Possible Implications of a Vortex Gas Model and Self-Similarity for Tornadogenesis and Maintenance

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Introduction

Recent radar studies indicate power laws for horizontal velocity in terms of distance from the axis of rotation [6] and the fractal nature of the tornado-related vorticity field with respect to the grid size [2]. As the power increases the likelihood of strong tornados appears to increase.

What are Power Laws?

A Power law is a relationship of the form $\zeta = Cr^b$, where C and b are constants, and ζ and r are physical quantities. In our case ζ is the velocity or the vorticity, and r is the distance to the axis of rotation or respectively, the length scale in which we view the data. They relate how the horizontal velocity or the vorticity change with distance from the axis of rotation or respectively, the length scale. Power laws suggest self similarity or fractal structure. Power laws can be obtained from radar data.

Filtering the Radar Data

Cai [2] filtered radar data from a particular elevation slice using a specific length scale. The filtering removed frequencies smaller than those that could be effectively represented in the given length scale. He then interpolated the data onto a grid with Δx and Δy corresponding to the particular length scale.

Pseudo-Vorticity

Pseudo-vorticity is the difference between the maximum outbound and the maximum inbound velocity, divided by the distance between the two. The following graph plots the log of the pseudo vorticity against the log of the length scale for nine different length scales. A strong tornado was on the ground at this time. For times leading up to tornado formation the slopes where > -1.6. The regression lines Cai calculated strongly fit the data over scales between that of the mesocyclone core and that of the "edge" of the mesocyclone, thereby indicating that a power law for vorticity vs. scale is valid. As those mesocyclones that produced tornadoes approached tornadogenesis, the slope of the line decreased.

Cai's Results for Strong Tornados [2]

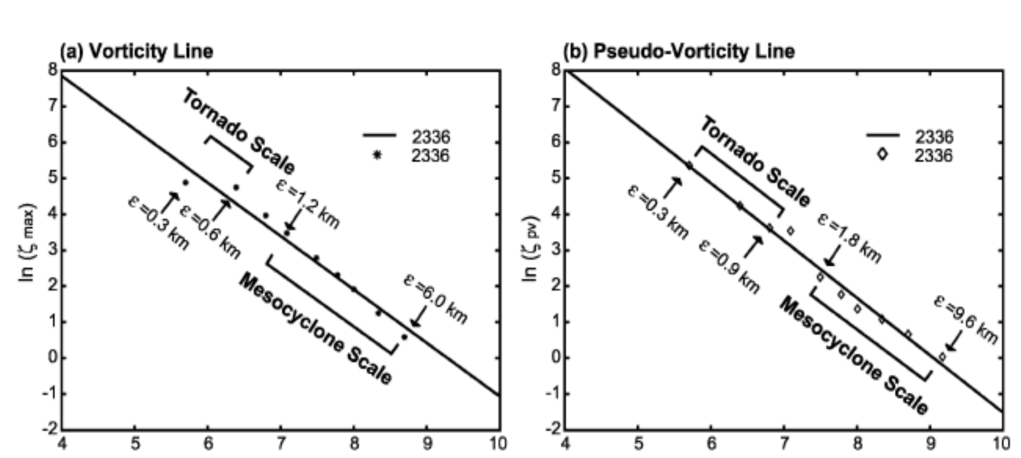

FIG. 6. Scale invariance between tornado and mesocyclone scales shown by the vorticity (a one-step Leise filter is applied to the wind field) and pseudovorticity (no Leise filter is applied to the single-Doppler velocity) line for the Kellerville tornadic mesocyclone at 0.8 km AGL at 2236 UTC. The Kellerville mesocyclone had an F3 tornado embedded inside it at this time. (a) Vorticity line. (b) Pseudovorticity line.

The above graph yields a power law relating vorticity and length scale, $\zeta = Cr^{-1.6}$.

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Wurman's Results for Strong Tornados [6]

Using data obtained from mobile Doppler radar, Wurman found power laws relating velocity and distance to the axis or rotation, $v = Cr^{-b}$, where $0.5 \leq b \leq 0.6$. His studies suggest a power law for the drop-off of the velocity at the tornado scale outside the tornado core. The tornadic flow would roughly approximate a modified Rankine vortex.

