Aspects of potential vorticity:

impermeability

coordinates

fluxes

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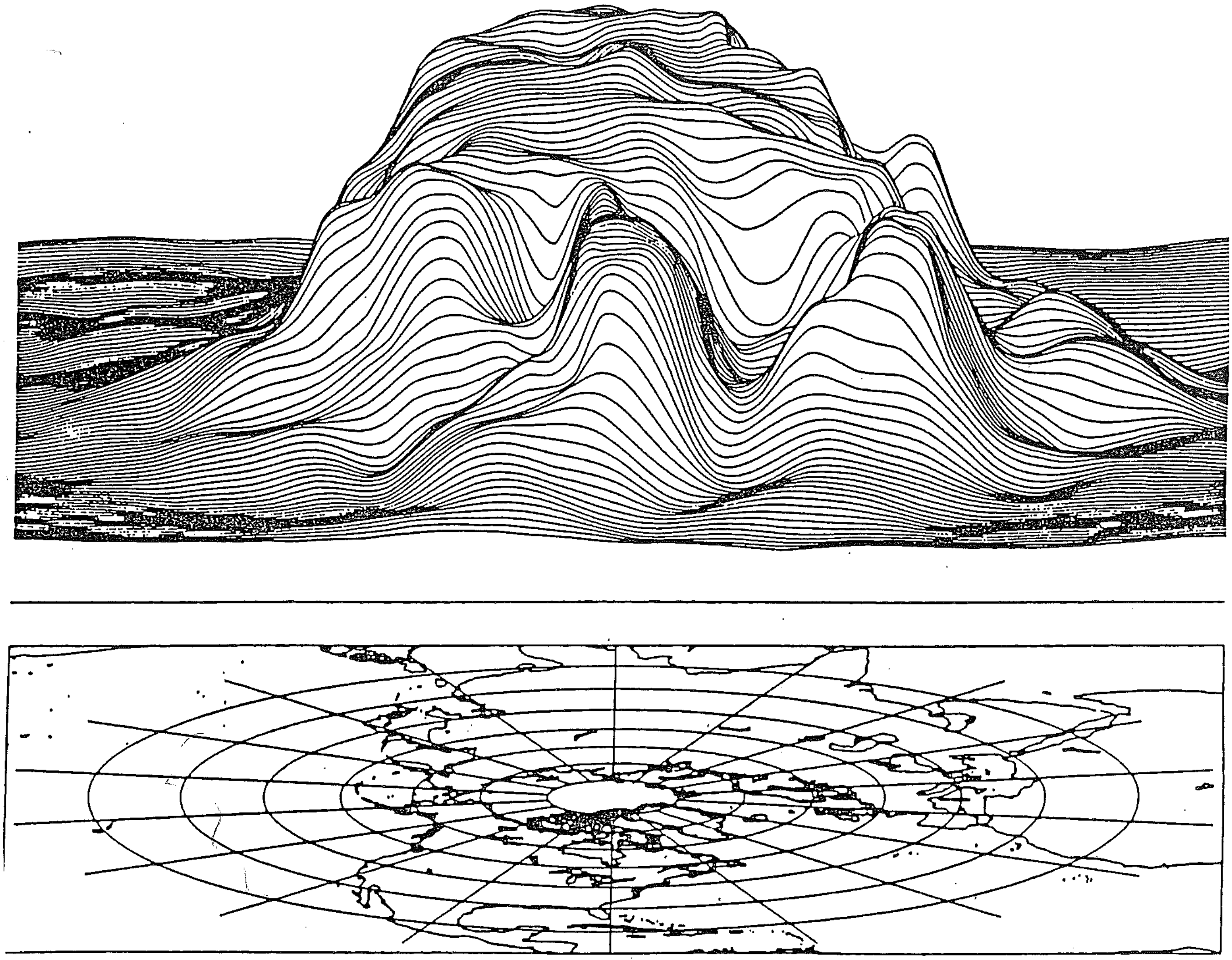


FIG. 1. Three-dimensional pattern that illustrates the topography of the 313 K isentropic surface over the Northern Hemisphere as viewed from 80° W longitude, 1200 GMT, 12 November 1977. (After Nagle, 1979.) · Johnson 1989

There can be no net transport of Rossby-Ertel potential vorticity across any isentropic surface

Haynes and McIntyre 1987

$$PV = \underset{\sim}{\omega}_a \cdot \nabla \chi / \rho$$

potential vorticity

$$P = \underset{\sim}{\omega}_a \cdot \nabla \chi = \nabla \cdot (\underset{\sim}{\omega}_a \chi)$$

mass weighted PV

Examples

$$\chi = \Theta ; \qquad P = \underset{\sim}{\omega}_a \cdot \nabla \Theta$$

$$\chi = p ; \qquad P = \underset{\sim}{\omega}_a \cdot \nabla p$$

$$\chi = z ; \qquad P = \zeta + 2\Omega \sin \varphi$$

Ertel (1947)

