7.1 The Schneider Electric Numerical Turbulence Forecast Verification using In-situ EDR observations from Operational Commercial Aircraft

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1. Introduction

Schneider Electric delivers a numerical Enhanced Flight Hazard (EFH) turbulence product (Lennartson and McCann, 2014) that can be used for both flight planning (Strategic) as well as flight-following (Tactical) use. It is very important that an aviation turbulence forecast show reasonable accuracy spatially, temporally, and quantitatively to be a credible source for operational flight decision support. Shown in this document will be a volume evaluation of the EFH product to test its credibility in operational use by comparing it with turbulence observations and with verification of another well recognized numerical turbulence forecast, the Graphical Turbulence Guidance (GTG); Version 2.5, Sharman et al., 2006). The EFH and GTG approach turbulence forecasting very differently. The EFH is a deterministic forecast, and GTG is a weighted ensemble of diagnostic forecasts. Verification was conducted by AvMet Applications, Inc. (AvMet). An analysis of EFH and GTG forecasts was conducted using data for a three month period from August 2014 to October 2014.

In the following sections, the Schneider Electric EFH forecast system will be briefly described; the verification methodology explained; the verification results and analysis shown; and finally, the conclusions and direction of future work are presented.

2. Description of the EFH turbulence forecast

The EFH numerical turbulence forecast is a deterministic forecast derived from a numerical weather prediction model. The forecast is model agnostic and focuses on four primary sources of turbulence: mountain wave, boundary layer, upper-level clear air, and convective turbulence. Output from all modes is integrated into one eddy dissipation rate (EDR; Cornman et al., 1995) value, the rate at which turbulent energy dissipates into the atmosphere. Figure 1 shows conceptually how the forecast turbulence sources can change throughout a flight path and where they can potentially be enhanced where two or more sources are present at a given point at a given altitude.

