Large-eddy simulation of efficiency of momentum transport in spatially developing urban boundary layer

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Urban heat island and air pollution have become serious issues in urban spaces.
To arrive at solutions to these problems with the design of buildings and urban spaces, it is necessary to predict the urban climate accurately.
Background and objective of the research

The problems of numerical models to predict the urban climate

- **Micro-scale models**: The prediction accuracy of RANS models in an urban boundary layer. (Yoshie et al. (2012))

In this research, we focus on the investigation and the improvement of the prediction accuracy of RANS models in an urban boundary layer.

**Micro-scale models**
- (Building scale)
- (City block scale)
Yoshie et al. (2012) show that RANS model (standard k-ε model) underestimates streamwise mean wind velocity above the top of the urban canopy in an urban boundary layer.

Yoshie et al. (2012) show that RANS model (standard k-ε model) overestimates mean concentration of contaminant at the urban canopy layer in an urban boundary layer.
Xie and Castro (2006) show that RANS model (standard k-ε model) overestimates Reynolds stress $<u'w'>$ above the top of the urban canopy in an urban boundary layer.

This result shows that the overestimation of Reynolds stress $<u'w'>$ is the reason why RANS model (standard k-ε model) underestimates streamwise mean wind velocity above the top of the urban canopy in an urban boundary layer.
The reason why Reynolds stress $<u'w'>$ is overestimated in RANS model (standard $k$-$\varepsilon$ model) in an urban boundary layer.

- Eddy viscosity coefficient $\nu_t$ is overestimated in an urban boundary layer.
  → It is necessary to investigate the model coefficient $C_\mu$ of eddy viscosity coefficient $\nu_t$.

- Gradient diffusion approximation is not correct in an urban boundary layer.
  → It is necessary to investigate the budget of Reynolds stress $<u'w'>$. 

\[
< u'w' > = -\nu_t \frac{\partial < u >}{\partial z}
\]

\[
\nu_t = C_\mu \frac{k^2}{\varepsilon}
\]
In this research to investigate the prediction accuracy of RANS model (standard $k$-$\varepsilon$ model) in an urban boundary layer, the accuracy of

- Eddy viscosity coefficient $v_t$
  (Model coefficient $C_\mu$ of eddy viscosity coefficient $v_t$)

- Gradient diffusion approximation
  (Budget of Reynolds stress $<u'w'>$)

are investigated using large-eddy simulation (LES)
Target point: urban canyon center in 5th row (point 1), 15th row (point 2), 30th row (point 3)

Analysis model is set up with reference to the wind tunnel experiment conducted by Uehara et al. (2000)
## Analysis model and analysis condition

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Results: Statistics of flow field

Urban canyon center in 5\textsuperscript{th} row (point 1), 15\textsuperscript{th} row (point 2), 30\textsuperscript{th} row (point 3)

- As the urban boundary layer spatially develops, streamwise mean wind velocity, streamwise turbulent intensity, turbulent kinetic energy, and Reynolds stress become small above the top of the urban canopy.
- The gradient of streamwise mean wind velocity becomes the largest at the top of the urban canopy.
- Streamwise turbulent intensity, turbulent kinetic energy, and Reynolds stress become the largest at the top of the urban canopy.
Results : Estimation of model coefficient $C_\mu$

$$
< u' w' > = -\nu_t \frac{\partial < u >}{\partial z} \quad (1) \quad -< u' w' > \frac{\partial < u >}{\partial z} = \varepsilon \quad (2)
$$

$$
\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (3) \quad C_\mu = \left( \frac{< u' w' >}{k} \right)^2 = 0.09 \quad (4)
$$

(1) : Two dimensional shear flow and gradient diffusion approximation
(2) : Local balance of turbulent kinetic energy
(3) : Eddy viscosity coefficient $\nu_t$ derived from equation (1) and equation (2)
(4) : Model coefficient $C_\mu$ of eddy viscosity coefficient $\nu_t$

• Model coefficient $C_\mu$ is estimated from the results of large-eddy simulation using equation (4).
Above the top of the urban canopy (from $z/H=1.0$ to 2.5), model coefficient $C_\mu$ becomes smaller than the value 0.09 that is used in standard $k$-$\varepsilon$ model regardless of the spatial development of the urban boundary layer.