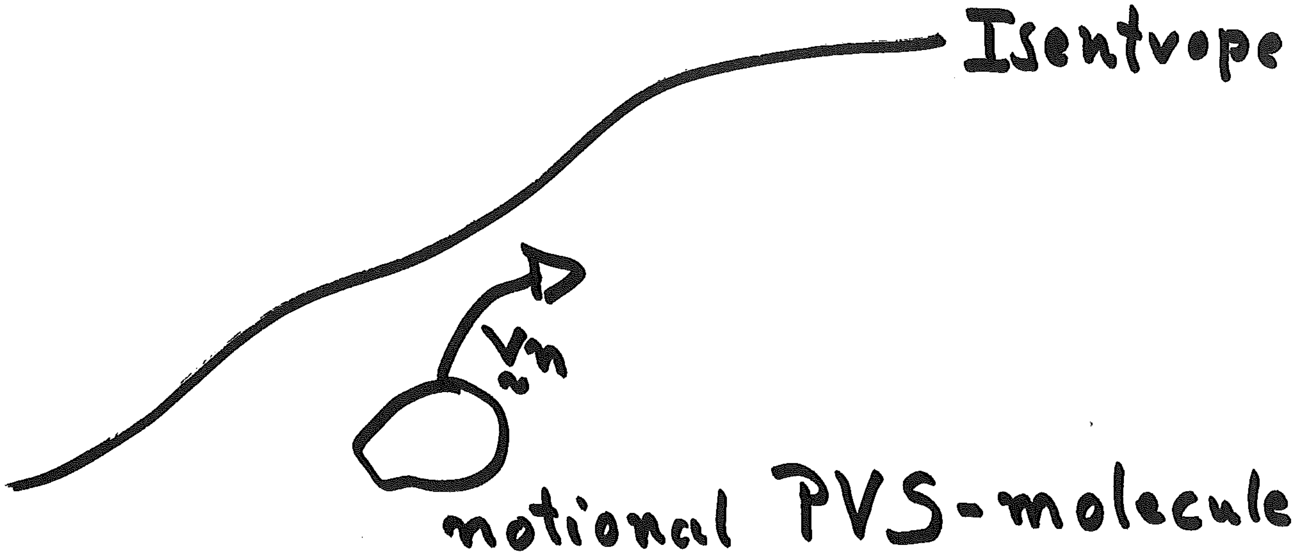
$$\frac{\partial P}{\partial t} = - \underset{\sim}{\omega}_a \cdot \nabla \dot{\chi} - \nabla \chi \cdot \left(\nabla \frac{1}{\rho} \times \nabla p \right)$$
$$- \nabla \cdot (\underset{\sim}{v} P) + \nabla \chi \cdot (\nabla \times F)$$

$$\frac{\partial P}{\partial t} = - \nabla \cdot \underset{\sim}{j}$$

$\underset{\sim}{j}$?

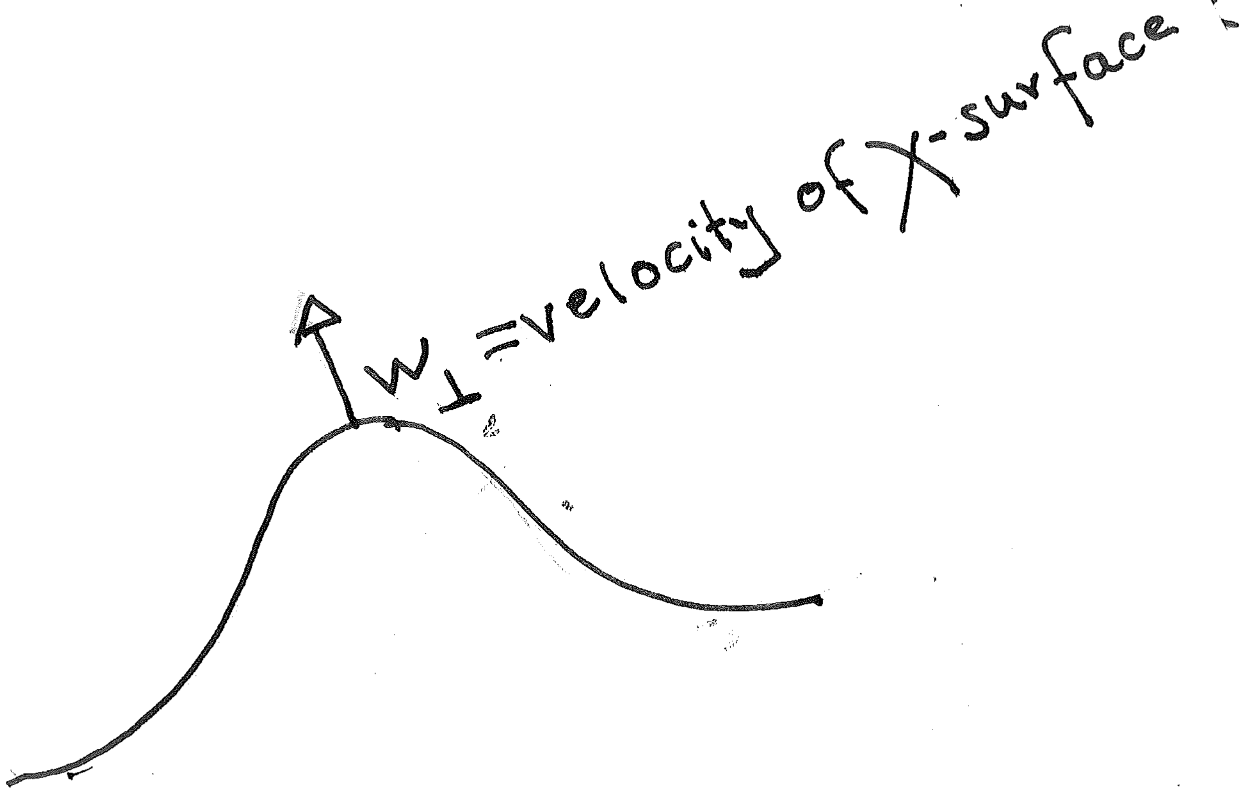
notional velocity

$$\underset{\sim}{v}_n = \underset{\sim}{j}/\rho$$



HM 90: The 'notional velocity' can be pictured as the velocity with which PVS-molecules would move'

no net transport?



