ANALOGIES OF OCEAN/ ATMOSPHERE ROTATING FLUID DYNAMICS WITH GYROSCOPES

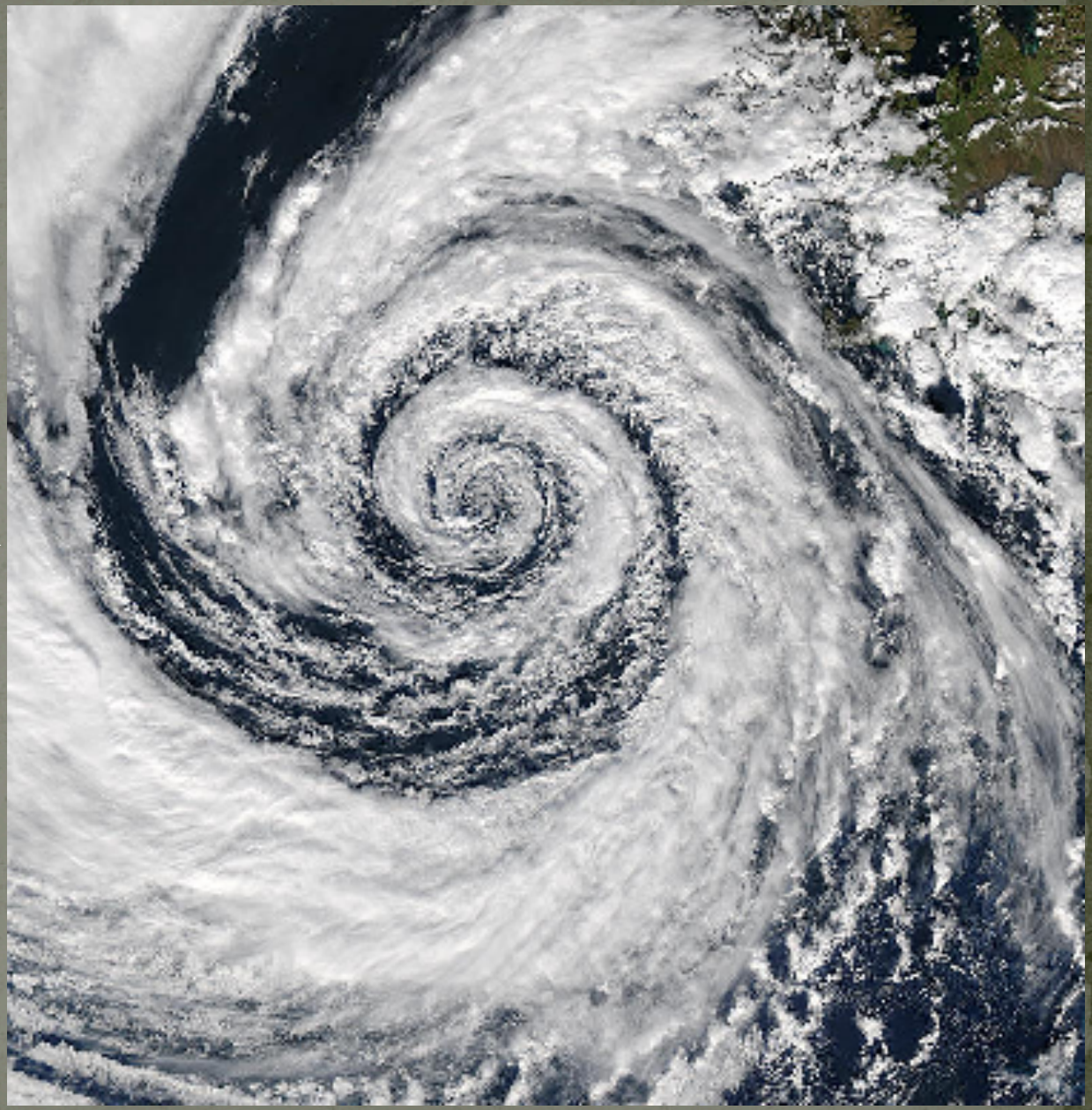
Teaching Opportunities

BY THOMAS W. N. HAINE AND DEEPAK A. CHERIAN

Aspects of geostrophic flow and inertial oscillation in the ocean and atmosphere are analogous to the motion of a rapidly spinning gyroscope.

AFFILIATIONS: HAINE—Earth and Planetary Sciences, The Johns Hopkins University, Baltimore, Maryland; CHERIAN—Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts
CORRESPONDING AUTHOR: Thomas W. N. Haine, 329 Olin Hall, The Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218
E-mail: thomas.haine@jhu.edu

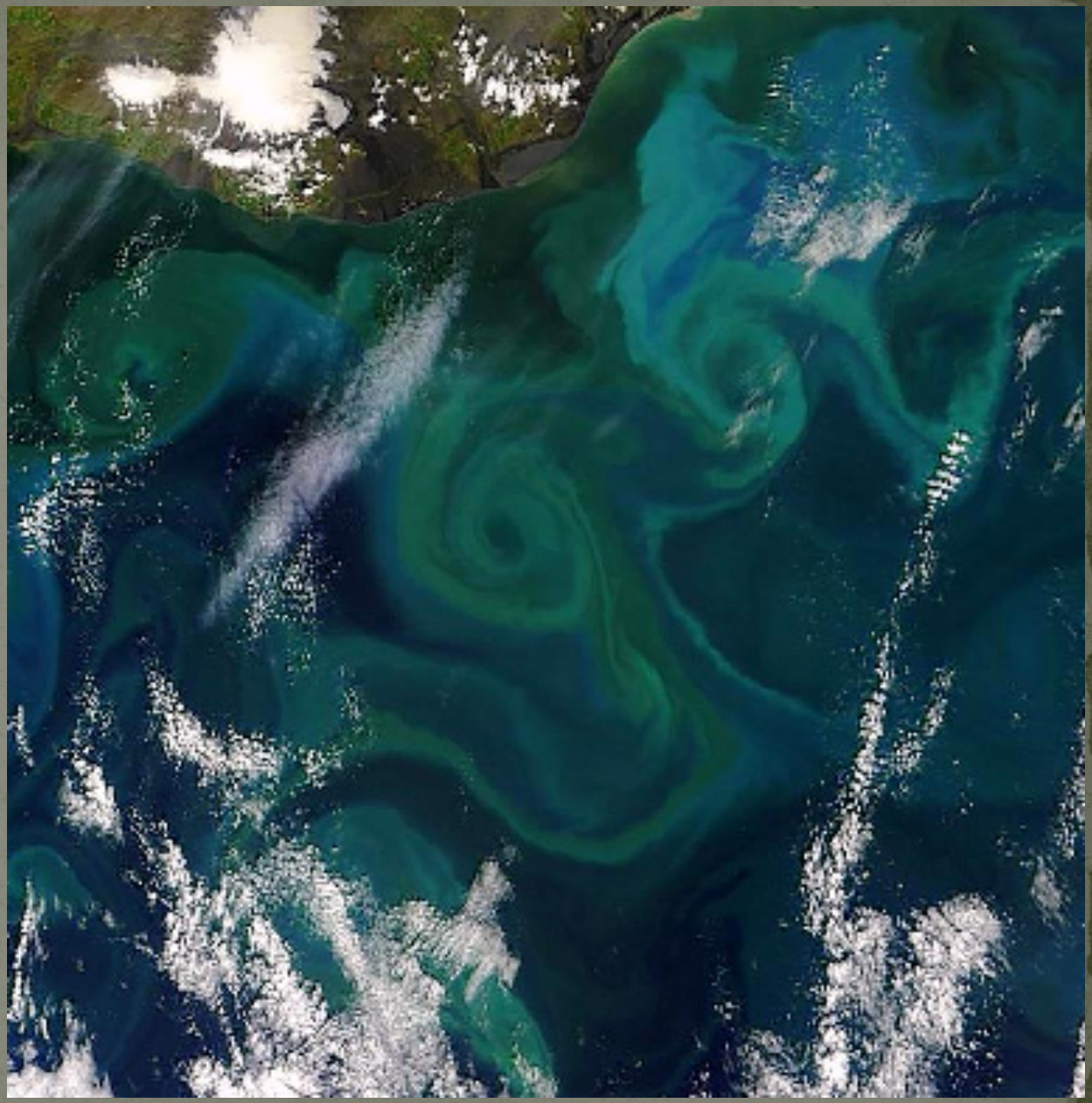
Geostrophic flow is ubiquitous:



Atmospheric cyclone over the Irminger Sea and southwest Iceland at 1410 UTC 4 Sep 2003, as seen by the NASA Aqua satellite and visualized by clouds. The diameter of the system is around 750 km.

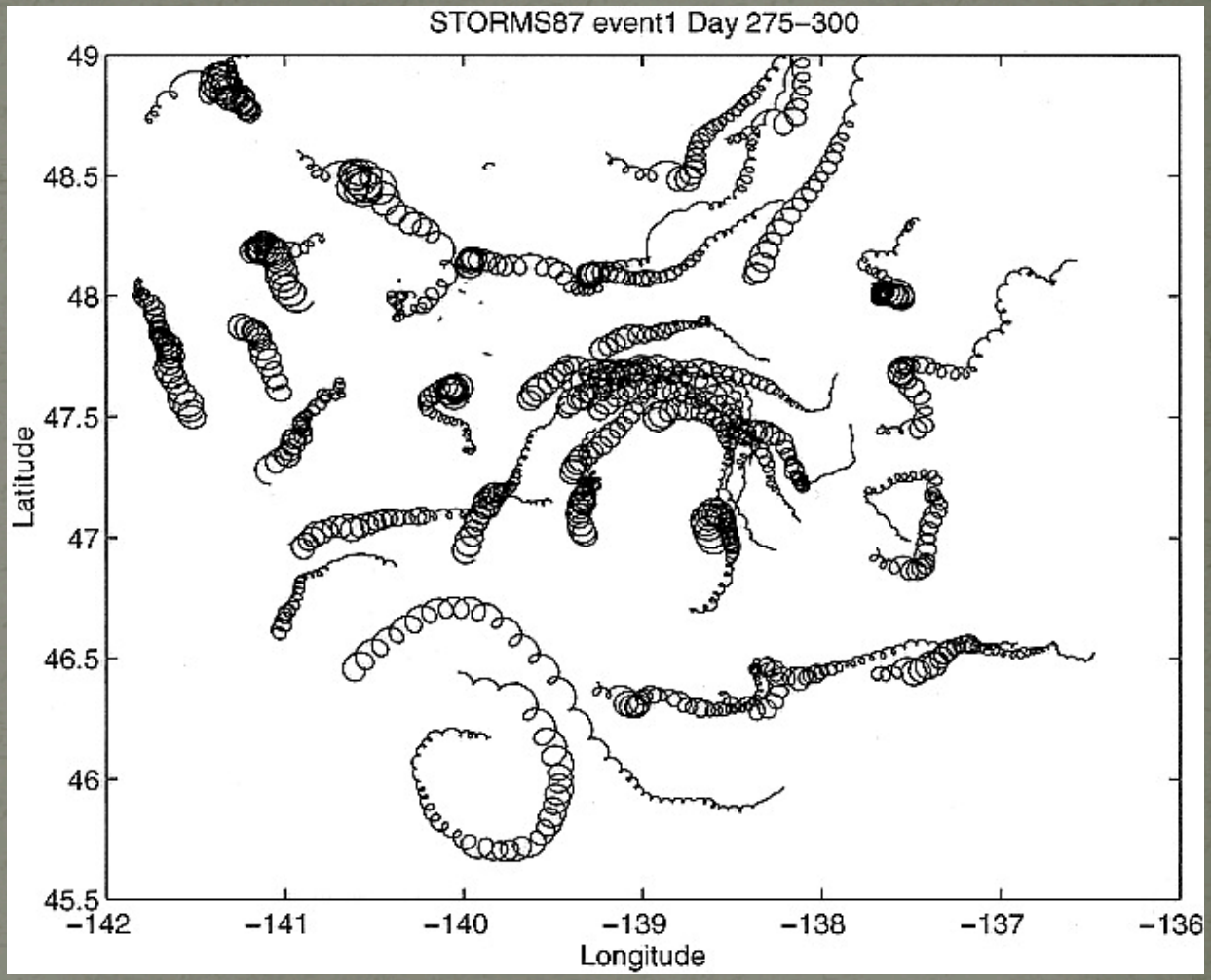
Geostrophic flow requires small Rossby no.

Geostrophic flow is ubiquitous:



Surface ocean geostrophic turbulence as seen by streaks of green phytoplankton south of Iceland at 1250 UTC 21 Jun 2004 by the NASA Terra satellite. The diameter of the cyclone in the center of the frame is about 40 km.

