A New Method for Calculating Total Vertical Motion in Isentropic Space

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Background

A full isentropic vertical motion (ω_θ) has long been difficult to calculate in an operational forecasting environment (Market et al. 2000). However, recent interactions with the Springfield, MO (SGF) National Weather Service Office spurred the development of the current method of estimating the total ω_θ. During the winter of 2012-13, this expression was developed into its current form, and initial tests were undertaken to test it's usefulness. Using output from NCEP's operational NAM-WRF model, individual points were evaluated for isentropic surfaces with condensation pressure differences < 20 mb and regions beneath where model precipitation occurred. The results of these initial trials are detailed in the present work.

Broader trials are currently underway. The ensuing work will address fields of ω values over large areas, compared statistically to the fields from dynamically consistent numerical model solutions. These results will be presented at a later date.

Prior Work on ω_θ

Isentropic Omega Equation (Moore 1993) : A detailed, illustrated treatise on the nature of ω_θ.

$$\omega_\theta = \underbrace{\left(\frac{\partial p}{\partial t}\right)_\theta}_{A} + \underbrace{V \bullet \nabla_\theta p}_{B} + \underbrace{\frac{\partial p}{\partial \theta}\frac{d\theta}{dt}}_{C}$$

- Term A – local tdy of the θ sfc
- Term B – pressure transport
- Term C – diabatic heating

Balanced Terms (Saucier 1955):

- On the synoptic scale, Terms A and C should balance one another.
- Therefore, the pressure transport term (B) should account for all vertical motions adequately.
- Not quite accurate….

Evaluation of All Terms (Market et al. 2000):

"The total mean ω_θ was of the same sign, but larger in magnitude than just the transport term by roughly a factor of two… The form of ω_θ where only the diabatic term is omitted *captured about **two thirds of the total** $\boldsymbol{\omega_\theta}$*; inclusion of the local pressure tendency term with the transport term gave a better result than the transport term alone."

Still, no workable operational solution emerged.

Study Purposes & Objectives

Develop and test a new expression for total vertical motion in isentropic space

(1) Derivation

(2) Evaluation

This work shows promise for improved, rapid, and relatively simple calculations of ω_θ for the operational environment.

Equation Development

Estimating the Diabatic Term:

Using the expression derived by Emanuel (1987) and revised by Cammas et al. (1994), we pose the diabatic heating as:

$$\frac{d\theta}{dt} = \omega\left(\frac{\partial\theta}{\partial p} - \frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}\right)$$

For an ascending atmosphere, we know there will be an adiabatic change and, beyond the LCL, a diabatic one:

Adiabatic — Diabatic

$$\frac{d\theta}{dt} = \omega\left(\underbrace{\frac{\partial\theta}{\partial p}}_{\text{Adiabatic}} - \underbrace{\frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}}_{\text{Diabatic}}\right)$$

So we:

1) multiply through by the dry static stability, and
2) assume that the background vertical motion is approximated by the adiabatic vertical motion alone:

$$\frac{d\theta}{dt}\frac{\partial p}{\partial\theta} = \omega_{adia}\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial p} - \frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}\right)$$

We substitute ω_{adia} for the total ω, because in a synoptic scale environment, layered clouds and stratiform precipitation form because of upglide and the ascent of isentropic surfaces. The thinking here is akin to the background forced ascent that must take place in order for a parcel to reach its LCL and LFC and lead ultimately to convection. In the same way that we approach Γ_s as merely Γ_d with a modification for condensing moisture and latent heat release, we view the total vertical motion as the background, dynamically-forced ascent with a similar "correction" for condensing moisture and latent heat release.

Equation Development

This is reasonable, as the diabatic (latent) contribution cannot engage without the broader, synoptic (adiabatic) forcing present first for lifting to saturation. So…

$$\frac{d\theta}{dt}\frac{\partial p}{\partial\theta} = \omega_{adia}\frac{\partial p}{\partial\theta}\left(\frac{\partial\theta}{\partial p} - \frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}\right)$$

$$\frac{d\theta}{dt}\frac{\partial p}{\partial\theta} = \omega_{adia} - \omega_{adia}\frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}\frac{\partial p}{\partial\theta}$$

Now calculate $\left.\frac{\partial p}{\partial t}\right)_\theta$ and $\vec{V}\bullet\vec{\nabla}p$ to get ω_{adia} .

The result is treated as the diabatic contribution, $\frac{d\theta}{dt}\frac{\partial p}{\partial\theta}$

which then gets added onto the existing ω_{adia}

to achieve a total isentropic vertical motion ω_θ thusly:

$$\omega_\theta = \underbrace{\left(\frac{\partial p}{\partial t}\right)_\theta + V \bullet \nabla_\theta p}_{} + \frac{\partial p}{\partial\theta}\frac{d\theta}{dt}$$

$$\omega_\theta = \omega_{adia} + \overbrace{\frac{\partial p}{\partial\theta}\frac{d\theta}{dt}}$$

$$\omega_\theta = \omega_{adia} + \left[\omega_{adia} - \omega_{adia}\frac{\gamma_m}{\gamma_d}\frac{\theta}{\theta_e}\frac{\partial\theta_e}{\partial p}\frac{\partial p}{\partial\theta}\right]$$

Data and Methods

Source for gridded fields:

- NCEP's NAM WRF model
  - Thinned 80 km gridded fields
  - 6-hourly output

Selection of areas to evaluate:

- Regions of significant modeled precipitation
  - >0.20" (5 mm) over 6 hours
- Isentropic surface well above ground level
  - ~700 mb
- Isentropic surfaces saturated or near to saturation
  - Condensation pressure differences of <50 mb

Cases selected:

- Case 1: 21 Nov 2012 @ 18Z (30 hr NAM sol'n)
  - 300 K being evaluated
- Case2: 01 Dec 2012 @ 00Z (36 hr NAM sol'n)
  - 292 K being evaluated
- Case 3: 17 Jan 2012 @ 18Z (18 hr Nam sol'n)
  - 300 K being evaluated

Results

Comparing new ω with accepted:

- Evaluations
  - Manual
  - On single points

Case values selected:

- Case 1: 21 Nov 2012
  - Modeled: **-3.0 μb s^{-1}** Calculated: **-3.1 μb s^{-1}**
- Case2: 01 Dec 2012
  - Modeled: **-3.0 μb s^{-1}** Calculated: **-3.2 μb s^{-1}**
- Case 3: 17 Jan 2012
  - Modeled: **-5.5 μb s^{-1}** Calculated: **-6.7 μb s^{-1}**
- Proposed method ***over***estimated ω by 3%, 7%, and 22% in Cases 1, 2, and 3, respectively.

Summary & Future Work

•Isentropic omega has long been difficult to calculate in an operational environment.

•A new method is thus proposed that treats the entire vertical motion as a dry process with a moist "correction."

The method proposed here appears to capture the entire vertical motion, and with only modest over-estimation

•We are currently testing this approach on a daily basis, for broad areas and on several different isentropic surfaces to determine its broader utility. Any questions / comments can be sent to: mjs5h7@mail.missouri.edu

Literature Cited

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