Impermeability:

$$V_{n\perp} = W_\perp$$

S: ~~isosurface~~

the flux naturally takes a form such that it represents always zero transport across moving surfaces S

Of course, one can always make the surfaces S look permeable by adding an identically non divergent vector field to the flux. But that is arguably a needless complication

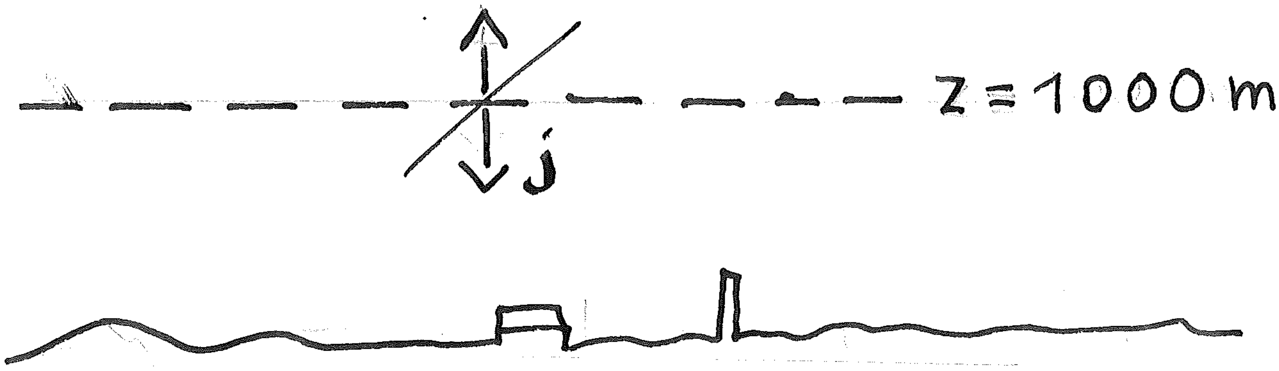
Mc Intyre: Encyclopedia

Danielsen (1990): 'In defense of Ertel's PV ...'

$$PV \Longleftrightarrow P$$

Viudez 1999: IT assigns to fictitious P. particles a suitable velocity

$$\chi = z \; ; \; P = \zeta + f$$



no net transport
of vorticity across a
constant height surface

Example:

homogeneous f-plane flow;

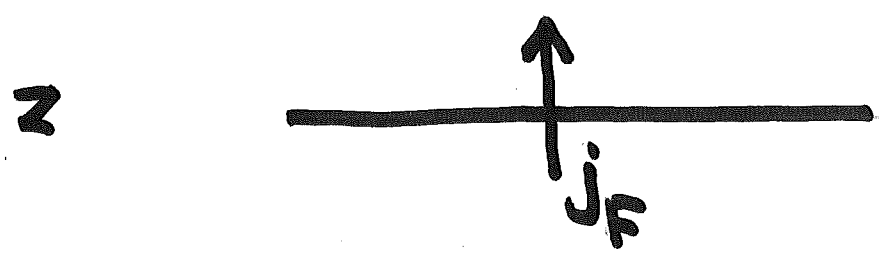
$\chi = z$; $P = \zeta + f$

$$\frac{\partial \zeta}{\partial t} + \ldots\ldots = K \frac{\partial^2 \zeta}{\partial z^2}$$

$$\downarrow$$

$$-\nabla \cdot (j_F)$$

$$j_F = -\left(0, 0, K \frac{\partial \zeta}{\partial z}\right)$$

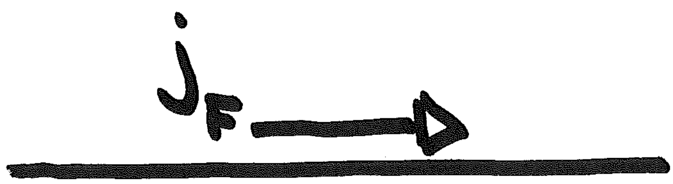


Permeability

$$-\nabla\cdot j_F = K\frac{\partial^2 \zeta}{\partial z^2} = K\frac{\partial^2}{\partial z^2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$j_F = -K\left(\frac{\partial^2 v}{\partial z^2}, -\frac{\partial^2 u}{\partial z^2}, 0\right)$$

z:



Impermeability

Is there a P-flux at all?