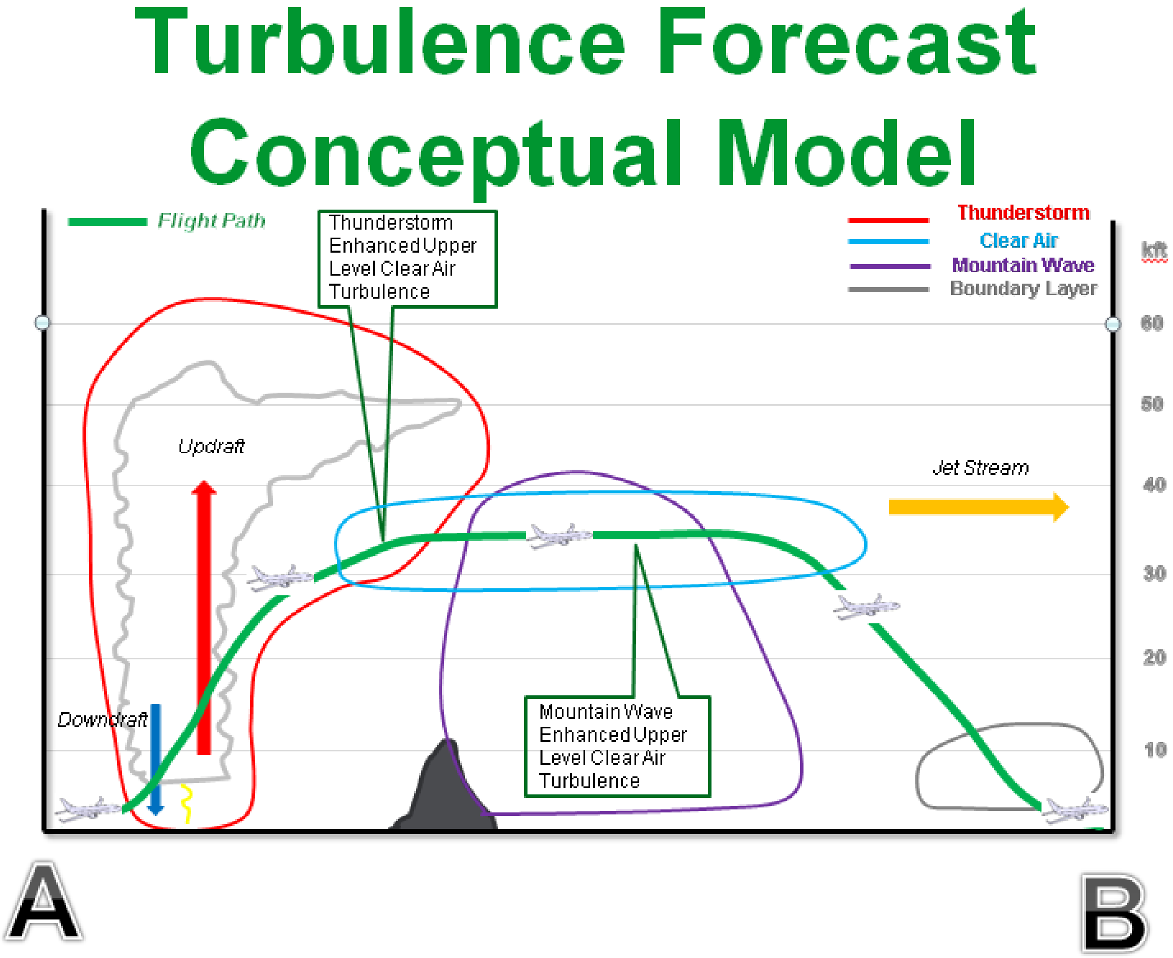


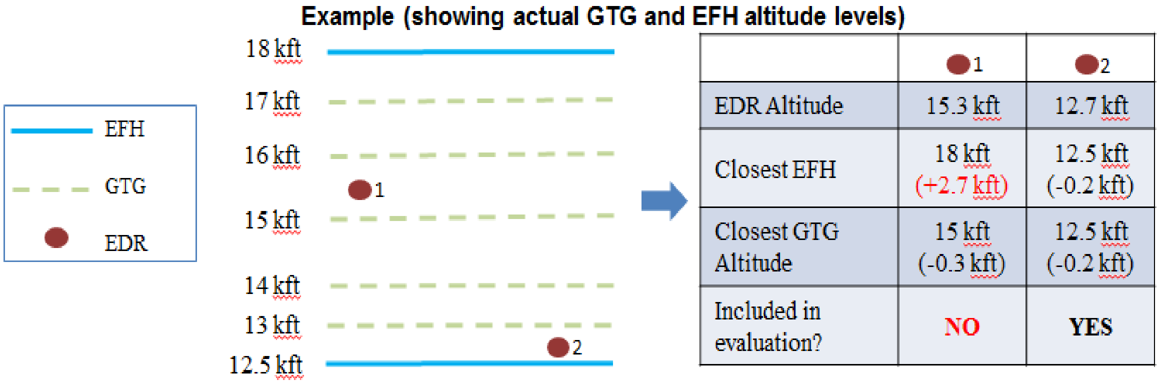
Figure 1: *Conceptual model featuring the four modes of the turbulence forecast: mountain wave, boundary layer, upper level clear air, and convective. Also illustrated is the integration of the modes where constructive interference can enhance turbulence.*

2.1 Mountain Wave

The mountain wave component of the turbulence forecast calculates the turbulence potential of waves breaking over high terrain. This calculation takes into consideration the attributes of the mountain(s) such as asymmetry and concavity as well as the wind direction at the mountain top level. Also included are the effects of a hydraulic jump and the reflection/resonance of terrain-induced mountain waves (McCann, 2006).

2.2 Boundary Layer

Boundary-layer turbulence results from the interaction of the lower atmosphere with the earth's surface. The turbulence values are calculated from the surface to the variable-from-point-to-point top of the boundary layer defined as where boundary-layer EDR becomes zero (McCann, 2001).



