Evolution of Snow-size Spectra by the Growth Processes of vapor Deposition, Aggregation and Riming

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A steady-state snow growth model (SGM) is formulated by microphysical growth processes of vapor deposition, aggregation and riming. SGM is capable of predicting the temporal and vertical evolution of ice particle size distribution (PSD) by using radar reflectivity (Z_w), supersaturation, temperature, liquid water content (LWC) and ice particle shape dependent mass-dimension power laws, and by solving the zeroth- and second- moment conservation equations with respect to mass. It appears that the riming process is essential in characterizing the snowfall rates and leads to the snowfall rates significantly greater than those produced by the vapor deposition and aggregation alone. Moreover, alteration in cloud condensation nuclei (CCN), due to aerosols, can modify cloud droplet SD (size distribution) and therefore change the snowfall rate. So, snowfall rate is sensitive to the shape of cloud droplet SD.

Ice particle growth rates are uniquely formulated in the SGM in terms of ice particle mass-dimension (m-D) power laws ($m = \alpha D^{\beta}$), and in this way the impact of ice particle shape on particle growth rates and fall speeds is accounted for. These growth rates appear qualitatively consistent with empirical growth rates, with slower (faster) growth rates predicted for higher (lower) β values. It is well known that for a given ice particle habit, the m-D power law for the smallest ice particles differs considerably from the power law for the largest particles, with β being much larger for the smallest crystals. Our recent work quantitatively predicts β and α for frontal clouds as a function of maximum dimension D where the m-D expression is a second-order polynomial in log-log space. By tailoring the m-D power law to the relevant PSD moments, the SGM ice particle growth rates and fall speeds are represented more accurate and realistic. It is speculated that by implementing this new m-D treatment in any cloud resolving model or climate model, the ice particle growth rates will become more accurate. The predicted size spectra by SGM are in good agreement with observed spectra from aircraft measurement during Lagrangian spiral descents through frontal clouds.

1. Theoretical Achievements

Since the lowest radar reflectivity (Z_w) over complex topography is often considerably above cloud base, radar quantitative precipitation estimates (QPE) often underestimate the precipitation at ground level. In this study, we develop a snow growth model (SGM) which is capable of being initialized by Z_w at the lowest reliable radar echo and consequently improves QPE at ground level.

It is well known that mass-dimension (m-D) and area-dimension (A-D) power laws (e.g. $m=\alpha D^{\beta}$) for small ice particles differ considerably from the power law for the large particles. To overcome this problem, β and α are predicted for frontal clouds as a function of D (Mitchell *et al.* 2014):

$$\beta = a_1 + 2a_2 \ln(D) \tag{1}$$

$$\alpha = \frac{exp\{a_0 + a_1 \ln(D) + a_1[\ln(D)]^2\}}{D^{\beta}} \tag{2}$$

Where a_0, a_1, a_2 are constants for a certain cloud type. These equations are derived from 2nd-order polynomial fit in log(*D*)-log(*m*) space:

$$\ln(m) = a_0 + a_1 \ln(D) + a_2[\ln(D)]^2 \tag{3}$$

Although riming effect on D is negligible, it considerably changes the m and A. Here, a method is introduced to calculate rimed mass and area from unrimed mass and area, and from maximum mass and area that can be obtained by riming. First, rimed ice water content (IWC) is calculated as:

$$\frac{\Delta IWC}{\Delta z} = \int_{150}^{D_{max}} A(D)N(D)\overline{E_d}QdD \tag{4}$$

Where *N* is number concentration, E_d is collision efficiency, and *Q* is liquid water content (LWC). From that, intercept in m-D power law can be calculated:

$$\frac{\alpha}{\alpha_u} \approx \frac{IWC}{IWC_u} \tag{5}$$

Where subscript *u* denotes unrimed conditions. In this calculation, it is noted that $D \approx D_u$ and $\beta \approx \beta_u$ where the latter is based on empirical results, discussed in Mitchell *et al.* (2014). Also discussed in that paper is the empirical result for maximum rimed mass being 3.5 times unrimed mass for dendrites. From the calculations of mass, a riming factor (R) is defined for projected area which between 0 and 1, and then rimed projected area, γ, δ is calculated.

