A New Formulation for Equivalent Wind Speed and Power Calculations Using Data from the High Resolution Doppler Lidar

by

Aditya Choukulkar[1*], Yelena Pichugina[1], Ronald Calhoun[2], Robert Banta[3], Alan Brewer[3], Michael Hardesty[1]

[1] *Cooperative Institute for Research in Environmental Sciences, Boulder, CO*

[2] *Environmental Remote Sensing Group, Arizona State University, Tempe, AZ*

[3] *National Oceanic and Atmospheric Administration, Boulder, CO*

Abstract

The spurt of growth in the wind energy industry has led to the development of many new technologies to study this energy resource and improve the efficiency of wind turbines. One of the key factors in wind farm characterization is the prediction of power output of the wind farm which is a strong function of the turbulence in the wind speed and wind direction. A new formulation for calculating the expected power from a turbine in the presence of wind shear, turbulence, directional shear and direction fluctuations is presented. The power predictions using this formulation are compared to previous formulations.

---

[*] Corresponding author: Aditya Choukulkar, CIRES/NOAA, 325 Broadway, Boulder, CO 80305. Email: aditya.choukulkar@noaa.gov

1. Introduction

The recent surge in the technological development in the field of wind energy has seen development of turbines capable of producing power in the multi-MW range. With increased size and power capacity of wind turbines, come concerns of accurately predicting the power output from these turbines. Advances in wind turbine technology have resulted in wind turbines with hub-heights over 100 m and rotor diameters in excess of 80 m. Traditionally, wind measurements have been carried out using meteorological towers which provide point measurements at hub heights. With the increase in rotor swept area and hub heights, the impact of wind shear and turbulence intensity become increasingly relevant and point measurements from meteorological towers no longer are good representations of the wind interacting with the turbine.

Direct implications of these developments are in the areas of forecasting and wind turbine control. No longer is it enough to measure/predict wind speed at hub-height in order to estimate wind power production. In addition to the variability in wind speed and direction with height, the turbulence parameters such as fluctuations of velocity components and direction fluctuations are important for determining the wind energy content. The effect of wind speed turbulence and its effect on wind power production has been a subject of considerable research with Elliot and Cadogan (1990) showing the influence of wind shear and turbulence on wind turbine power curves. Wagner et al. (2009) showed that significant error in power measurement could result if the effect of wind shear was ignored and demonstrated the use of equivalent wind speed method for power curve measurement.

Wind speed turbulence is often accompanied by fluctuations in wind direction. The power output is calculated assuming the wind interacts with the rotor parallel to its axis. Misalignment of the wind direction with the rotor axis can significantly reduce the power output. Akhmatov (2007) describes the influence of wind direction on wind power fluctuations. Pedersen (2004) using the 3D actuator disk model showed that the power output is related to the yaw angle error through a $\cos^2$ relationship. But these studies do not quantify the effect of fluctuations in the wind direction. In addition, it is not uncommon to expect shear in wind direction along with shear in wind speed across the rotor swept area. Therefore, the combined effects of the shear in wind direction and wind speed need to be considered for accurate assessment of the wind power output. In this paper, theoretical formulations will be presented which account for the effects of direction fluctuations and shear along with wind speed turbulence and shear and their effects on power output. The rest of the paper is structured as follows. In section 2, we will derive an expression that accounts for turbulence in velocity and direction as well as shear in velocity and direction. An expression for equivalent wind speed will be developed. In section 3 the effect of these on wind power production will be quantified.