	●1	●2
EDR Altitude	15.3 kft	12.7 kft
Closest EFH	18 kft (+2.7 kft)	12.5 kft (-0.2 kft)
Closest GTG Altitude	15 kft (-0.3 kft)	12.5 kft (-0.2 kft)
Included in evaluation?	**NO**	**YES**

Figure 2: *Observation 1 is an example of a rejected sample and observation 2 is an example of an accepted sample. The solid light blue lines represent EFH levels and dashed light green lines represent GTG levels.*

2.3 Upper-Level Clear-Air Turbulence

Upper-level Clear-Air Turbulence (CAT) is computed by applying Lighthill-Ford spontaneous imbalance theory to identify gravity waves that locally alter the environment's wind shear and stability. The altered state may be enough to lower the Richardson number to less than 0.25 thus initiating Kelvin-Helmholtz instability (Knox et al., 2008).

2.4 Convective

The convective turbulence component computes the turbulence related to vertical motions within convective clouds. This component is proportional to the updraft/downdraft strength (Byers and Braham, 1949). Furthermore, there are two additional thunderstorm features that are taken into account in the turbulence computation, gravity waves emitting outward from storm updrafts and the mountain wave-like turbulence associated with overshooting thunderstorm tops (Lennartson and McCann, 2014).

3 Verification Methodology

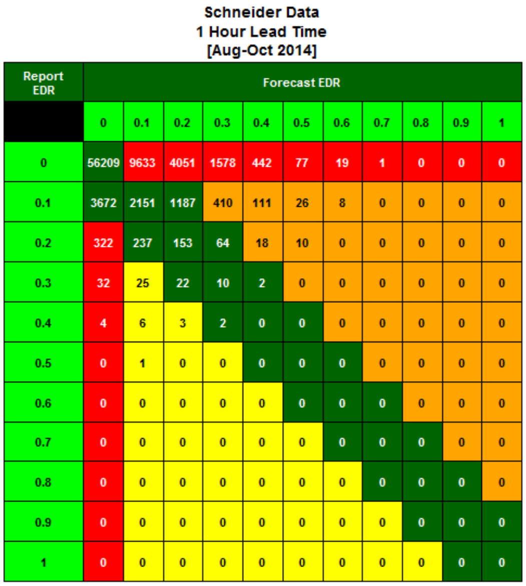
Forecasts were validated against 121,576 EDR turbulence observations from commercial airlines over a span of 68 days from August 2014 to October 2014. The forecasts were validated over the common EFH and GTG forecast lead times of 1, 2, 3, 6, 9 and 12 hours. Sampling from the forecasts was defined horizontally as the average EDR forecast value within a 50 mile radius of each observation. Vertically, samples were defined as a mutual forecast level from EFH and GTG within 1 kft of the observation (Figure 2).

All null values for both products were considered as EDR values of 0 forecast (i.e., no turbulence), and both forecast datasets were treated as direct representations of observed in-situ EDR values.

4 Verification Approach and Definitions

A combination of three methods to analyze the forecasts are used: a contingency table showing the correlation between forecast and observed EDR values, graphs showing daily statistics, and summary tables showing all relevant statistics from both forecast models.

The contingency table shows counts of forecast EDR versus observed EDR in discrete bins (Figure 3). Ideally, forecasts and observations are perfectly correlated and all data lie along the diagonal of the table. Off-diagonal elements indicate conditional biases in the forecasts.