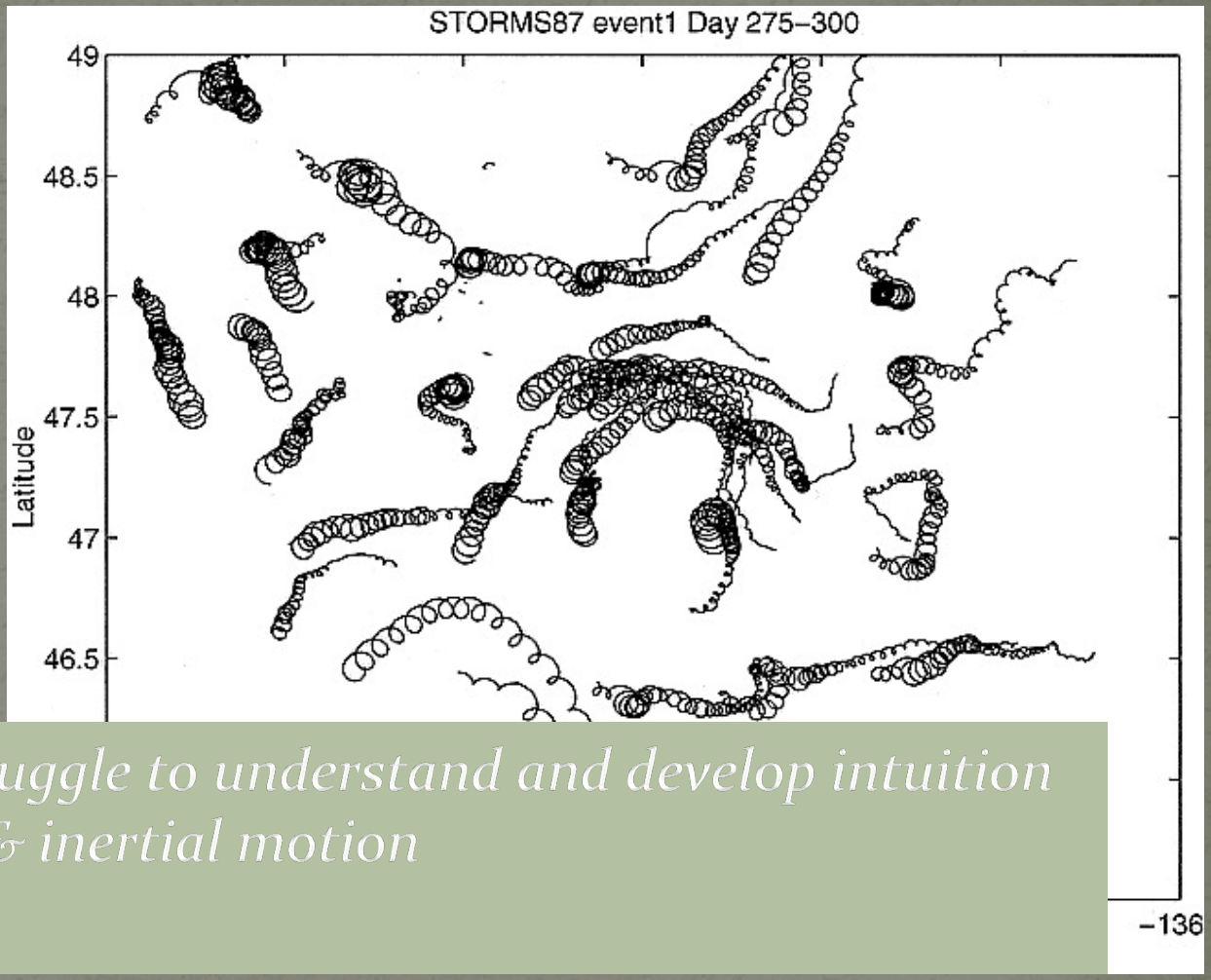
Inertial motion is ubiquitous:



Near-inertial motion of satellite-tracked surface drifters in the North Pacific Ocean (from van Meurs 1998).

Clockwise circles in phase with period 19h.

Inertial motion is ubiquitous:

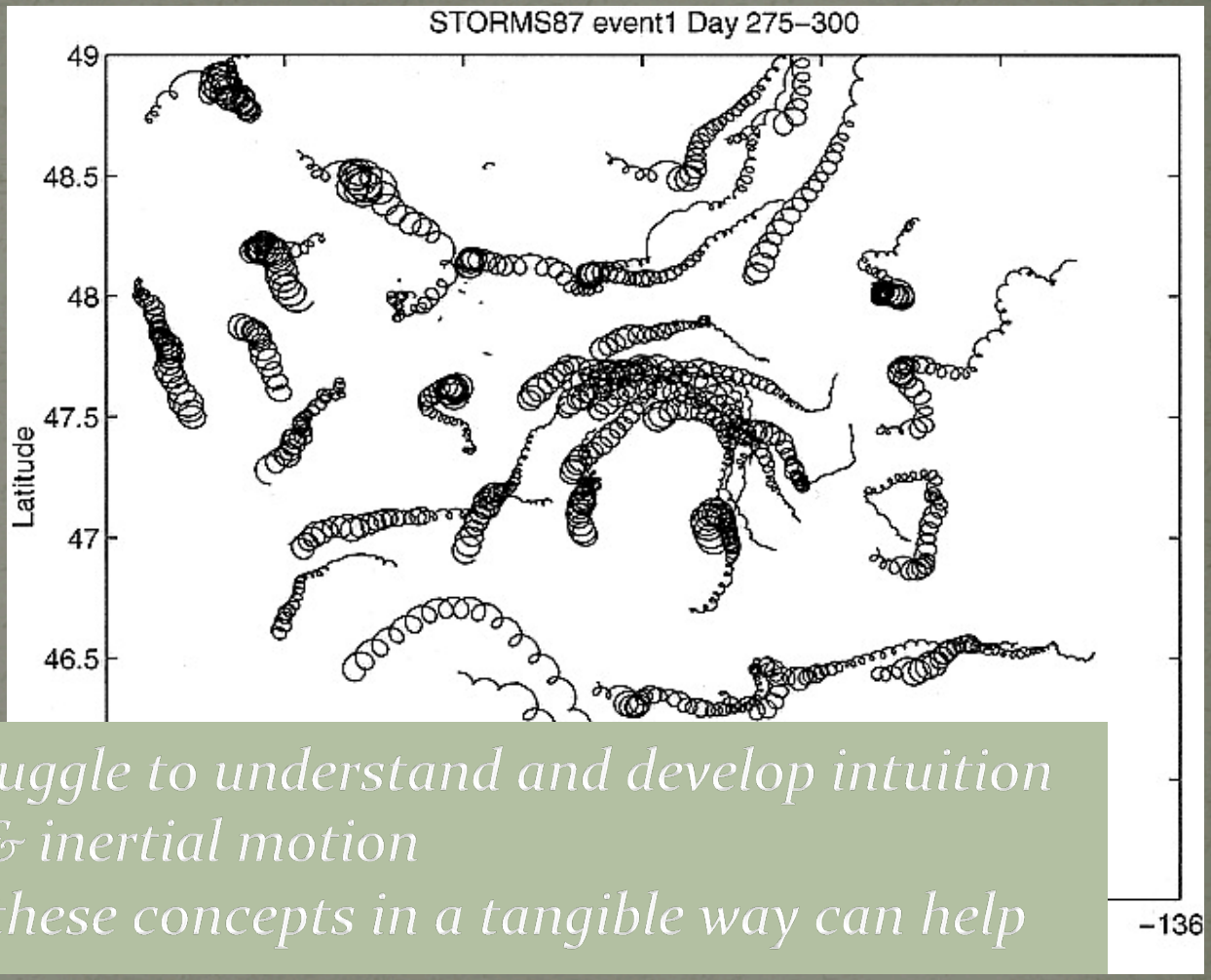


Near-inertial motion of satellite-tracked surface drifters in the North Pacific Ocean (from van Meurs 1998).