Bretherton and Schär (1993)

$$j_F^i = M F$$

no derivatives

Now
$$j_F^i = e_3 \times F$$

but

$$j_F^p = K \nabla_2 \times \left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}, 0 \right)$$

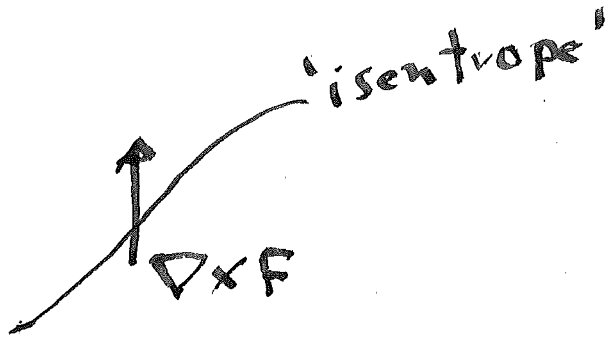
There are many equivalent forms of the flux

Friction

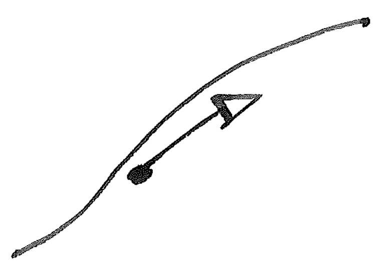
$$\nabla\chi\cdot(\nabla\times F) = \nabla\cdot(\chi\nabla\times F)$$

$$= \nabla\cdot(F\times\nabla\chi)$$

Permeable: $j_F = -\chi\nabla\times F$



Impermeable: $j_F = -F\times\nabla\chi$



$$\underset{\sim}{\omega}_a \cdot \nabla \dot{\chi} = \nabla \cdot (\dot{\chi} \underset{\sim}{\omega}_a) = \nabla \cdot (\underset{\sim}{v}_a \times \nabla \dot{\chi})$$

'Impermeable'

Permeable

Permeability

$$\nabla\chi\cdot\left(\nabla\frac{1}{\rho}\times\nabla \underset{\sim}{F}\right) = \nabla\cdot\left(\chi\left(\nabla\frac{1}{\rho}\times\nabla p\right)\right)$$

$$= -\nabla\cdot\left(p\left(\nabla\chi\times\nabla\frac{1}{\rho}\right)\right)$$

Impermeability

$\chi = \theta$

$\chi = p$

4 choices; 1 imperm, 3 perm

$\chi = z$ 12 choices; 2 imp, 10 perm

Conclusion

There are many equivalent forms of the P-flux. Some satisfy the impermeability theorem the majority does not

Electric analogy

(e.g. Schneider et al 2002)

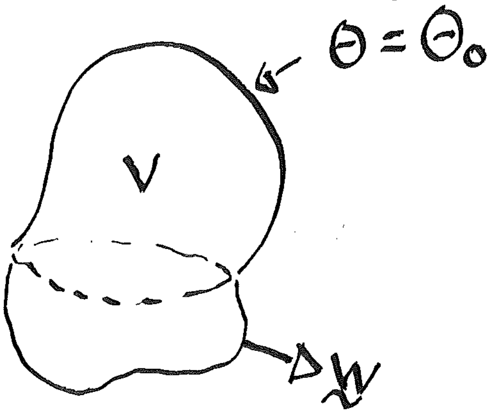
$P \;\hat{=}\;$ electric charge density

$j \;\hat{=}\;$ electric current density

$$\left.\begin{matrix}\omega_a \Theta \\ v_a \times \nabla \Theta\end{matrix}\right. \;\hat{=}\; \text{electric displacement}$$

There is no P-flux but many vectors j

$$\frac{d}{dt}\int_V P\,dv = \int_S (-\underset{\sim}{j} + P\underset{\sim}{w})\cdot n\,ds$$