2. Model Setup

The SGM in study is based on the model by Mitchell *et al.* (2006) with significant improvement. Riming process is added to the model, in addition to the previous processes of vapor deposition and aggregation. The vertical profile of LWC and relative humidity over ice ($\mathrm{RH_i}$) is shown in Fig. 1. Our SGM is based on zeroth and second moment equations with respect to mass:

$$\frac{\partial N}{\partial z} = \frac{bN}{\lambda}\frac{\partial \lambda}{\partial z} - \frac{1}{V_{Nf}}\left[kCS^{k-1}\frac{dS}{dt} + \frac{\partial (wN)}{\partial z}\right] - \frac{\pi E I(\nu, b) N^2}{8\Gamma(b+\nu+1)\Gamma(\nu+1)\lambda^2} \qquad (6)$$

$$\frac{\partial \lambda}{\partial z} = \frac{\lambda}{(2\beta+b)N}\frac{\partial N}{\partial z} - \frac{1}{V_{Zf}}\frac{f(T,p,S)\lambda^{\beta}2\Gamma(\beta+\nu+2)}{\alpha(2\beta+b)\Gamma(2\beta+\nu+1)}$$

$$- \frac{\pi E I(\beta,\nu,b)N}{4(2\beta+b)\Gamma(2\beta+b+\nu+1)\Gamma(\nu+1)\lambda}$$

$$- \frac{2\gamma \bar{E}_d \square Q\, \Gamma(\beta+b+\delta+\upsilon+1)}{\alpha(2\beta+b)\Gamma(2\beta+b+\upsilon+1)\lambda^{\delta-\beta-1}} \qquad (7)$$

Where λ is slope parameter in Gamma size distribution, I is Gauss' hypergeometric function, E is aggregation efficiency, V_{Nf} is number concentration weighted fallspeed, a and b are fallspeed power law coefficients, V_{Zf} is radar reflectivity weighted fallspeed, and finally f is diffusional mass growth rate.

Compared to Mitchell *et al.* (2006), all the features explained in Sec. 1 are added to our SGM. Moreover, the thermal effect of diffusional growth has been added to the calculation of mass growth rate. Also, fall speeds are now calculated based on Mitchel & Hymsfield (2005) to avoid certain kinks in variables during the change in flow regimes.

3. Results

Fig. 2 shows the vertical profile of ice particle mass for rimed, unrimed, and maximum rimed condition. Similar results are observed for projected are (figure not shown). α, β, γ and δ are constant in most models which is not realistic, but they are variable in our model. This agrees with observations and theory, since in the absence of riming, particles are more compact at cloud top and become more branched during the decent. This is due to aggregation and diffusion. Riming causes an increase in mass and projected area.

Fig. 3. Depicts the vertical profile of snowfall rate for various combinations of processes in SGM. From cloud top to cloud base, increase in $\overline{D}$ leads to increase in fall speed and consequently decrease in N, since mass flux is conserved. Compared to deposition, aggregation produces larger particles with faster fall speed and lower number concentration. Moreover, riming increases ice particle fall speed and significantly enhances the *IWC*. So, snowfall rate has increased dramatically.

4. Conclusions

By using the new m-D and A-D relationships based on 2nd-order polynomial fit, the model characteristics are represented more accurately and realistically, compared to power laws. Such new approach can be used in various global and regional climate models to better represent the microphysical properties of cloud ice particles.

Although riming often has a minor impact on ice particle size, its impact on ice particle mass and projected area can be considerable. A new method is introduced in this study that calculates the rimed mass fraction of an ice particle, based on both field observations (of rimed and unrimed snowfall) and theory. The treatment for riming is explicit, accounting for the dependence of collision efficiency on droplet and ice particle size using both hydrodynamic theory and experimental measurements. Snowfall rates are found to be sensitive to the median mass dimension of the cloud droplet size distribution.

