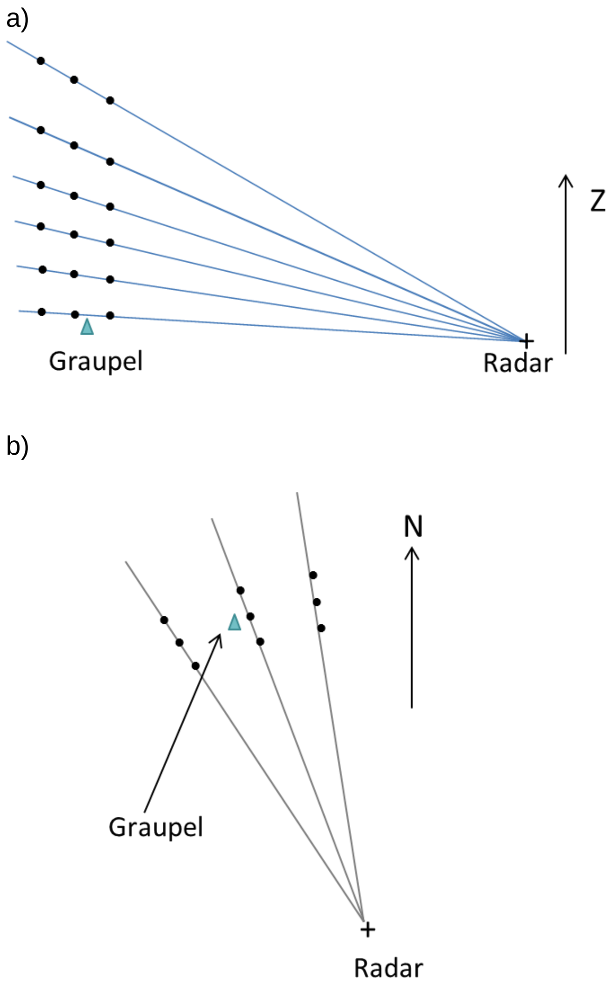


Figure16. Scheme showing the points used to build the plot in figure 17 a) RHI showing all the tilts used; the highest elevation is 6.2°; b) view from above (multiple azimuths).

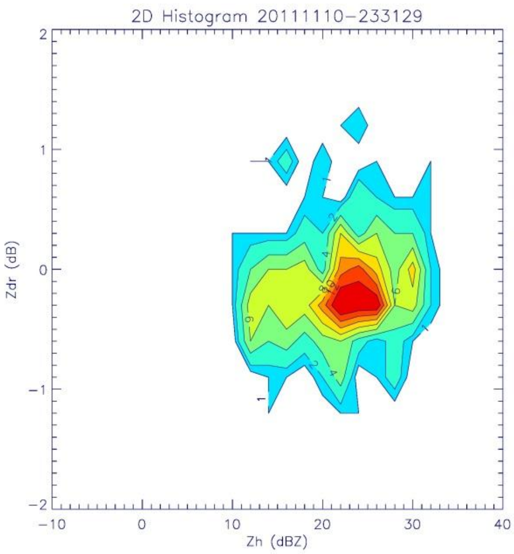


Figure 17. Distribution of the radar points in the $Z_h$-$Z_{dr}$ space at 2331 UTC on November 10, 2011.

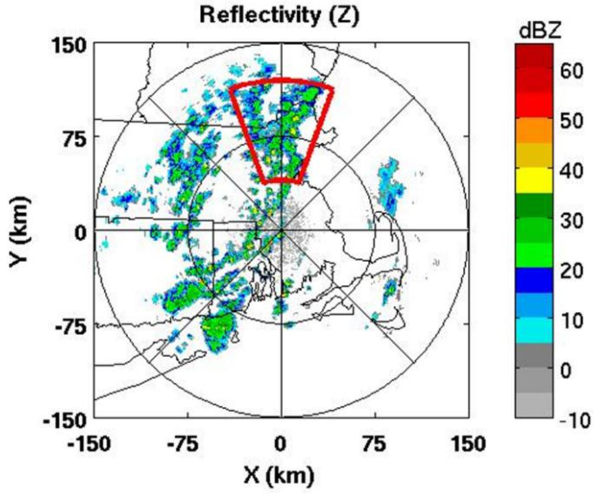


Figure 18. Reflectivity $Z_h$ on April 12, at 1557 UTC, the time the graupel was observed at MIT Lincoln Laboratory. The sector in red shows the area where the conical graupel was observed. The radar scan elevation is 1.5°.