$$\int P\,dv = \int_S \omega_a \Theta \cdot n\,ds$$

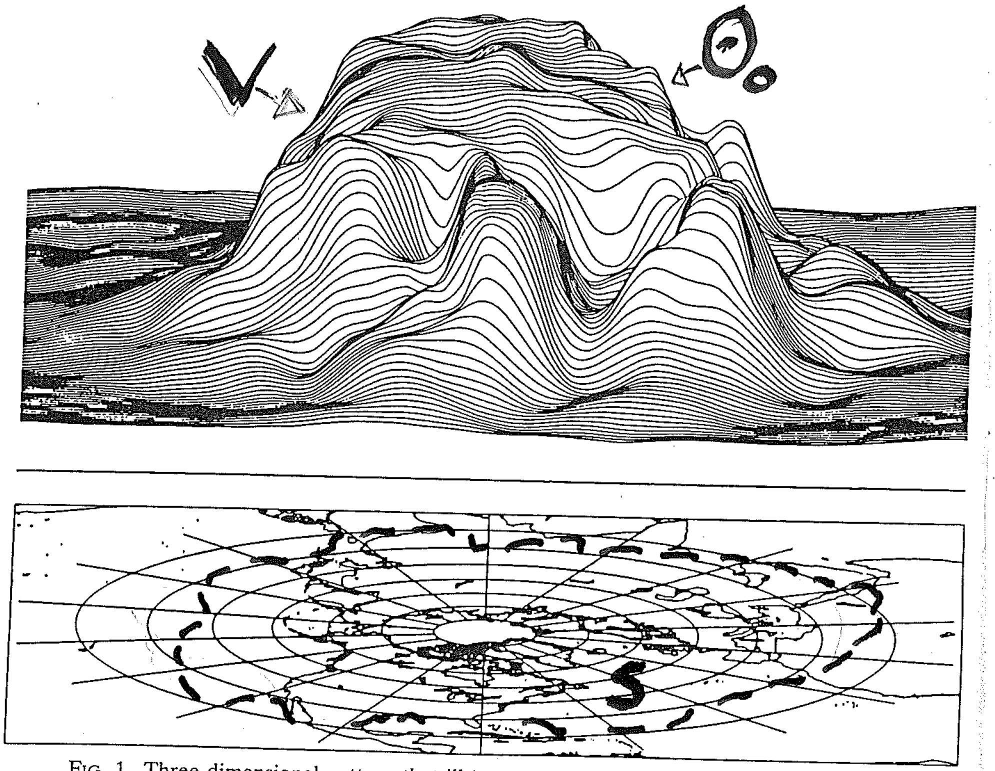
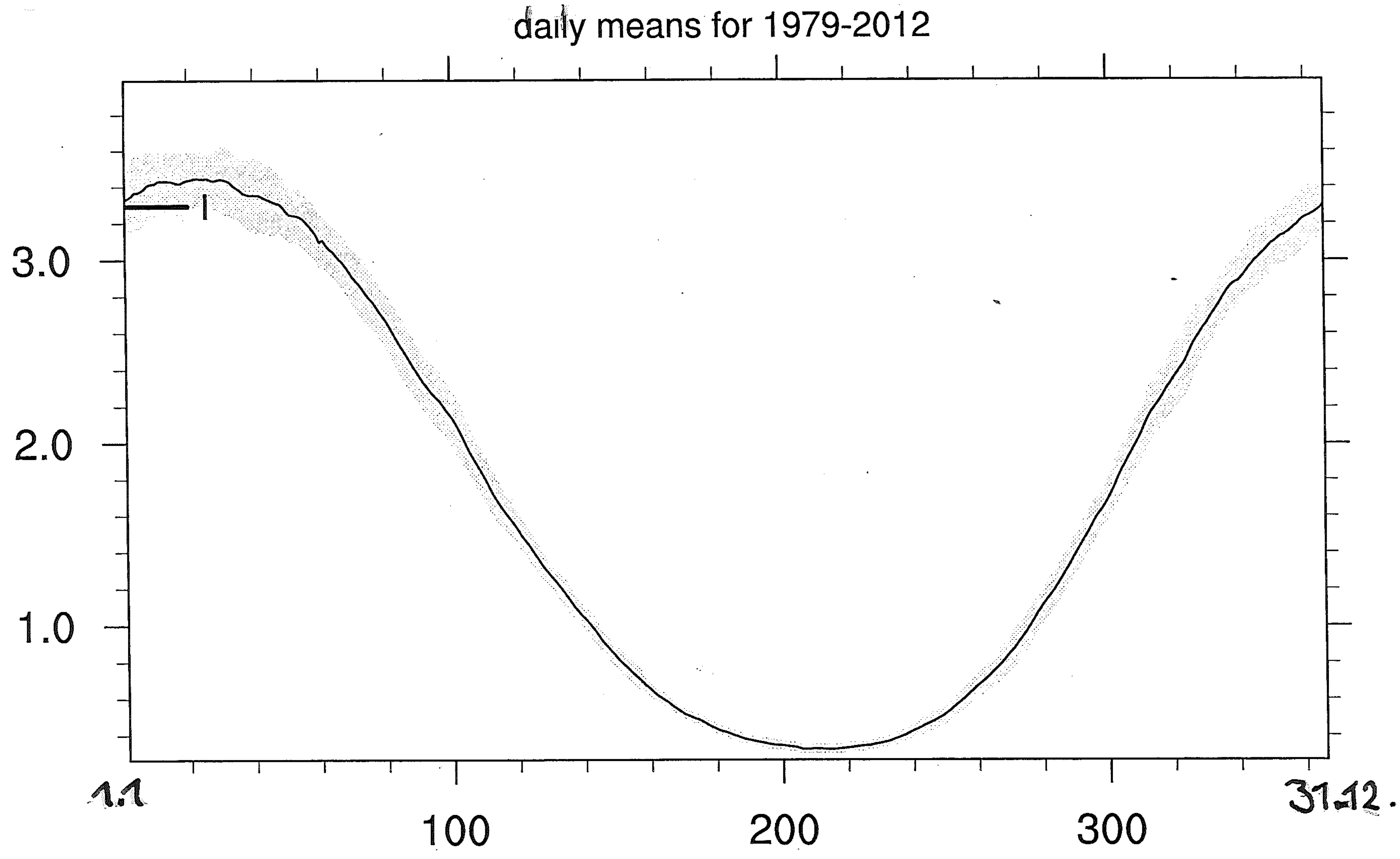


FIG. 1. Three-dimensional pattern that illustrates the topography of the 313 K isentropic surface over the Northern Hemisphere as viewed from 80° W longitude, 1200 GMT, 12 November 1977. (After Nagle, 1979.)

$$\int P\,dv = \int_S (\Theta_o - \Theta_s)(\zeta_s + 2\Omega \sin\varphi)\,ds$$

$$\frac{d}{dt}\int P\,dv = \int_S -(\zeta_s + 2\Omega \sin\varphi)\dot{\Theta}_s + e_3\cdot(\nabla\Theta_s \times F)\,ds$$