Kelvin-Helmholtz Instability [1]

The connection between thin vortex layers and vortex sheets

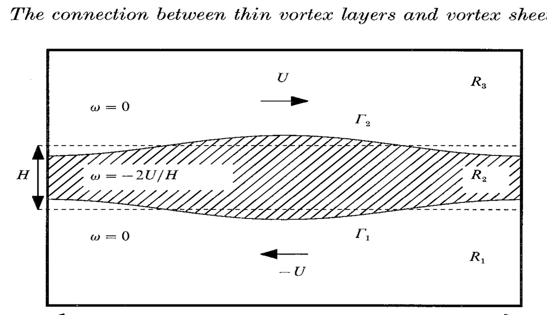


FIGURE 1. A schematic of the vortex layer.

Baker and Shelley [1] studied the roll-up associated to the thin vortex layer shown above. They found the area A depended on the thickness H, as $A = 8.58H^{1.55}$.

Helicity

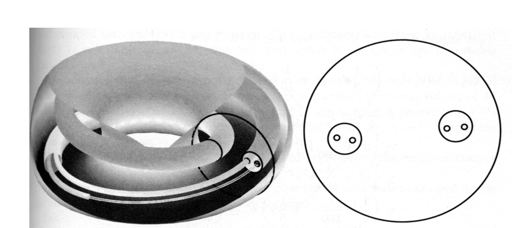
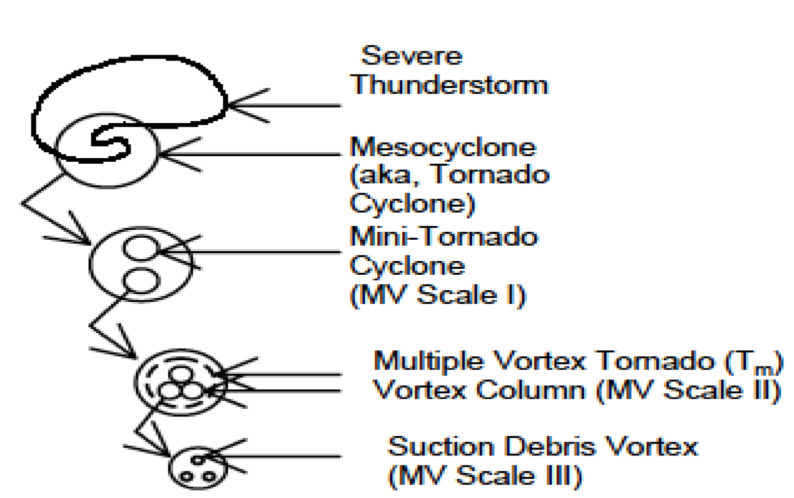


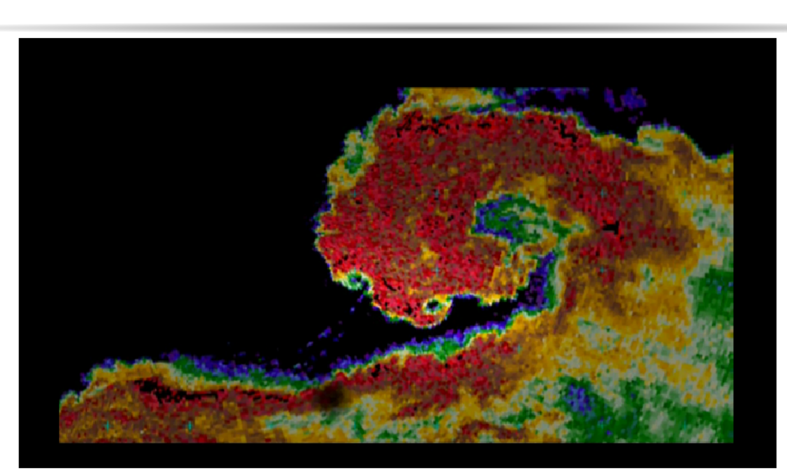
FIGURE 17.1.1. The Smale attractor and a cross section

Compare cross-sections to the hierarchy below. Borrowed from p. 533, Katok, A. and B. Hasselblatt, 1995: Introduction to the Modern Theory of Dynamical Systems, Encyclopedia of Mathematics and its Applications, Vol. 54. Cambridge University Press.