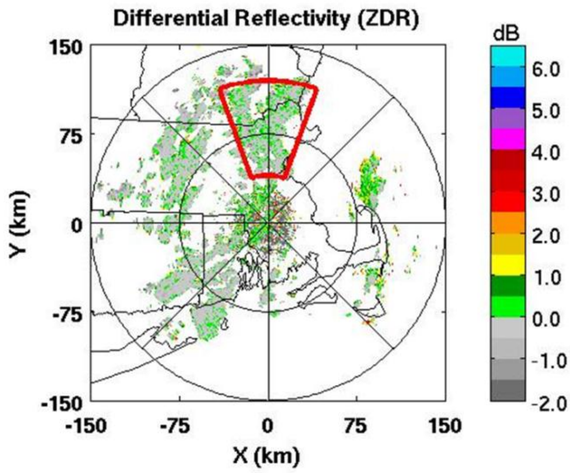


Figure 19. Same as figure 18, but for $Z_{dr}$.

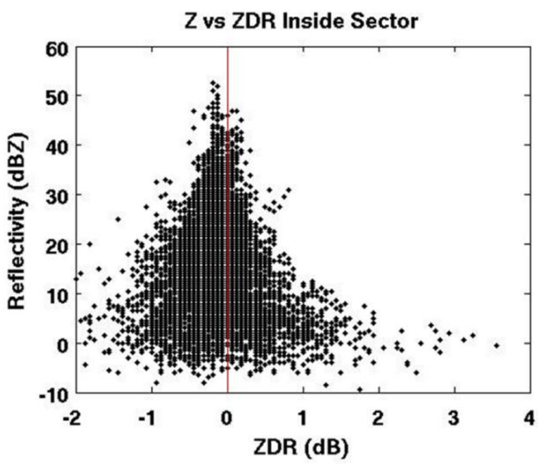


Figure 20.Scatterplot of $Z_h$ as a function of $Z_{dr}$ for the area inside the red sector.

## 5. DISCUSSION

Radar observations for the two cases presented show a clear negative $Z_{dr}$, combined with relatively high $Z_h$. The combined ranges of observed $Z_h$ and $Z_{dr}$ suggest the presence of graupel particles with small apex angles, in agreement with the observations of conical graupel at the ground.

In general, the only hydrometeor types that produce a negative $Z_{dr}$ are either conical graupel with small apex angles, very large and irregularly shaped hail (Straka *et al*., 2000), or vertically aligned ice crystals (example: Dolan and Rutledge, 2009), the latter only occurring under the influence of a strong electric field (Foster and Hallett, 2008). The presence of graupel, however, may indicate a modest electric field capable of aligning the crystals in the vertical. If one accepts presence of vertically aligned crystals, they could be a contributor to the total negative $Z_{dr}$ signature. However, in both cases presented here, that negative $Z_{dr}$ signature is certainly produced by conical graupel given the associated strong reflectivity.

One other thing worth mentioning is that $Z_{dr}$ will be negative only if it falls steadily with either apex or base down. List and Schemenauer (1971) made laboratory measurements of the oscillations of different conical graupel shapes for varying Reynolds numbers. For sufficiently small particles (typical Reynolds number lower then 200-500, depending on the shapes), all the shapes fall steadily. Once sizes increase, and the corresponding Reynolds number increases to 200-800, particles start oscillating. Eventually, at larger Reynolds numbers, particles will start to tumble. In such cases $Z_{dr}$ should be close to 0 dB.

Because graupel is by definition a rimed particle, the region where graupel grows is a region where there is supercooled water and thus presents a potential icing hazard to aviation. The ultimate goal is to determine or infer icing severity from the information provided in part by $Z_{dr}$. Cober and List (1993) have made experiments in wind tunnels and observed that the apex angle in conical graupel increases with increasing airflow velocity and droplet median volume diameter (all other conditions held fixed), and decreases as the in-situ temperature increases. With varying realistic conditions of temperature, droplet sizes, and fall speeds, they obtained a range of apex angles from 30° to 70°. Our calculations for the preferred model (curved base model, figure 11) indicate a $Z_{dr}$ of -1.2 dB at 30° and +0.8 dB at 70°, but with a mean that is definitely negative, consistent with the measurements.

Since the apex angles are a function of several conditions that play a role in icing, the detection of negative $Z_{dr}$ due to the presence of conical graupel should give an indirect assessment of icing conditions. In order to obtain a degree of icing severity from the shape of the conical graupel, more research is needed, and more cases analyzed, along with PIREPs (pilot reports) and thermodynamic data.

**Acknowledgements**

This work was sponsored by the Federal Aviation Administration under Air Force Contract No. FA8721-05-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the United States Government.

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