$\int P\,dv\ ;\ \Theta_s < 285\,K = \Theta_0 ;\ NH$



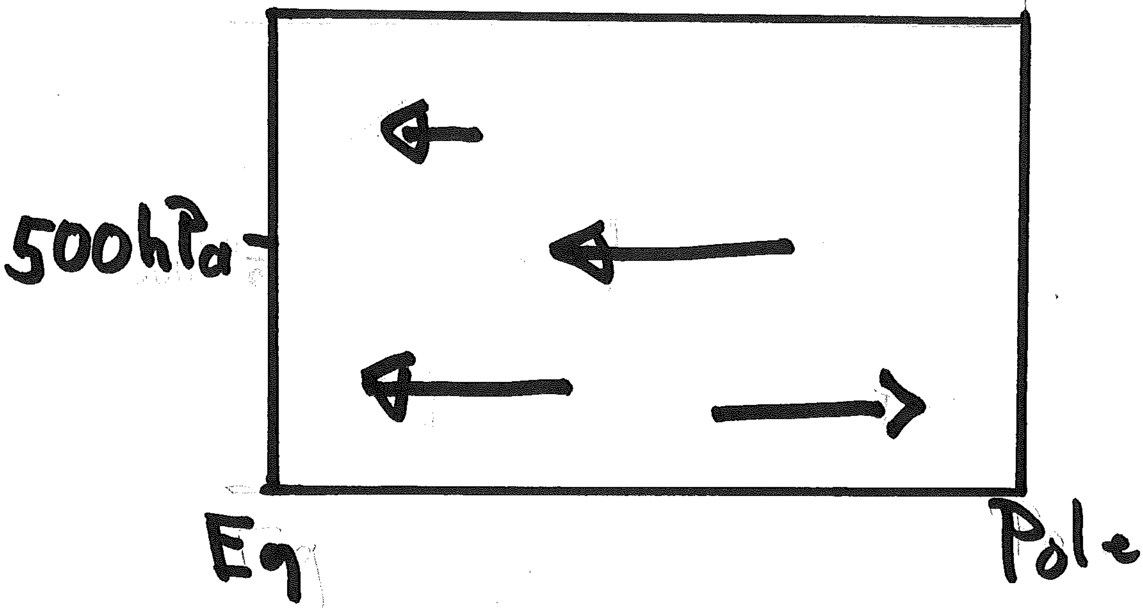
There is no net P-flux

but there is an advective P-flux

$\underset{\sim}{v} P$

Advective fluxes of PV

quasigeostrophic $\overline{v'q'_g}$

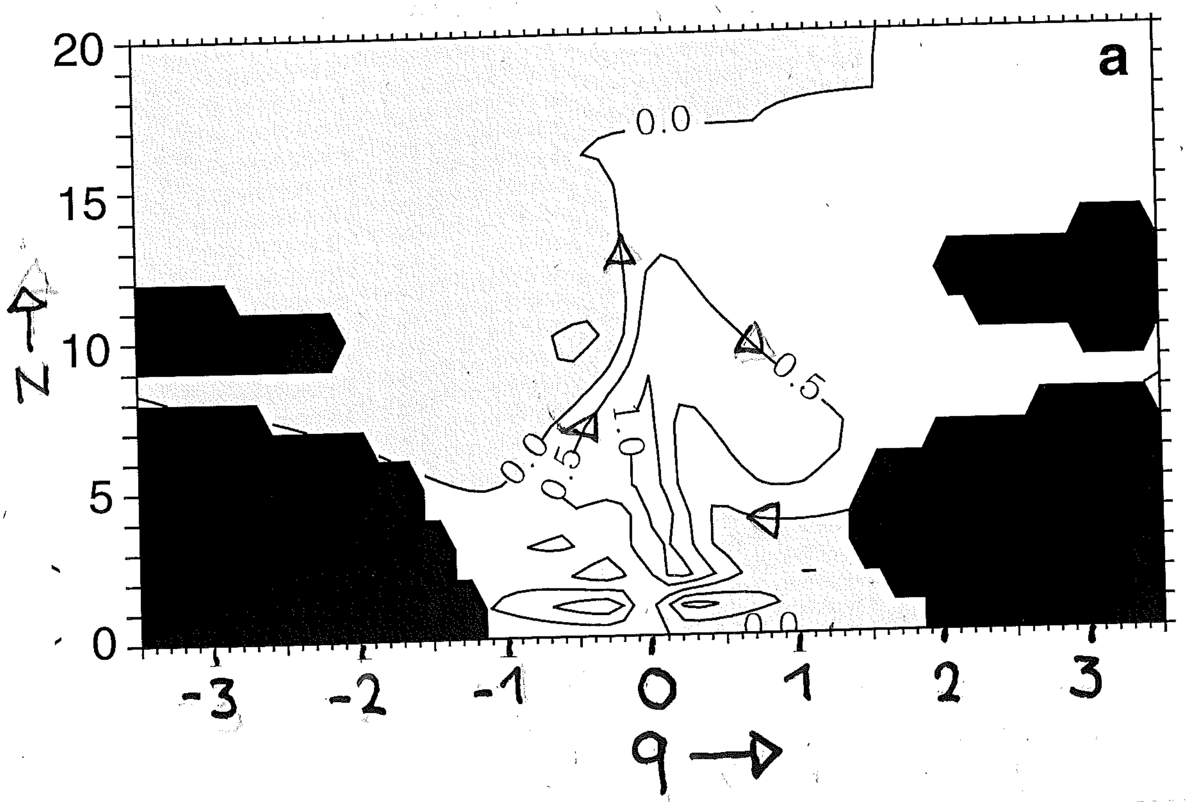


$(\overline{vP}, \overline{wP})$?