2. Theory

The effect of wind shear and wind direction on power output of wind turbines has been evaluated independently (Kaiser et al., 2007; Antoniou et al., 2009; Wagner et al., 2009 etc). However, these effects do not occur in isolation, but combined. Therefore the following formulation will attempt to quantify the combined effects of these parameters. To start off, the power available in the wind can be expressed in the form of wind energy flux or kinetic energy flux (Wagner et al., 2009) defined as:

$$P(t) = \frac{1}{2}\rho C_P A U(t)^3 \tag{1}$$

where ρ is the density of air, C_P is the coefficient of power, A is the rotor swept area and $U(t)$ is the wind speed. To account for the possibility of azimuthal angle variations in the airflow we re-write the equation as:

$$P(t) = \frac{1}{2}\rho C_P A\left[U(t)\cos\alpha(t)\right]^3 \tag{2}$$

where $\alpha(t)$ is the angle of the wind to the turbine axis. To account for the influence of turbulence in wind speed and direction, we write:

$$U(t) = \overline{U} + u' \text{ and } \alpha(t) = \overline{\alpha} + \alpha' \tag{3}$$

where, $\overline{U}$ and $\bar{\alpha}$ are the 10 minute means of the wind speed and wind direction respectively. Substituting equation (3) into equation (2) we get:

$$\begin{aligned} P(t) &= \frac{1}{2}\rho C_P A[\overline{U} + u']^3[\cos(\overline{\alpha} + \alpha')]^3 \\ &= \frac{1}{2}\rho C_P A[\overline{U} + u']^3\left[1 - \frac{(\overline{\alpha} + \alpha')^2}{2}\right]^3 \\ &= \frac{1}{2}\rho C_P A\overline{U}^3\left[1 + \frac{u'}{\overline{U}}\right]^3\left[1 - \frac{\overline{\alpha}^2}{2}\left(1 + \frac{\alpha'}{\overline{\alpha}}\right)^2\right]^3 \\ &= \frac{1}{2}\rho C_P A\overline{U}^3\left[1 + 3\frac{u'}{\overline{U}} + \frac{6}{2!}\left(\frac{u'}{\overline{U}}\right)^2\right]\left[1 - \frac{\overline{\alpha}^2}{2}\left(1 + 2\frac{\alpha'}{\overline{\alpha}} + \frac{\alpha'^2}{\overline{\alpha}^2}\right)\right]^3 \end{aligned} \tag{4}$$

The variation of the 10 minute mean power is considered most important for studying the wind farm characteristics. Therefore taking the 10 minute mean of the above equation:

$$\overline{P} = \frac{1}{2}\rho C_P A\overline{U}^3\left[1 + 3\left(\frac{\sigma_u}{\overline{U}}\right)^2\right]\left[1 - \frac{\overline{\alpha}^2}{2} - \frac{\sigma_\alpha^2}{2}\right]^3 \tag{5}$$

where σ_u^2 is the variance of wind speed and

σ_α^2 is the variance of direction.

From equation (5) we can write the turbulent equivalent wind speed as:

$$U_T = \sqrt[3]{\overline{U}^3\left[1 + 3\left(\frac{\sigma_u}{\overline{U}}\right)^2\right]\left[1 - \frac{\overline{\alpha}^2}{2} - \frac{\sigma_\alpha^2}{2}\right]^3} \tag{6}$$

The effect of wind shear can be accounted for by rewriting the wind speed in terms of the equivalent wind speed (see equation (4) in Wagner et al., 2009). Substituting the turbulent equivalent wind speed into the shear equivalent wind speed we get:

$$U_{eq} = \sqrt[3]{\frac{1}{A}\sum_{i=1}^{N} U_{Ti}^{3} A_i} = \sqrt[3]{\frac{1}{A}\sum_{i=1}^{N} \overline{U}_i^{3}\left[1+3\left(\frac{\sigma_{ui}}{\overline{U}_i}\right)^2\right]\left[1-\frac{\overline{\alpha}_i^{2}}{2}-\frac{\sigma_{\alpha i}^{2}}{2}\right]^3 A_i} \tag{7}$$

where σ_{ui}^2 is the variance of velocity fluctuations at *i*-th level

α_i is the angle of the wind with respect to the rotor axis at *i*-th level

$\overline{U}_i$ is the wind speed at the *i*-th level

$\sigma_{\alpha i}^2$ is the direction fluctuations at the *i*-th level

Therefore the equivalent wind energy content is given by:

$$\overline{P}_{eq} = \frac{1}{2}\rho C_P \sum_{i=1}^{N} \overline{U}_i^{3}\left(1+3\left(\frac{\sigma_{ui}}{\overline{U}_i}\right)^2\right)\left[1-\frac{\overline{\alpha}_i^{2}}{2}-\frac{\sigma_{\alpha i}^{2}}{2}\right]^3 A_i \tag{8}$$