There is some possibility that above the top of the urban canopy (from $z/H=1.0$ to 2.5), eddy viscosity coefficient $\nu_t$ is overestimated in standard $k$-$\varepsilon$ model and the accuracy of standard $k$-$\varepsilon$ model decreases regardless of the spatial development of the urban boundary layer.

$${\nu}_t = C_\mu \frac{k^2}{\varepsilon}$$
Results: Budget of Reynolds stress $<u'u'>$

$$\frac{\partial <u_i'u_j'>}{\partial t} = C_{ij} + P_{ij} + \Phi_{ij} + T_{ij}^{GS} + T_{ij}^{SGS} + \Psi_{ij} + D_{ij} - \varepsilon_{ij}^{GS} - \varepsilon_{ij}^{SGS}$$

$C_{ij} = -\frac{\partial \langle u_k' u_i'u_j' \rangle}{\partial x_k}$

Convection term

$P_{ij} = -\langle u_i'u_k' \rangle \frac{\partial \langle u_j' \rangle}{\partial x_k} - \langle u_j'u_k' \rangle \frac{\partial \langle u_i' \rangle}{\partial x_k}$

Production term

$\Phi_{ij} = \left\langle -p' \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right) \right\rangle$

Pressure strain correlation term

$T_{ij}^{GS} = -\frac{\partial \langle u_i'u_j'u_k' \rangle}{\partial x_k}$

GS turbulent diffusion term

$T_{ij}^{SGS} = -\frac{\partial \langle u_i' \tau'_{jk} \rangle + \langle u_j' \tau'_{ik} \rangle}{\partial x_k}$

SGS turbulent diffusion term

$\Psi_{ij} = -\frac{\partial \langle p' (u_j' \delta_{ik} + u_i' \delta_{jk}) \rangle}{\partial x_k}$

Pressure diffusion term

$D_{ij} = \nu \frac{\partial^2 \langle u_i'u_j' \rangle}{\partial x_k^2}$

Molecular diffusion term

$\varepsilon_{ij}^{GS} = 2\nu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} \right\rangle$

GS dissipation term

$\varepsilon_{ij}^{SGS} = -\left\langle \tau'_{ik} \frac{\partial u_j'}{\partial x_k} \right\rangle - \left\langle \tau'_{jk} \frac{\partial u_i'}{\partial x_k} \right\rangle$

SGS dissipation term

$$C_{13} + P_{13} + \Phi_{13} + T_{13}^{GS} + T_{13}^{SGS} + \Psi_{13} + D_{13} - \varepsilon_{13}^{GS} - \varepsilon_{13}^{SGS} + \text{Residual} = 0$$
Results: Budget of Reynolds stress $\langle u'w' \rangle$

As the urban boundary layer spatially develops, above the top of the urban canopy (from $z/H=1.0$ to 1.5), not only production term but also diffusion term becomes large and contribute to the increase of $\langle u'w' \rangle$.

There is some possibility that as the urban boundary layer spatially develops, above the top of the urban canopy (from $z/H=1.0$ to 1.5), the accuracy of gradient diffusion approximation of Reynolds stress $\langle u'w' \rangle$ decreases as the urban boundary layer spatially develops.
• Large-eddy simulation is performed to investigate the prediction accuracy of RANS model (standard k-ε model) in spatially developing urban boundary layer.

• Above the top of the urban canopy (from z/H=1.0 to 2.5), model coefficient $C_\mu$ becomes smaller than the value 0.09 that is used in standard k-ε model regardless of the spatial development.

• As the urban boundary layer spatially develops, above the top of the urban canopy (from z/H=1.0 to 1.5), not only production term but also diffusion term becomes large and contributes to the increase of $\langle u'w' \rangle$.

• There is some possibility that above the top of the urban canopy, the accuracy of eddy viscosity coefficient $\nu_t$ and the accuracy of gradient diffusion approximation of $\langle u'w' \rangle$ decreases.