Fractal hierarchy of multiple vortices, *BAMS*, **58**, 900–908

Tornado Fractal



©Josh Wurman. Radar reflectivity image of a hook echo with vortices moving into a tornado. This image exhibits geometric self similarity: smaller hook echos on a larger hook echo.

Vortex Gas Model

The vortex gas model [3] treats large collections of vortices as if they form a gas. The interaction of the vortices is governed by the laws of fluid dynamics. There is a notion of temperature (due to Lars Onsager): negative temperatures are hotter than positive temperatures, infinite temperatures are between the positive and negative temperatures ($-\infty = +\infty$), the closer a negative temperature is to zero the hotter it is.

- Vortices with negative temperature are smooth.
- Vortices with positive temperature are "balled up".
- Vortices with infinite temperature are fractal.
- As vortices stretch they cool down.

Vortex Gas Model and Tornados

The vortex gas model suggests, that as smooth slender vortices enter the developing tornado they stretch and cool down, supplying kinetic energy to the ambient flow in the tornado, thus heating it up. As the vortices are stretched further they would become fractal and transfer energy to small scales via the Kolmogorov 5/3 cascade. As this process repeats itself many times the ambient tornado vortex would achieves a quasi-equilibrium with its environment.

Suction Spots [4]

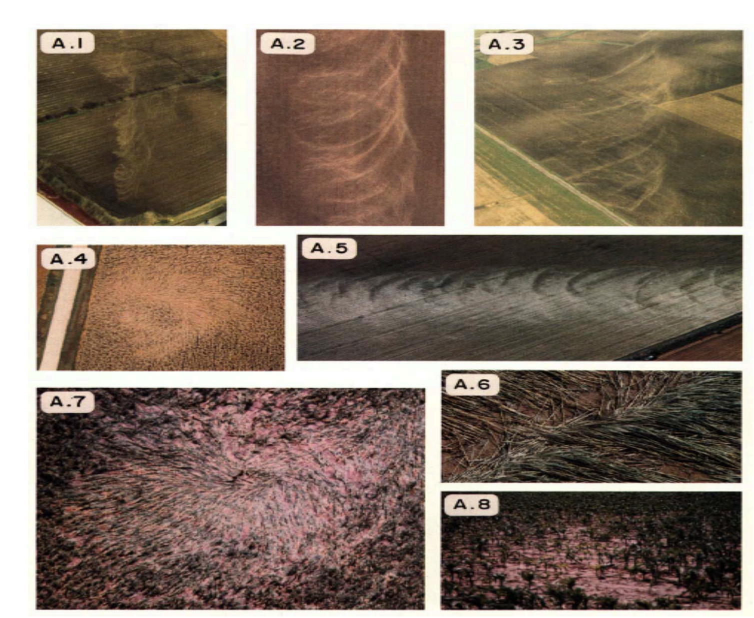


FIG. 14. Examples of ground marks left behind by suction vortices embedded inside tornadoes. Locations and dates of occurrences are: (A.1) Decatur, Illinois tornado 3 April 1974; (A.2) Magnet, Nebraska tornado, 6 May 1975; (A.3) Homer Lake, Indiana tornado, 3 April 1974; (A.4) Dubuque, Iowa tornado, 28 September 1972; (A.5) and (A.6) Pearsall, Texas tornado, 15 April 1973; (A.7) Mattoon Lake, Illinois tornado, 21 August 1977; (A.8) Grand Island, Nebraska tornado, 3 June 1980.

Tracks left by high-energy vortices within a tornado, some as narrow as 30 cm. Some of these paths appear to originate outside the tornado and intensify as they move into the tornado. These seem to make a partial revolution around the tornado then dissipate. Others appear to originate in the tornado. This suggests a Hopf bifurcation in the velocity field.