Clockwise circles in phase with period 19h

Students can struggle to understand and develop intuition for geostrophic & inertial motion

Inertial motion is ubiquitous:



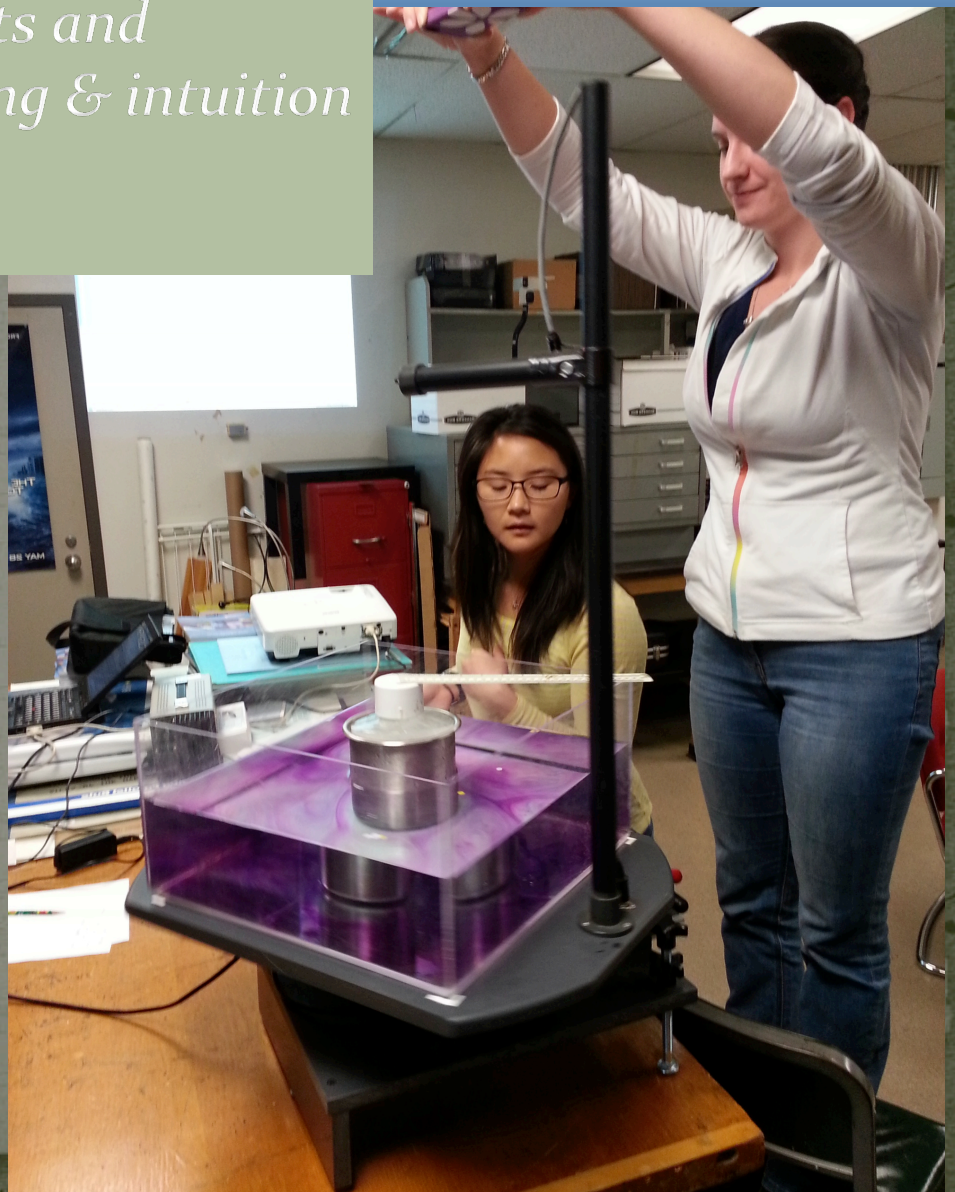
Near-inertial motion of satellite-tracked surface drifters in the North Pacific Ocean (from van Meurs 1998).

Clockwise circles in phase with period 19h

Students can struggle to understand and develop intuition for geostrophic & inertial motion
Demonstrating these concepts in a tangible way can help

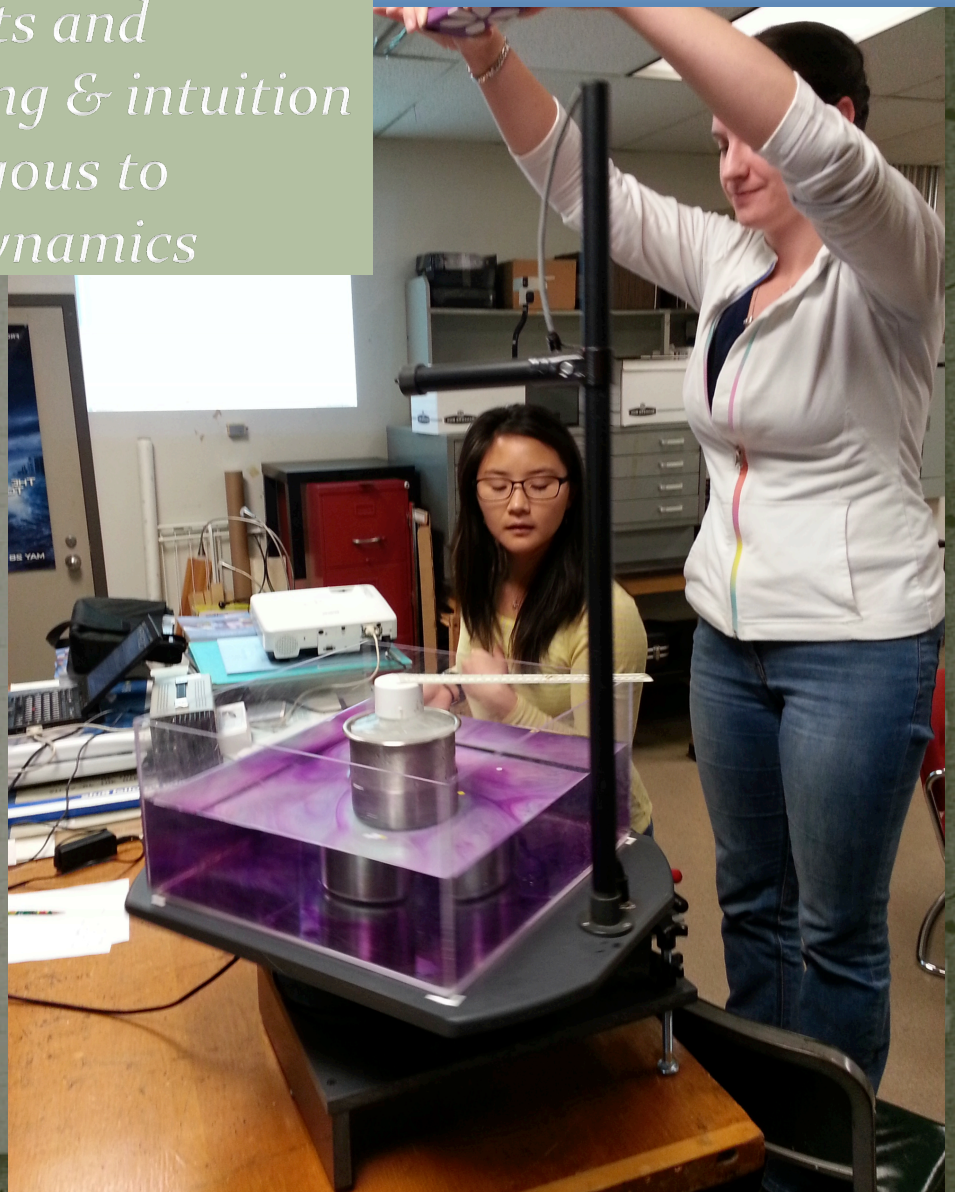
Weather in a Tank

We use laboratory tank experiments and gyroscopes to develop understanding & intuition