3. Quantifying the Impact of Turbulence on Wind Power

From the equations derived in Section 2, it is possible to quantify the effect of turbulence and direction fluctuations on wind energy content. The wind energy content is strongly correlated with the wind power produced by the turbines and hence gives a reasonable estimate of the expected turbine performance in those conditions. Based on typical values of wind and direction shear as well as turbulence intensity, the variations in wind energy content are estimated using the Equation (8). These variations are presented in Figure 1.

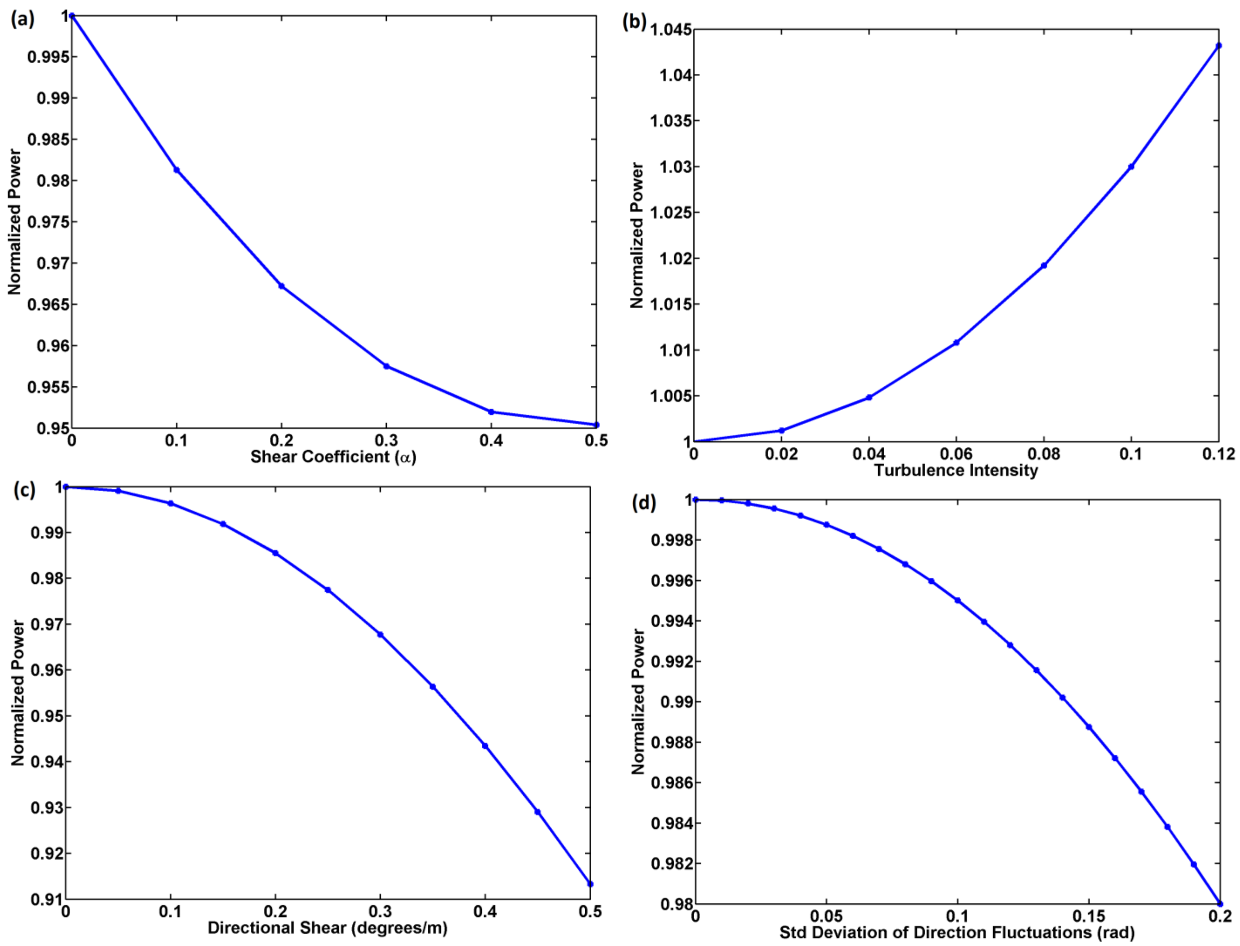

Figure 1. Modeling the theoretical effect of turbulence on wind energy content. (a) Theoretical impact of wind speed shear. (b) Theoretical impact of turbulence intensity. (c) Theoretical impact of direction shear. (d) Theoretical impact of directional fluctuations.

As seen from Figure 1(a), in theory higher values of turbulence intensity increase the energy content of the wind. Various studies (for example: Akhmotov 2007; Tindal et al., 2008) have shown that turbulence intensity increases power production at lower wind speeds and lowers power production near the rated wind speeds. Figures 1(b), 1(c) and 1(c) show that wind speed shear, direction fluctuations and wind direction shear reduce the available wind energy content. Therefore, it is clear that measurement and modeling efforts directed at forecasting expected wind power should aim to estimate these parameters accurately in the wind turbine rotor layer.