Schneider Data
1 Hour Lead Time
[Aug-Oct 2014]

Report EDR	Forecast EDR										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	56209	9633	4051	1578	442	77	19	1	0	0	0
0.1	3672	2151	1187	410	111	26	8	0	0	0	0
0.2	322	237	153	64	18	10	0	0	0	0	0
0.3	32	25	22	10	2	0	0	0	0	0	0
0.4	4	6	3	2	0	0	0	0	0	0	0
0.5	0	1	0	0	0	0	0	0	0	0	0
0.6	0	0	0	0	0	0	0	0	0	0	0
0.7	0	0	0	0	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0

Figure 3: *Ranges of forecast turbulence versus observed EDR values where dark green boxes are perfect hits, red boxes are misses, yellow boxes are under-forecasts, and oranges are over-forecasts.*

Cells in the table are classified into four categories: “perfect” hits, under-forecasts, over-forecasts and misses. The respective definitions for each box represented in Figure 3, and the color coding applied to each, are given below:

Condition for Perfect Hit (dark green):
Case 1:
(Observed EDR > 0.1) AND
(Forecast EDR – Observed EDR ≤ 0.1)

Case 2:
(Observed EDR ≤ 0.1) AND
(Forecast EDR ≤ 0.1)
Condition for Over-forecast Hit (orange):
(Observed EDR > 0.1) AND
(Forecast EDR – Observed EDR ≥ 0.2)

Condition for Under-forecast Hit (yellow):
(Observed EDR > 0.3) AND
Forecast EDR > 0.1) AND
(Observed EDR – Forecast EDR ≤ 0.1)

Condition for False Alarm (red):
(Observed EDR > 0.1) AND
(Forecast EDR ≤ 0.1)

From the data in the table, summary statistics can be derived to measure performance such as: perfect hit rate, over-forecast rate, under-forecast rate, overall hit rate, false alarm rate, and false alarm ratio and are defined below:

$$\text{Perfect Hit Rate} = \frac{\#\ \text{Perfect Hits}}{\#\ \text{EDR Observations}}$$

$$\text{Overforecasted Hit Rate} = \frac{\#\ \text{Overforecast Hits}}{\#\ \text{EDR Observations}}$$

$$\text{Underforecasted Hit Rate} =$$

$$\frac{\#\ \text{Underforecast Hits}}{\#\ \text{EDR Observations}}$$

$$\text{Hit Rate} = \frac{\#\ \text{Hits}}{\#\ \text{EDR Observations}}$$

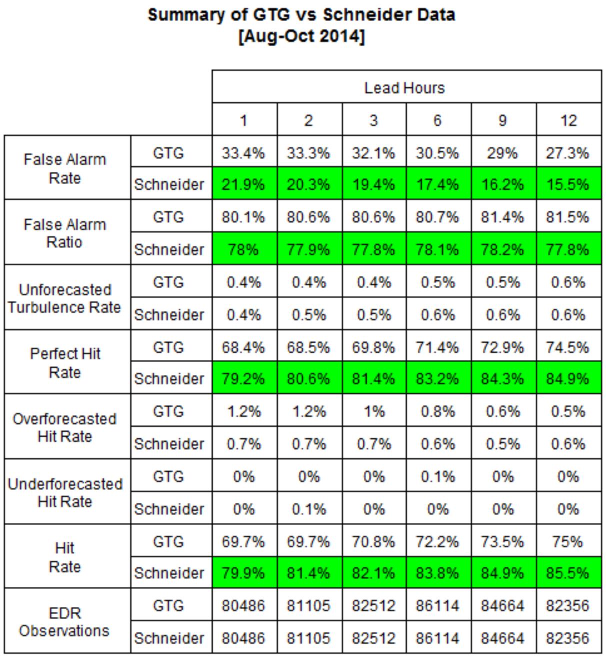
$$\#Hits = \#PerfectHits + \#OverforecastHits + \#UnderforecastHits$$

$$FalseAlarmRate = \frac{\#FalseAlarms}{\#ObservedEDR \leq 0.1}$$

$$FalseAlarmRatio = \frac{\#FalseAlarms}{\#ForecastEDR > 0.1}$$

$$UnforecastedTurbulenceRate = \frac{\#UnforecastedTurbulenceOccurrances}{\#EDRObservations}$$

$$\#UnderforecastTurbulenceOccurrences = \#(ForecastEDR \leq 0.1)\ AND\ \#(ObservedEDR > 0.2)$$