Weather in a Tank

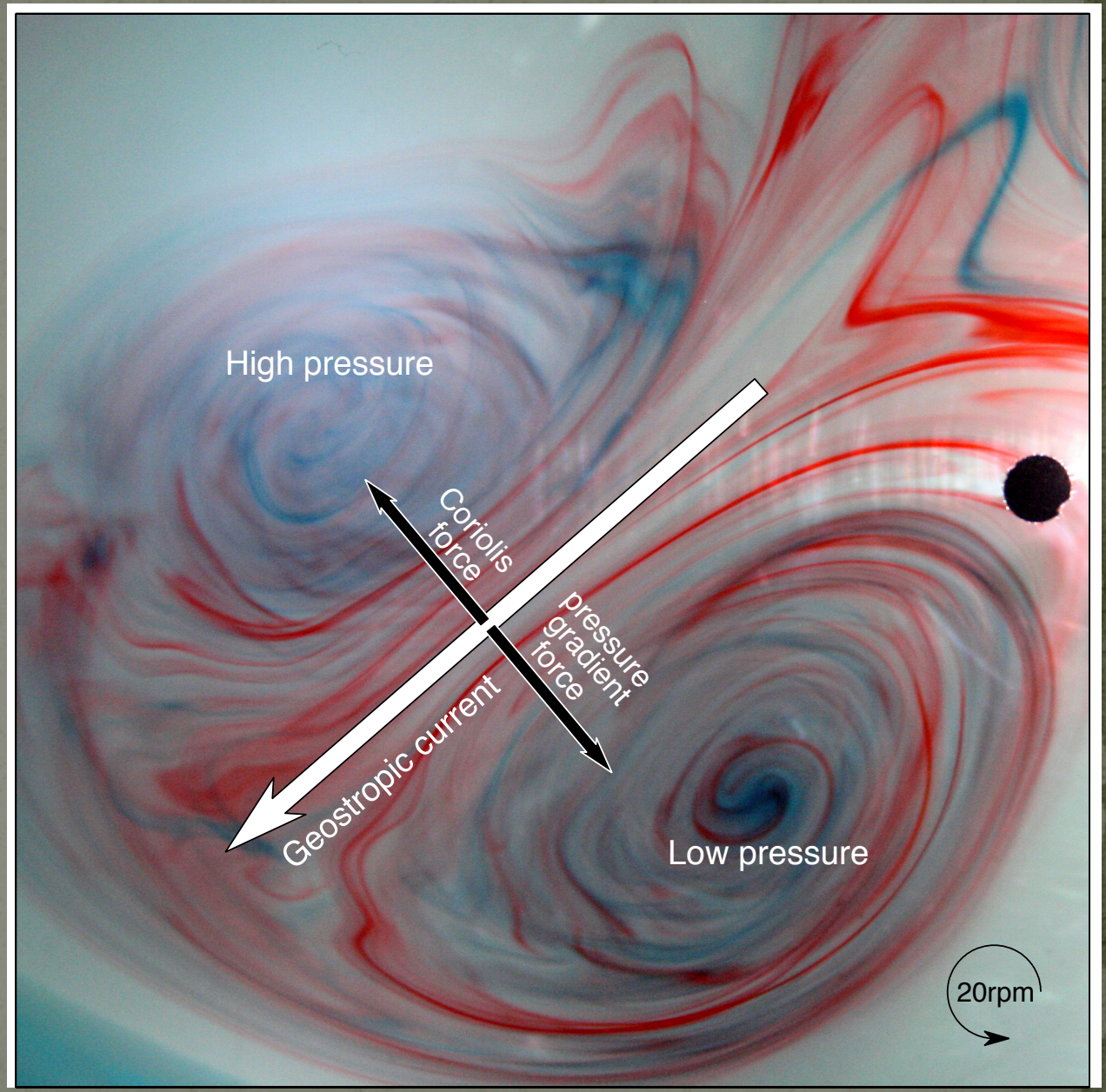
We use laboratory tank experiments and gyroscopes to develop understanding & intuition
Gyroscopes are dynamically analogous to ocean/atmosphere rotating fluid dynamics



Geostrophic flow

Current speed α pgf/Ω and at right angles to pgf

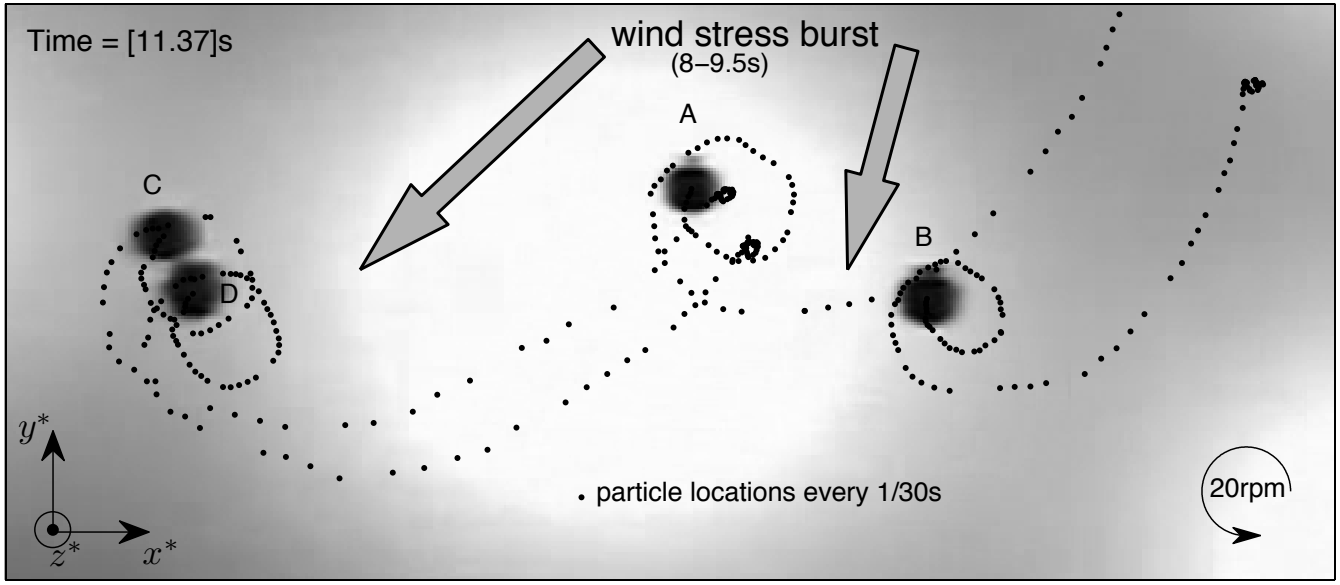
Geostrophic motion in a rotating tank of water. The image, taken from above, shows streaks of dye in a tank of water 10 cm deep, rotating at 20 rpm. The diagonal measures about 17 cm. The black dot is a floating piece of paper from a hole punch. Only part of the 40-cm square tank is shown.