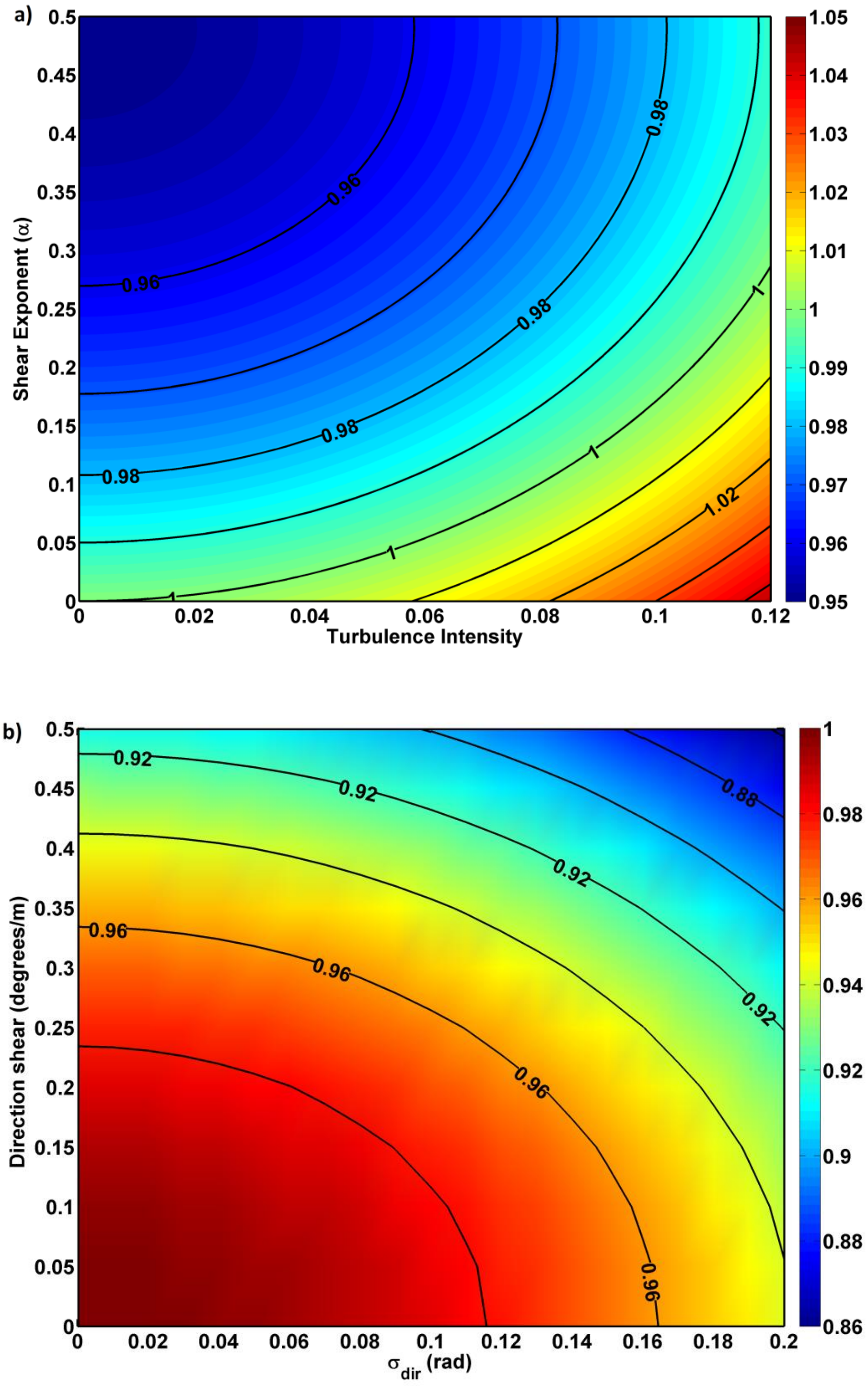

Figure 2. Combined effects of turbulence parameters on available wind energy content. The wind energy content is normalized with respect to the mean value in the absence of turbulence or shear. (a) Combined

effect of wind speed shear and turbulence intensity. (b) Combined effect of directional shear and direction fluctuations.

Since, these four effects do not occur individually, but occur in combination, studying the combined effects of these parameters is essential to understand the variability in available wind energy. Figure 2 shows the variation of wind energy content due to the combined effects of various turbulence parameters. It can be seen that the combined influence of turbulence, wind speed shear, wind direction shear and direction fluctuations can introduce power deficits of up to 10%. Therefore, measurement and forecasting efforts should aim to quantify these parameters in the turbine rotor layer with as high resolution in time and space as possible. Such methods can be important tools in wind turbine control to make the optimal control decision which will result in the maximum recovery of wind energy content. The availability of this information combined with lidar's capability to make measurements several kilometers upstream of the wind turbine location can enable a planned control response.

4. Power Calculations using Lidar Data

The High Resolution Doppler Lidar (HRDL) was deployed to the National Renewable Energy Laboratory (NREL) National Wind Technology Center (NWTC) to measure the winds both upwind and downwind of a research wind turbine. This measurement campaign was part of the Turbine Wake and Inflow Characterization Study (TWICS), a joint field program involving National Oceanic and Atmospheric Administration (NOAA), University of Colorado, Cooperative Institute for Research in Environmental Sciences (CIRES), Lawrence Livermore National Laboratory (LLNL) and NREL. The HRDL performed various scan configurations to envelop the vertical and horizontal extents of the wake from the turbine.