Modeling Suction Spots

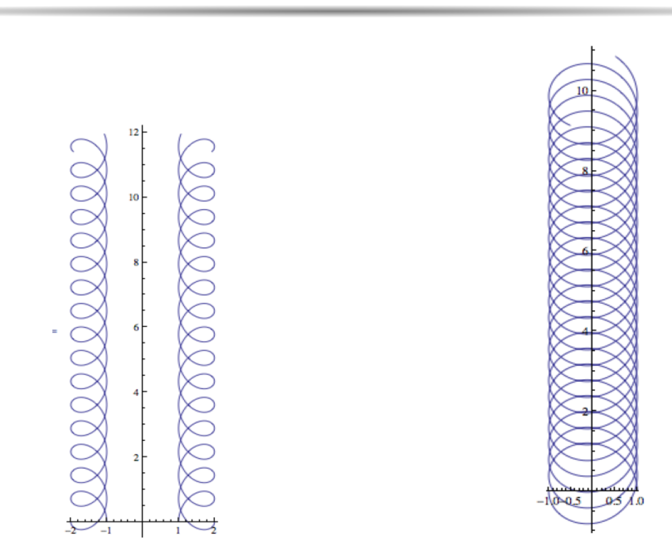


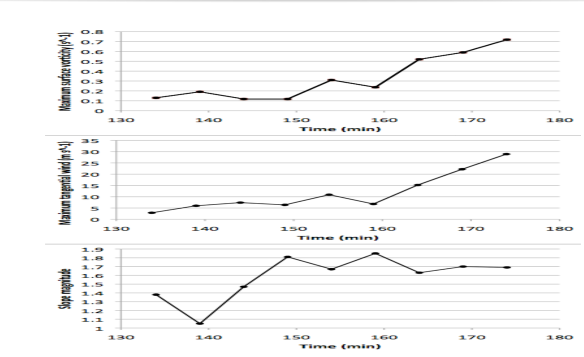
FIGURE 6. Graphs of numerical solutions of suction vortex tracks

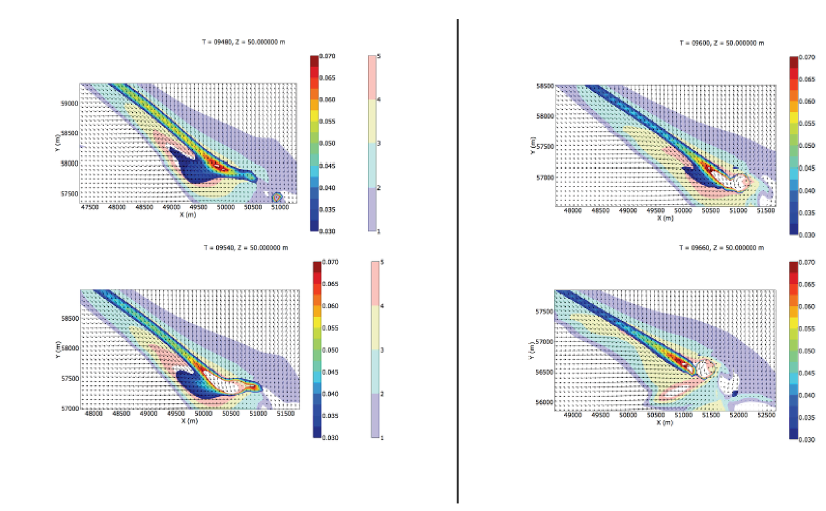
Suction spots modeled using the 2-D vortex gas theory.

More Results of [6]

Studies by Wurman and collaborators of mobile Doppler radar data obtained from intercepts of tornadic storms seem to support the vortex gas model method of tornadogenesis and maintenance, with high energy single-cell vortices being produced by shear either inside or outside the vortex. These vortices appear to be associated with transient intense updrafts.

Time series simulations





Numerical model (CM1) at University of OK and NSSL produced results consistent with [2] and [6].

Energy Spectrum and Power Laws of [2, 6]

Results in [6] suggest that subvortices moving in tornados have negative temperature and their horizontal cross-sections are fractal. Assuming the cross-sectional dimension of tornados and vorticity obey a power law, we can describe the energy spectrum in the form $E(k) \sim k^{-\gamma}$; for $T < 0$ we obtained $\gamma \geq 2$. In the case $T > 0$ we have $\gamma = 0$. The energy loss in the vortex filament as the temperature goes from $T < 0$ to $T > 0$ (vortex cooling down) results in an increase of the kinetic energy in the surrounding flow. Energy evolution in time is governed by $\partial_t E(k) + 2k^2 R^{-1} E(k) = Q(k)$ [3]. For small k the energy increase will be consistent with an increase in γ thus indicating a transfer of energy from smaller to larger scales.

References

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