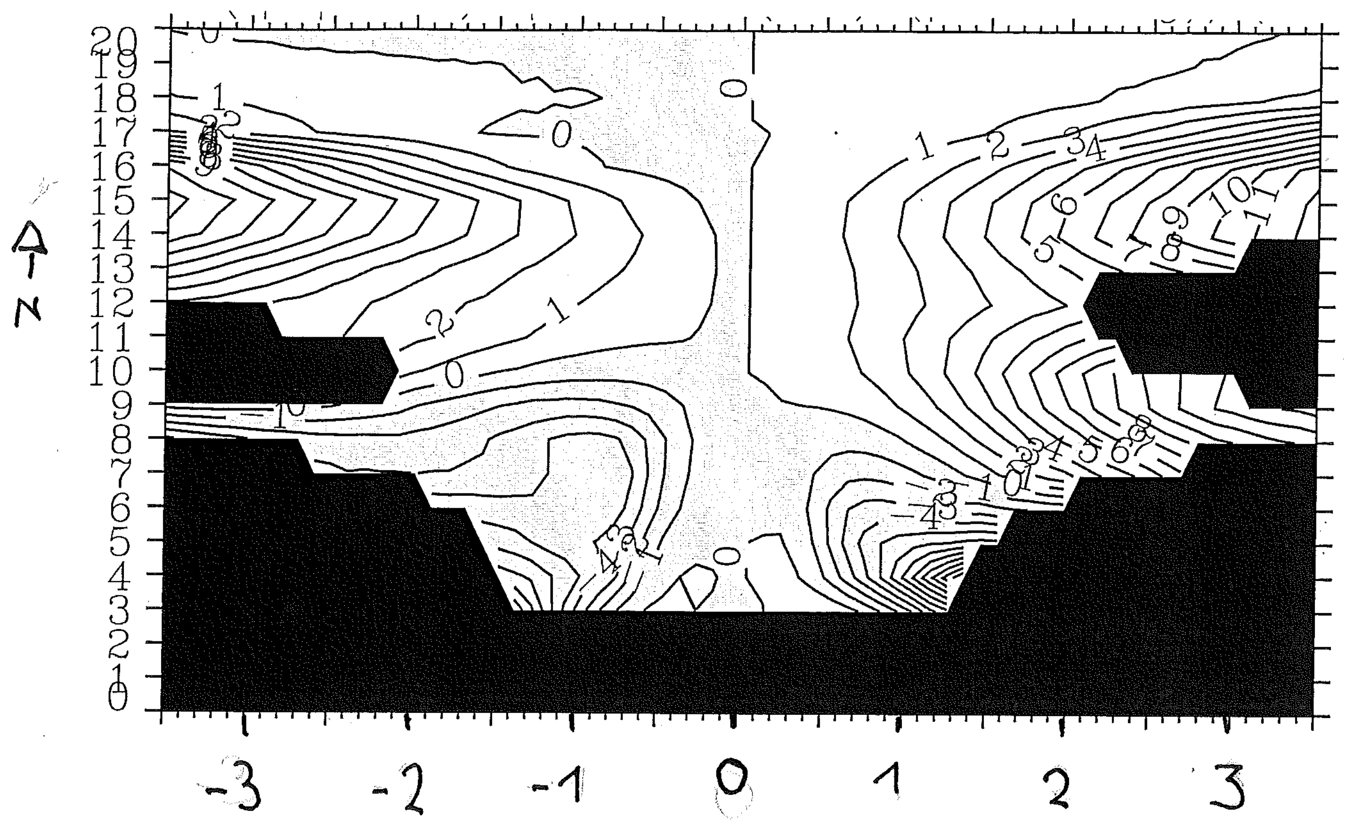
$$q = (\zeta + f)\frac{\partial\theta}{\partial z}\rho^{-1} \leftarrow \text{'PV'}$$

Coordinates: (λ, q, z)