The HRDL developed at NOAA can provide measurements with 30 m range resolution either through fixed elevation horizontal slice scans or fixed azimuth vertical slice scans. The HRDL can produce data rich three dimensional measurements over a wind farm site allowing identification of fluid dynamic processes occurring over a wide range of scales (Pichugina 2012; Banta 2013). Although this experiment was designed for wake characterization, the data can be used for estimation of wind turbine inflow parameters as well.

Table 1. Scanning strategy during the TWICS field campaign. The complete sets of scans took 30 minutes to compete and were repeated continuously throughout the operating period.

Number of scans	Scan Name	AZ start	AZ end	El start	El end	Time (seconds)
1	2-5ppi	0	360	2	5	196
1	15-45ppi	0	360	15	45	130
2	RhiSurv	112	202	0	180	105
2	RhiTurbBob	125	131	-1	12	204
1	ppi4lvl	105	140	0	6	405
2	RhiTurbBob	125	131	-1	12	205
1	ppi4lvl	105	140	0	6	555

The scanning strategy during the TWICS field campaign is shown in Table 1. Of these scans, the “ppi4lvl” scans were best suited to provide data for this analysis. In addition, data was not available continuously as the experiment was focused on studying wakes during night time stable boundary layer conditions. The optimal interpolation technique (Choukulkar et al., 2012) was used to retrieve the horizontal wind fields for these scans. Next the wind fields were binned into vertical bins each of heights 10 m to create a vertical profile of wind speed and directions. From this data the wind shear and the direction shear can now be estimated. The wind shear and direction shear calculated within the rotor layer of the turbine at the TWICS site is shown in Figure 3. As seen from the figures, there is presence of considerable shear in wind speed and direction throughout the observation period.