Inertial motion

Inertial oscillation in rotating tank of water at 20rpm

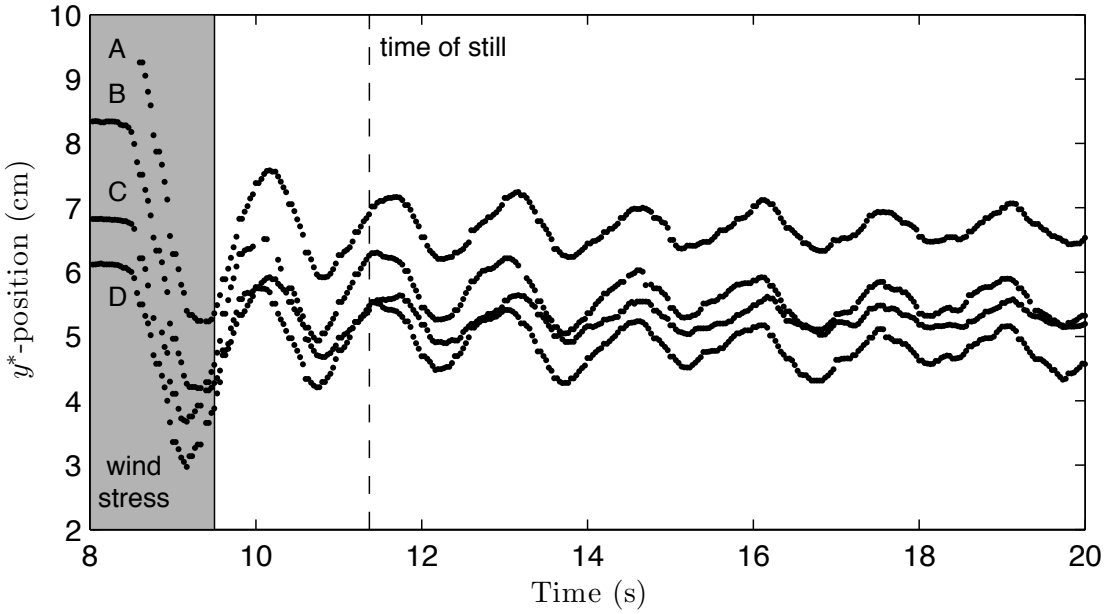
Inertial motion



Current speed $\alpha\ \Omega$

Clockwise circles in phase

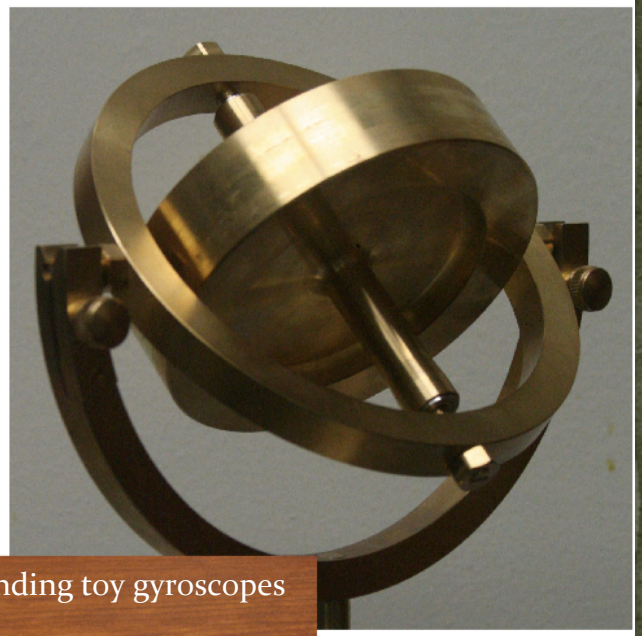
Snapshot of 4 floating particles (A-D) with locations tracked over time. The dimensions of the image are approximately 7 cm by 23 cm and the water depth is 10 cm.



Gyroscopes

MITAC mechanically driven, gimbal-mounted gyroscope

Brass gimbal-mounted lecture gyroscope



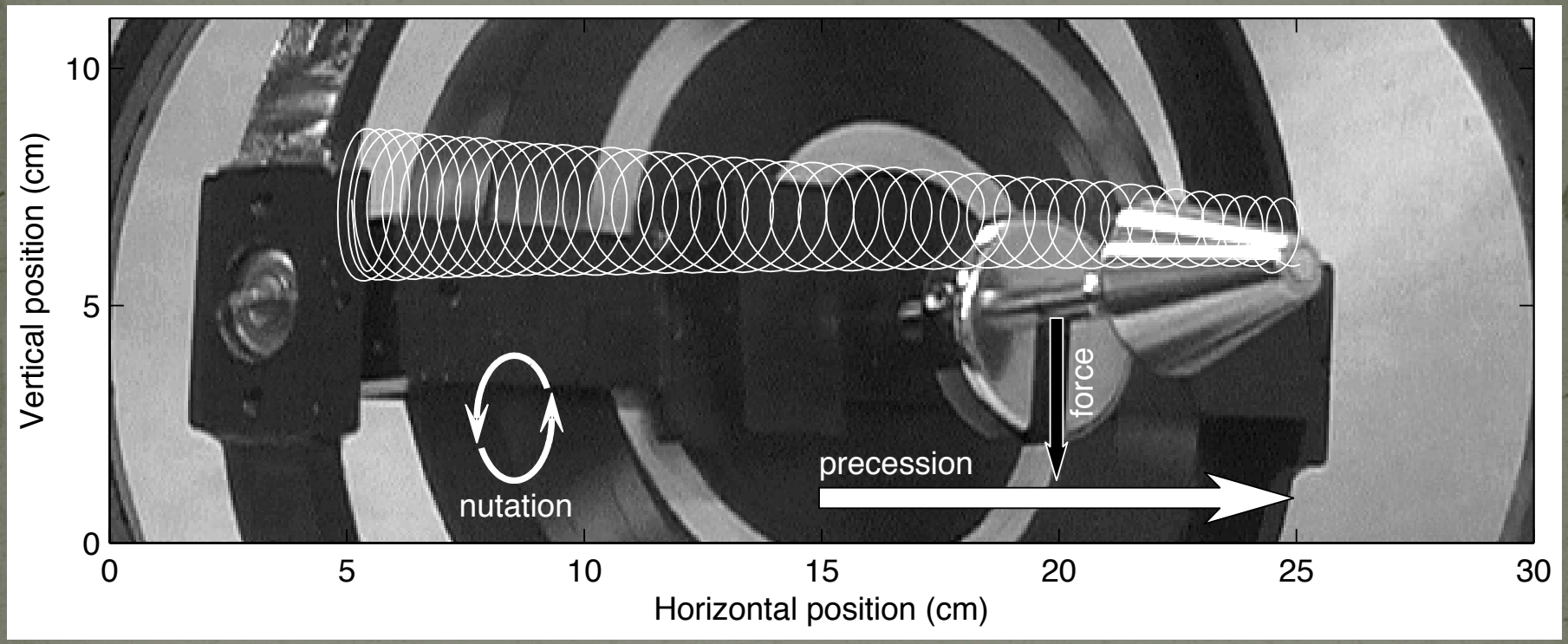
Freestanding toy gyroscopes

Gyroscope motion

MITAC gyroscope exhibits nutation and precession.
Rotation rate = 225rpm.