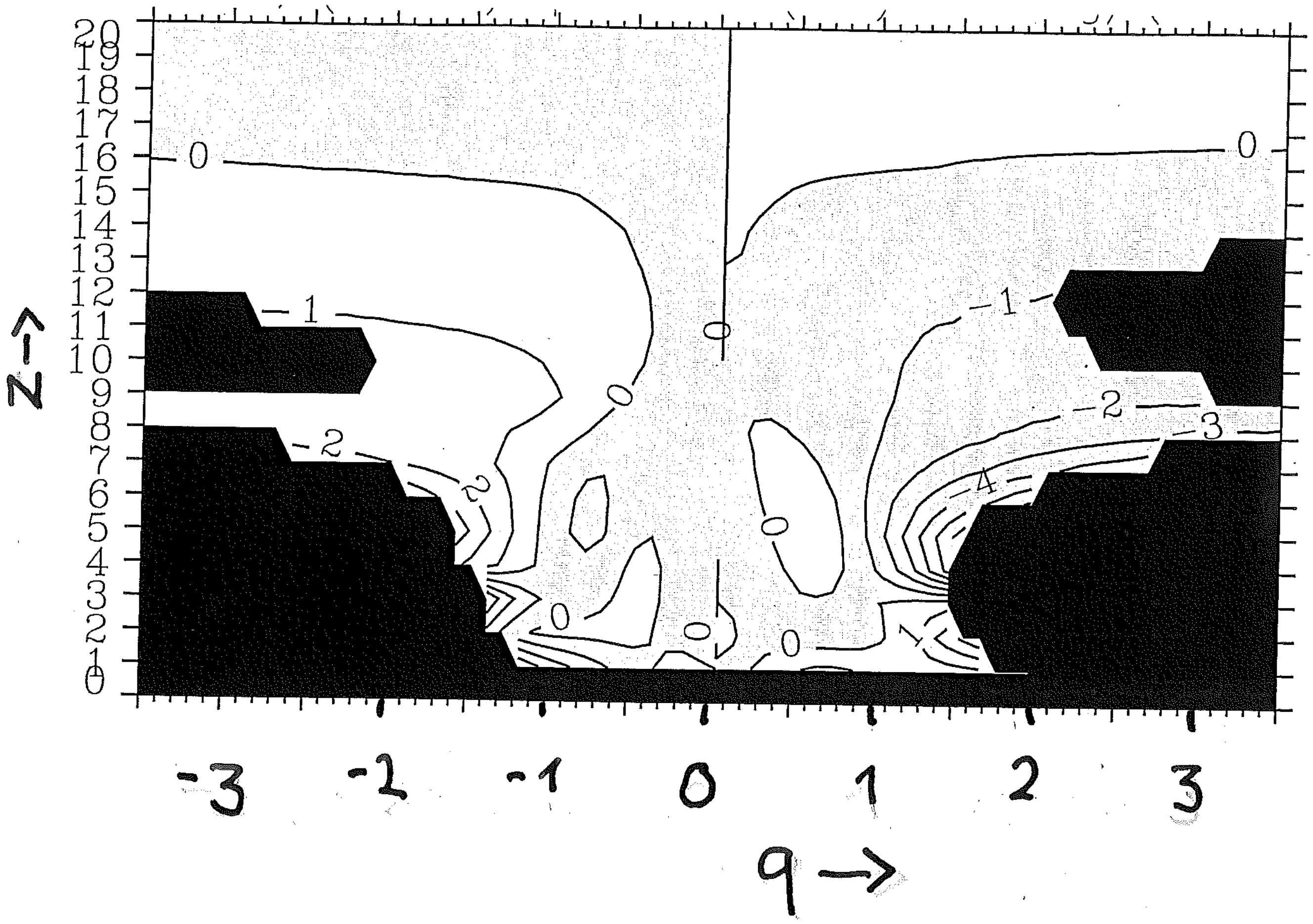
mass streamfunction



Advective q-flux: $\overline{\dot{q}\rho q}$, $\overline{w\rho q}$



horizontal: $\overline{\dot{q}\rho q}$



vertical $\overline{w\rho q}$

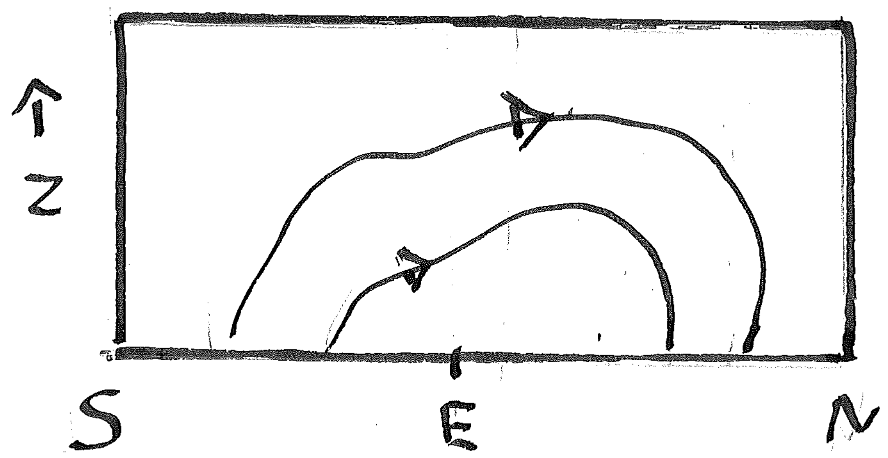
Flux climatology

$\overline{}$: longterm, zonal mean

$$\frac{\partial P}{\partial t} = -\nabla \cdot (j)$$

$$\overline{\nabla \cdot j} = 0$$

Stream function?



But: $\nabla \cdot (\chi \nabla \times F) = \nabla \cdot (F \times \nabla \chi)$

$$\chi = \Theta \; ; \; -\int_0^\infty \overline{\nabla \cdot j} = \overline{j}_n \Big|_{z=0}$$

Two forms of frictional flux

1) $\overline{-\Theta_s \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)}$

2) $\overline{-\Theta_s \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)} = \overline{\frac{\partial}{\partial y} (F_x \Theta_s)}$

Conclusions

The ›electric analogy‹ demonstrates that a flux of mass weighted PV cannot be defined

The majority of j vectors does not satisfy the impermeability theorem

The advective P- flux is mainly directed downward in the NH troposphere

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