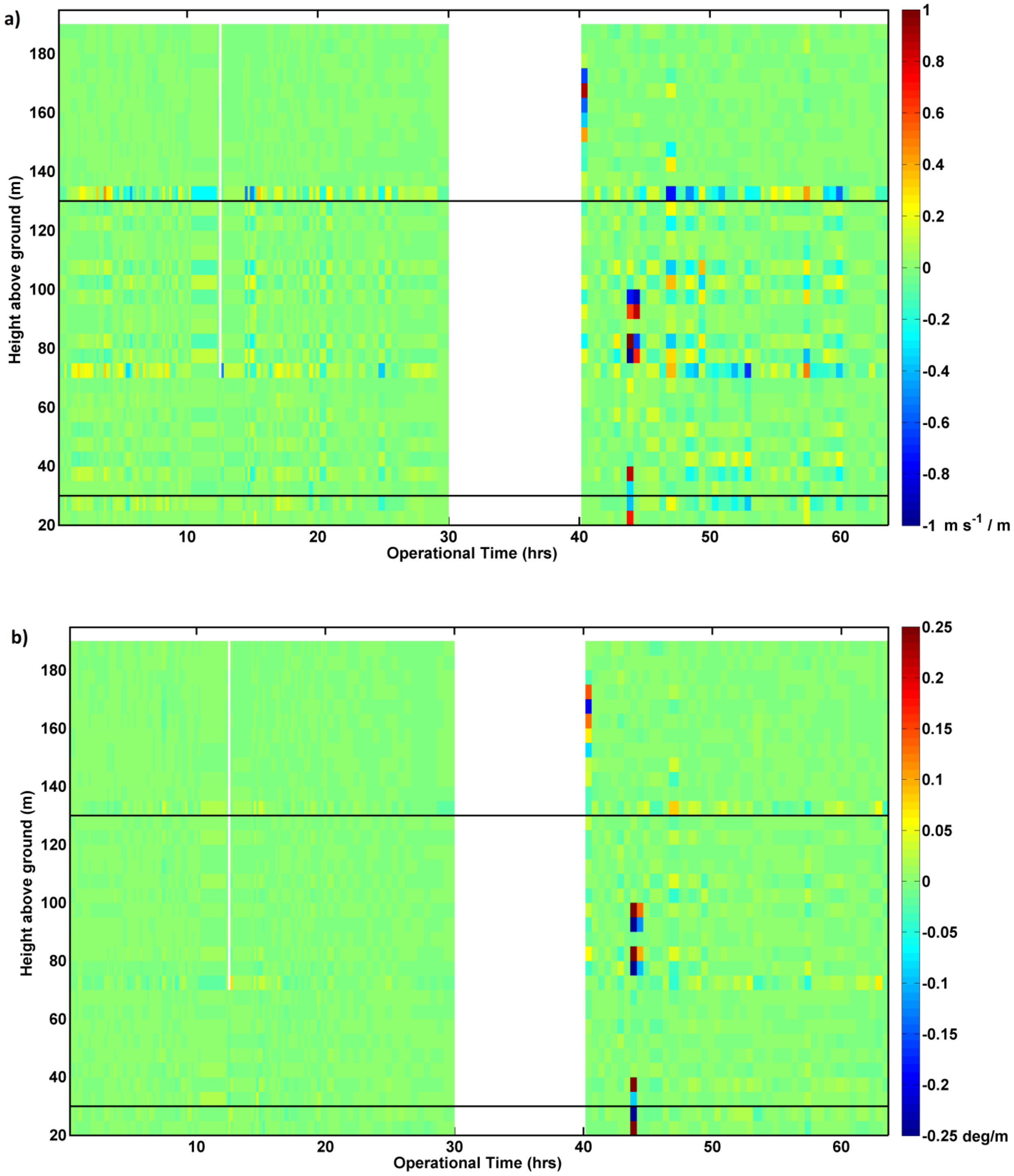

Figure 3. Wind speed and direction shear from the TWICS data-set. (a) Wind speed shear for the operational period between April 16th to 28th. (b) Wind direction shear for the operational period between April 16th to 28th. The black lines indicate the rotor swept area.