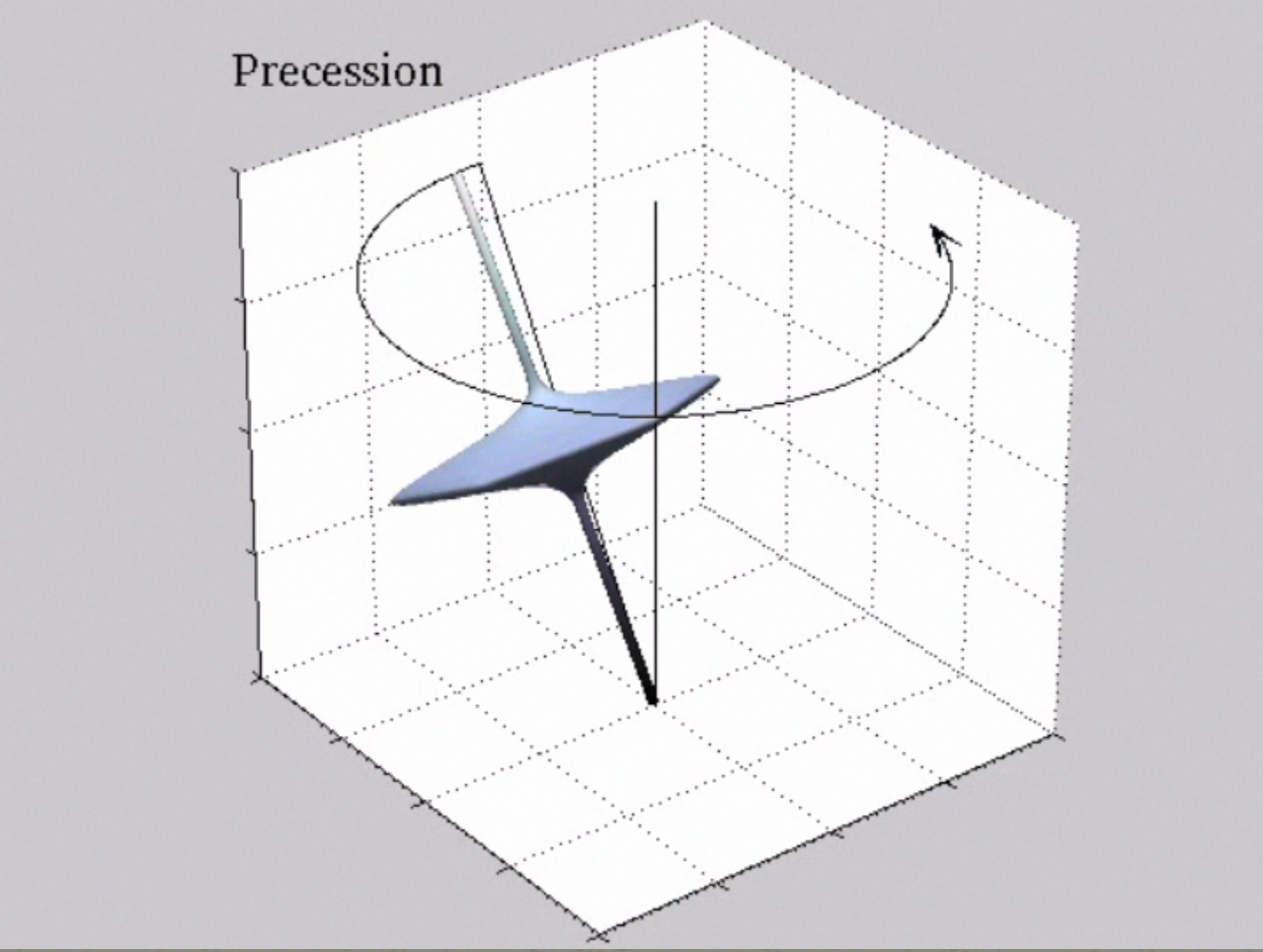
Gyroscope motion

Tip motion = precession + nutation



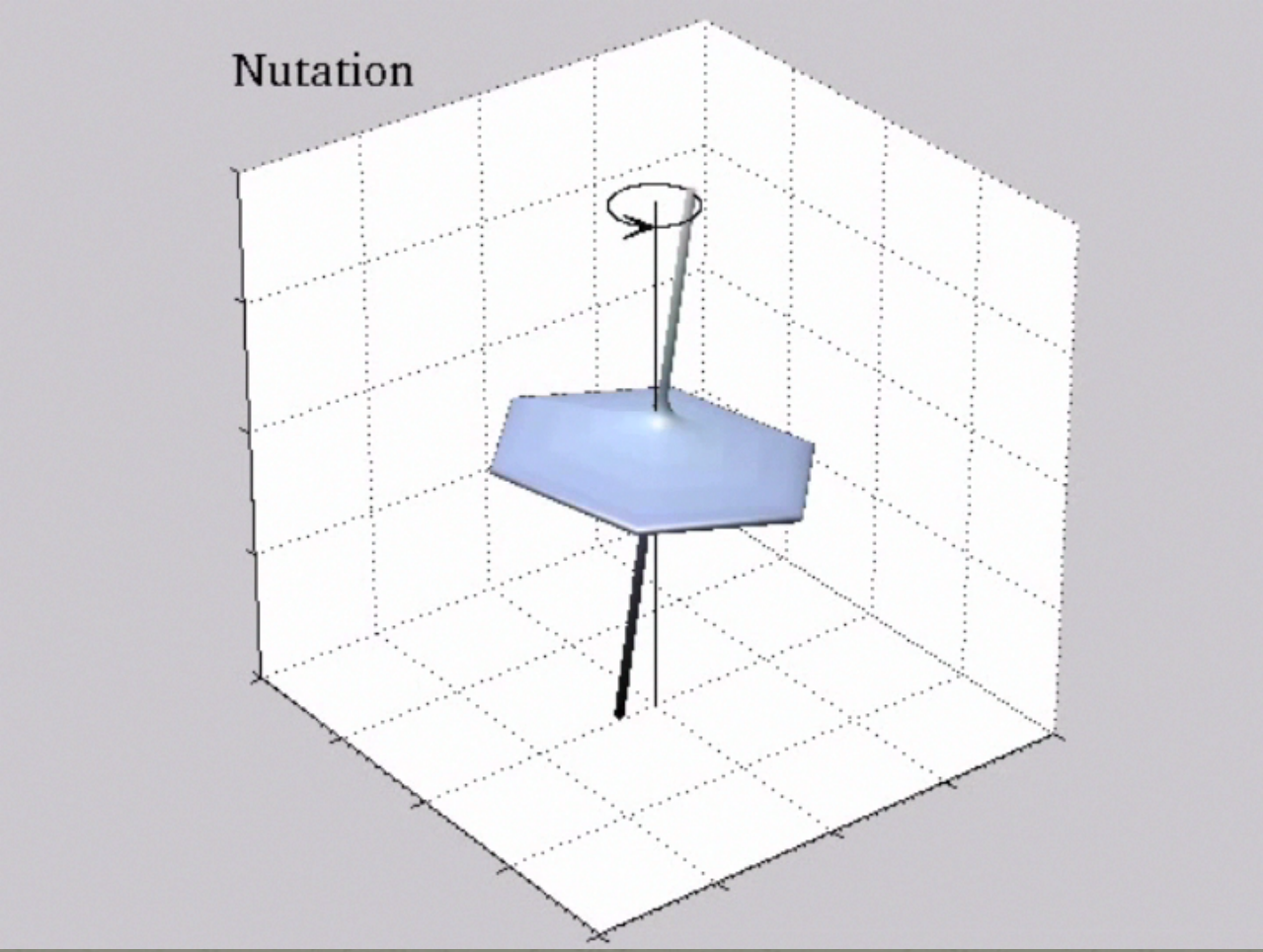
MITAC gyroscope under torque rotating at 225 rpm. A weight (visible behind the conical tip) generates a gravitational force, hence a torque, and the motion is started by a firm push of the hand. The white line traces the motion of the tip over 12 s.