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References

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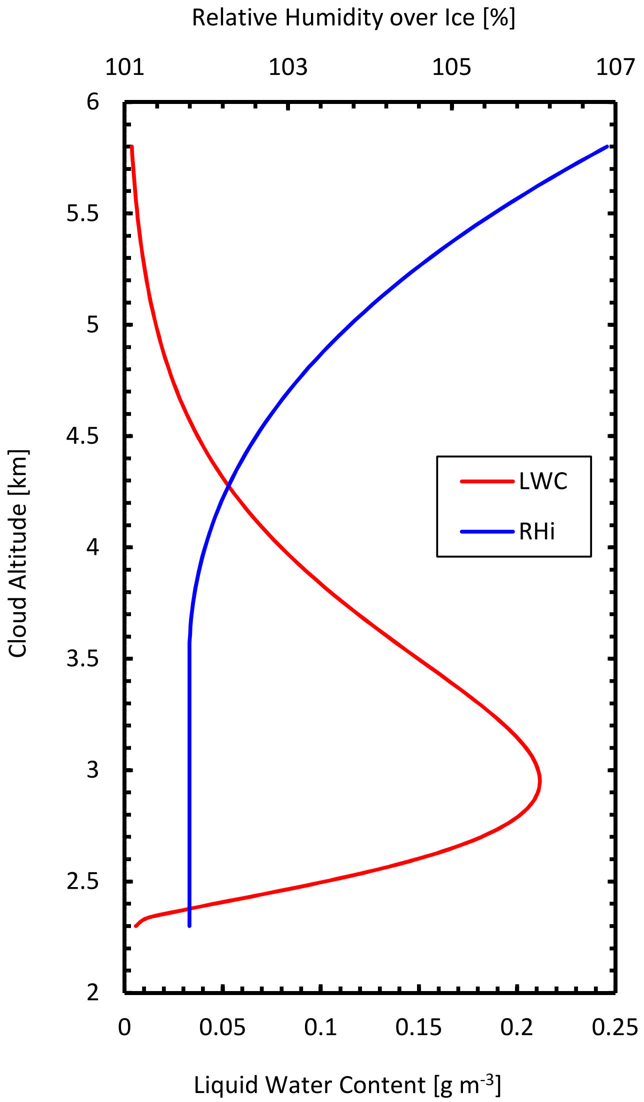

Fig. 1. Vertical profile of liquid water content and relative humidity over ice, used to initialize the SGM.

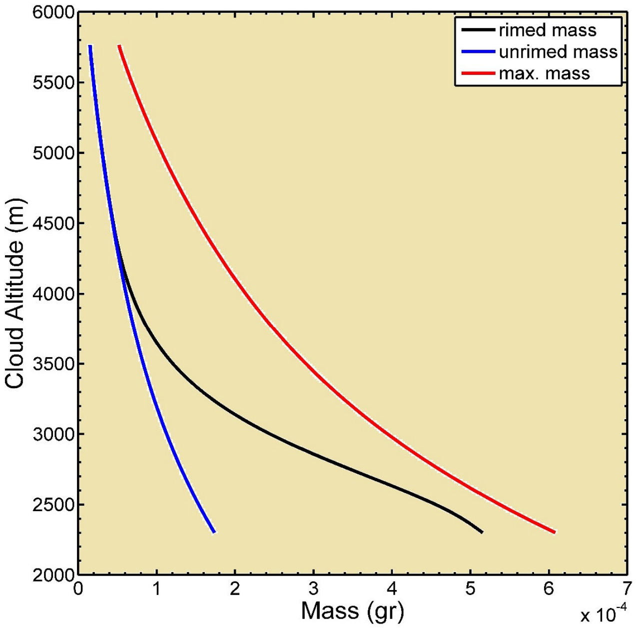

Fig. 2. Vertical profile of ice particle mass for rimed, unrimed, and maximum rimed condition.

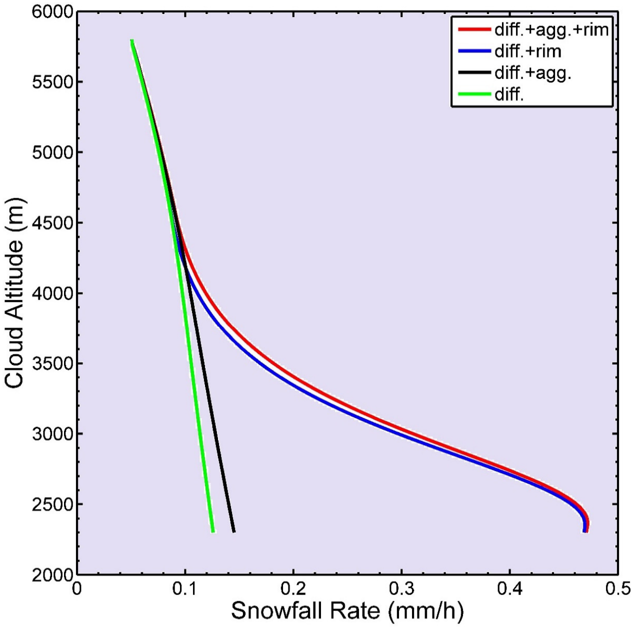

Fig. 3. Vertical profile of snowfall rate for various combinations of processes in SGM.