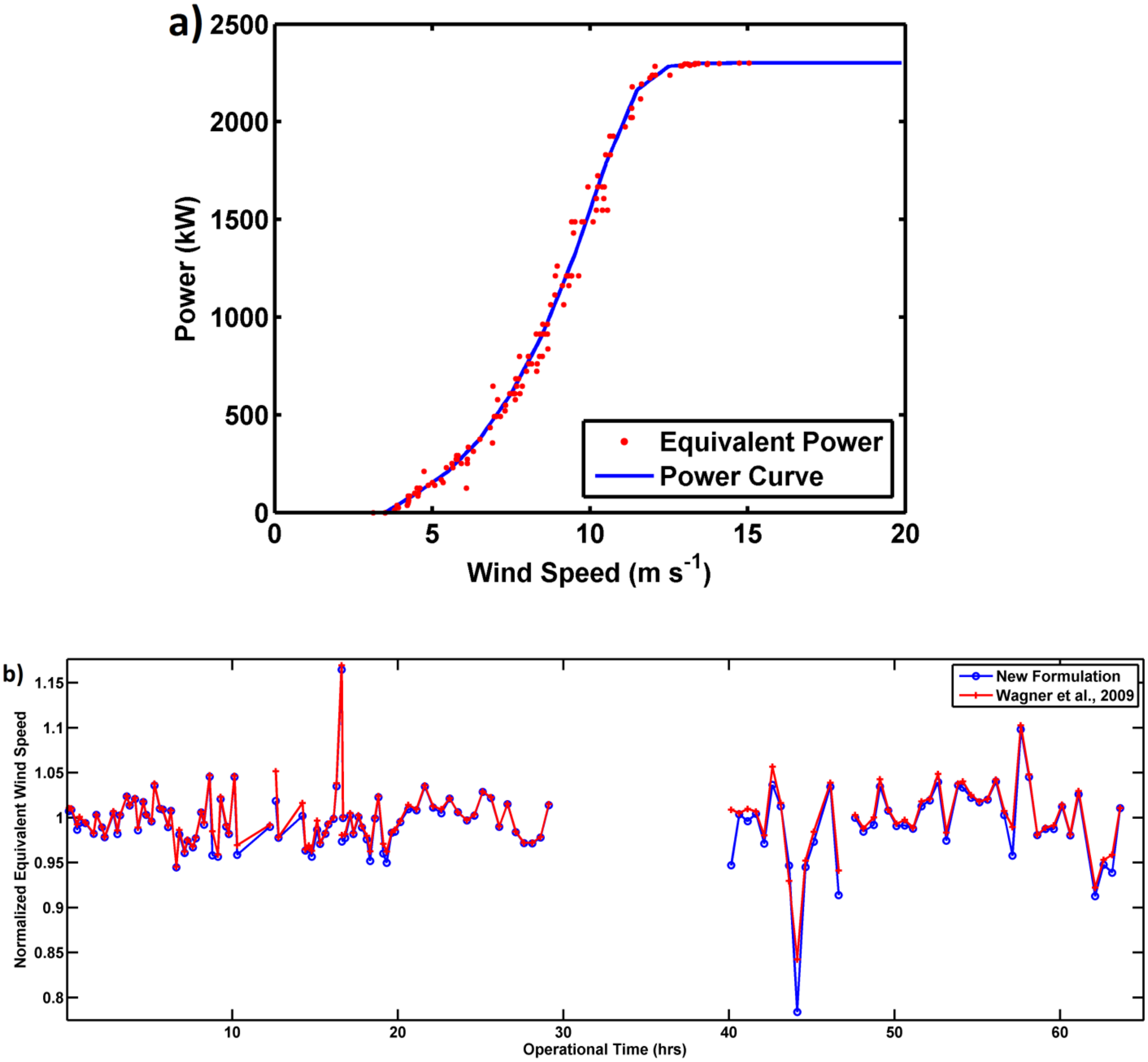

Figure 4. Impact of variations in wind speed and direction on power. (a) Comparison of power curve with Equivalent power computed using the new formulation. (b) Comparison of equivalent wind speed from Wagner et al (2009) and Equation (7) presented in Section 2.