Precession



$$\omega_p = \frac{mgl}{I_3\omega_3} \quad \text{(precession frequency)}$$

Nutation



$$\omega_n = \frac{I_3 \omega_3}{I} \quad \text{(nutation frequency),}$$

Link between equations of motion:

f-plane RSW equations:

$$\frac{du}{dt} - fv = M$$

$$\frac{dv}{dt} + fu = 0.$$

Geostrophy ⇔ precession
Inertial motion ⇔ nutation

Gyroscope equations:

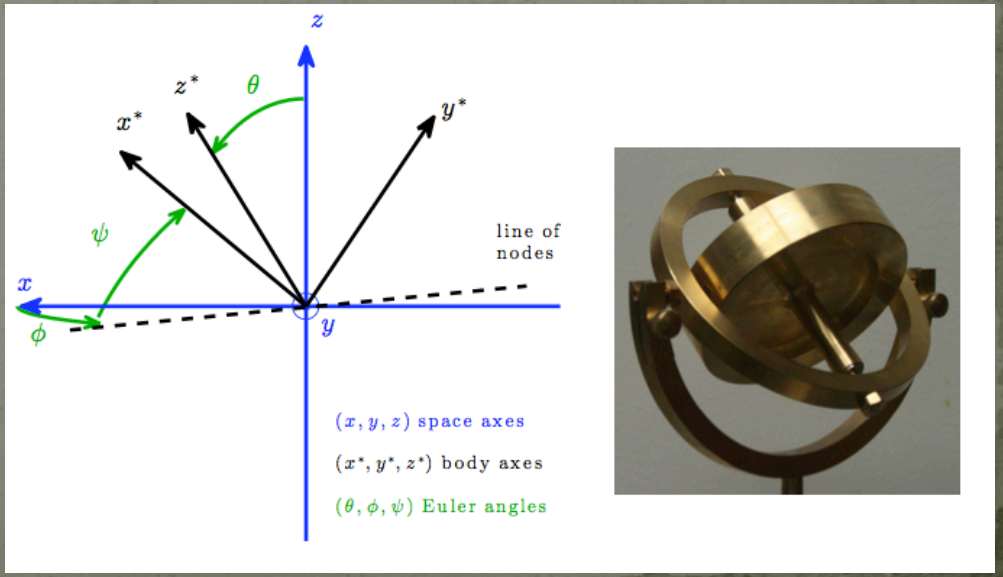
$$\frac{d\dot{\theta}}{dt} + \omega_n \dot{\phi} \sin\theta_0 = \omega_p \omega_n \sin\theta_0$$

$$\frac{d\dot{\phi}}{dt} \sin\theta_0 - \omega_n \dot{\theta} = 0,$$

Link for $\sin\theta_0 = 1$:

$$(u, v, f, M) \Leftrightarrow (\dot{\theta}, \dot{\phi}, -\omega_n, \omega_p \omega_n).$$

Gyroscope angle coordinates:



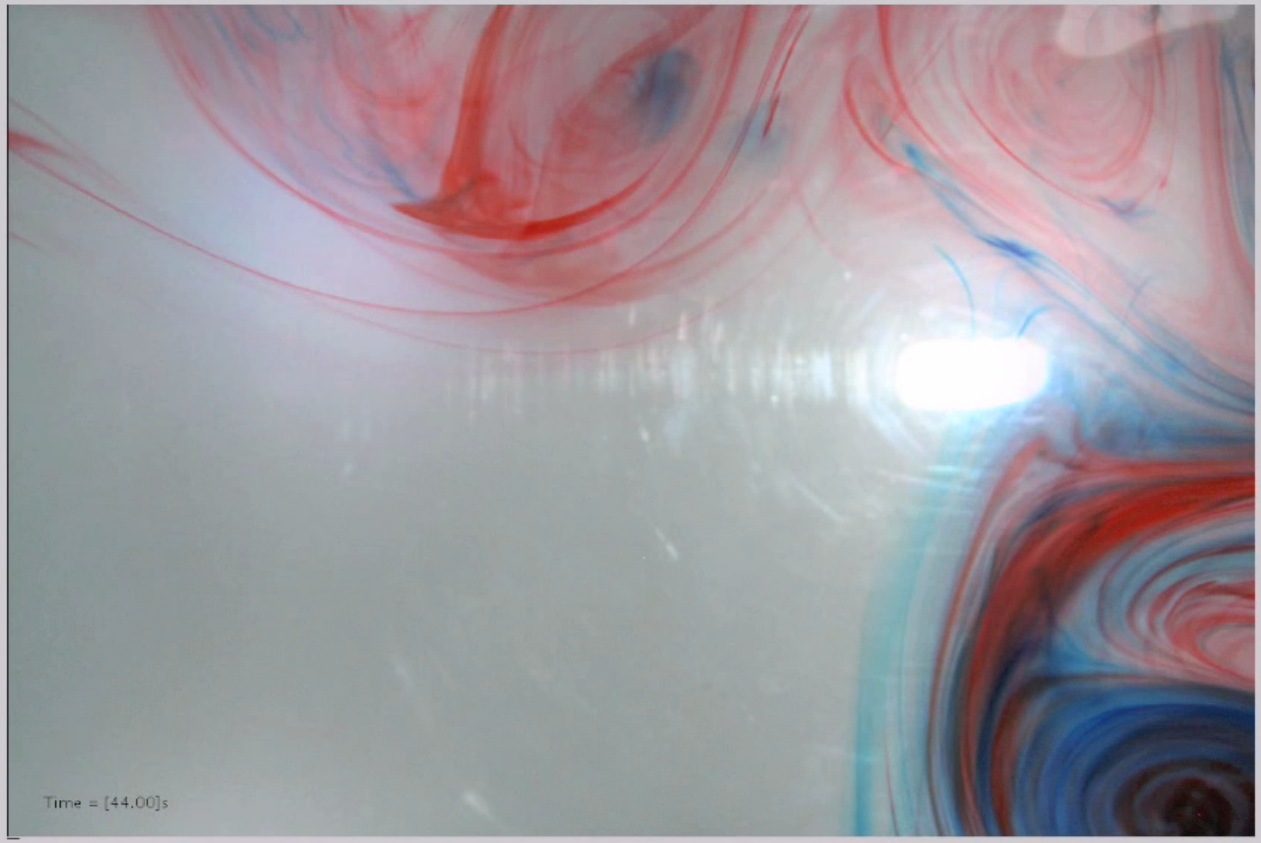
Conclusions

1. Gyroscopes are tangible devices with analogous dynamics to the ocean & atmosphere:

- *Analogous free and forced motions:*
  Geostrophy ⇔ precession
  Inertial motion ⇔ nutation
- *Role of non-dimensional number*
  Rossby no. ⇔ gyroscope no.
- *Geostrophic adjustment*
- *Nonlinear effects*

2. More pedagogical assessment is needed

Geostrophic flow



Precession & Nutation