These measurements are used to estimate the rotor equivalent wind speed using the formulation presented in this paper. The rotor equivalent wind speed is then used to estimate the power produced a Siemens 2.3 MW wind turbine using a standard power curve. As seen from Figure 4(a), the formulation predicts a significant scatter in the power production compared to the prediction from the power curve. In addition, comparison with the power estimated using the Wagner et al. (2009) formulation, as shown in Figure 4(b), indicates that the new formulation predicts significantly less available wind power. These theoretical conclusions need to be verified using measurements in order to test the validity of the assumptions made in these theoretical formulations.

5. Conclusion and Future Work

An expression for equivalent wind power has been derived which captures the effect of turbulence on available wind energy content. Through this equation the available wind power can be estimated more accurately. Using this equivalent wind power equation, the effect of wind shear, turbulence intensity, direction shear and direction fluctuations on wind power are estimated. In addition, it is shown that this equation allows estimation of the combined effect of various turbulence parameters. This can be used to develop optimal control strategies that can deploy the control action that produces the maximum power in the given conditions.

The results from the TWICS field study show the importance of accounting for the various wind parameters in estimating the wind energy content. Future work will entail comparing the wind power estimations based on these calculations to the actual wind power produced by the turbine.

6. References

Akhmatov V. (2007): Influence of wind direction on intense power fluctuations in large offshore wind farms in the North Sea. *Wind Engineering*, **31**(1), 59-64.

Antoniou I., Pedersen S.M., Enevoldsen P.B. (2009): Wind shear and uncertainties in power curve measurement and wind resources. *Wind Engineering*, **33**(5), 449-468.

Banta R.M. (2008): Stable boundary layer regimes from the perspective of a low-level jet. *Acta Geophysica*, **56**(1), 58-87.

Banta R.M., Pichugina Y.L., Kelly N.D. Hardesty R.M., Brewer W.A. (2013): Wind energy meteorology: Insight into wind properties in the turbine-rotor layer of the atmosphere from high resolution Doppler lidar. *Bulletin of the American Meteorological Society*, **94**, 6, 883-902.

Choukulkar A., Calhoun R., Billings B., Doyle J.D. (2012): A Modified Optimal Interpolation Technique for Vector Retrieval for Coherent Doppler LIDAR. *Geoscience and Remote Sensing Letters, IEEE*, **9** (6), 1132-1136.

D. L. Elliot, J. B. Cadogan, Effects of wind shear and turbulence on wind turbine power curves, EWEC 1990, September 10–14, Madrid, Spain.

Kaiser K., Langreder W., Hohlen H., Højstrup J. (2007): Turbulence correction for power curves. *Wind Energy*, 159-162.

Pedersen T.F. (2004): On wind turbine power performance measurement at inclined airflow. *Wind Energy*, **7**, 163-176.

Pichugina, Y.L., Banta R.M., Brewer W.A., Sandberg S.P., Hardesty R.M. (2012): Doppler Lidar–Based Wind-Profile Measurement System for Offshore Wind-Energy and Other Marine Boundary Layer Applications. *Journal of Applied Meteorology and Climatology*, **51**, 327–349.

Tindal A., Johnson C., LeBlanc M., Harman K., Rareshide E., Graves A.M. (2008): Site-specific adjustments to wind turbine power curves. *Proceedings of the American Wind Energy Association WINDPOWER Conference* (Houston, TX).

Wagner R., Antoniou I., Pedersen S.M., Courtney M.S., Jørgensen H.E. (2009): The influence of the wind speed profile on wind turbine performance measurements. *Wind Energy*, **12**(4), 348-362.