Summary of GTG vs Schneider Data
[Aug-Oct 2014]

		Lead Hours					
		1	2	3	6	9	12
False Alarm Rate	GTG	33.4%	33.3%	32.1%	30.5%	29%	27.3%
	Schneider	21.9%	20.3%	19.4%	17.4%	16.2%	15.5%
False Alarm Ratio	GTG	80.1%	80.6%	80.6%	80.7%	81.4%	81.5%
	Schneider	78%	77.9%	77.8%	78.1%	78.2%	77.8%
Unforecasted Turbulence Rate	GTG	0.4%	0.4%	0.4%	0.5%	0.5%	0.6%
	Schneider	0.4%	0.5%	0.5%	0.6%	0.6%	0.6%
Perfect Hit Rate	GTG	68.4%	68.5%	69.8%	71.4%	72.9%	74.5%
	Schneider	79.2%	80.6%	81.4%	83.2%	84.3%	84.9%
Overforecasted Hit Rate	GTG	1.2%	1.2%	1%	0.8%	0.6%	0.5%
	Schneider	0.7%	0.7%	0.7%	0.6%	0.5%	0.6%
Underforecasted Hit Rate	GTG	0%	0%	0%	0.1%	0%	0%
	Schneider	0%	0.1%	0%	0%	0%	0%
Hit Rate	GTG	69.7%	69.7%	70.8%	72.2%	73.5%	75%
	Schneider	79.9%	81.4%	82.1%	83.8%	84.9%	85.5%
EDR Observations	GTG	80486	81105	82512	86114	84664	82356
	Schneider	80486	81105	82512	86114	84664	82356

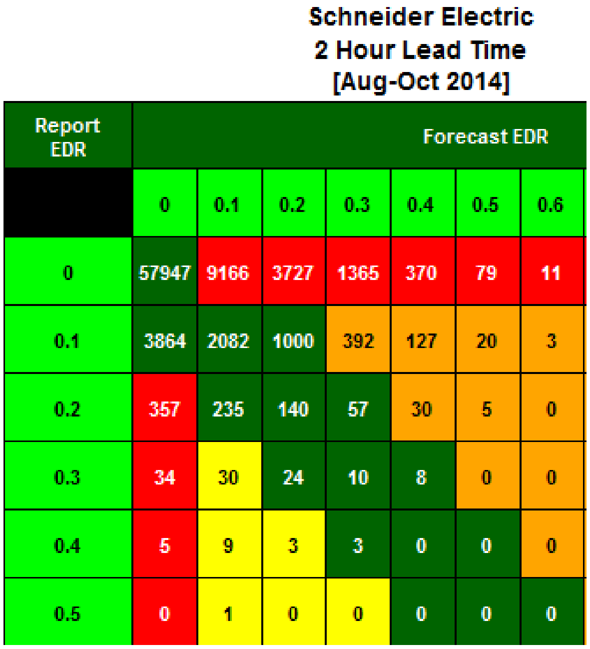
Figure 4: *Summary Statistics Table showing all relevant statistics to evaluate forecast quality. These statistics include all days from the evaluation period for EFH (Schneider) and GTG.*

5 Summary of Verification Results

Data were analyzed in aggregate and in several subsets including by forecast lead time, EDR value, and proximity to convection.

Figure 4 in section 4, shows in green that EFH has the advantage over GTG consistently for false alarm rate and ratio and also in perfect hits and overall hits when evaluating the entire period. The reason the hits are as elevated as they are is in large part because ~85% of the observations used in the evaluation are zero. So in this evaluation period, forecasting EDR below 0.1 can make a big difference in the overall statistics.

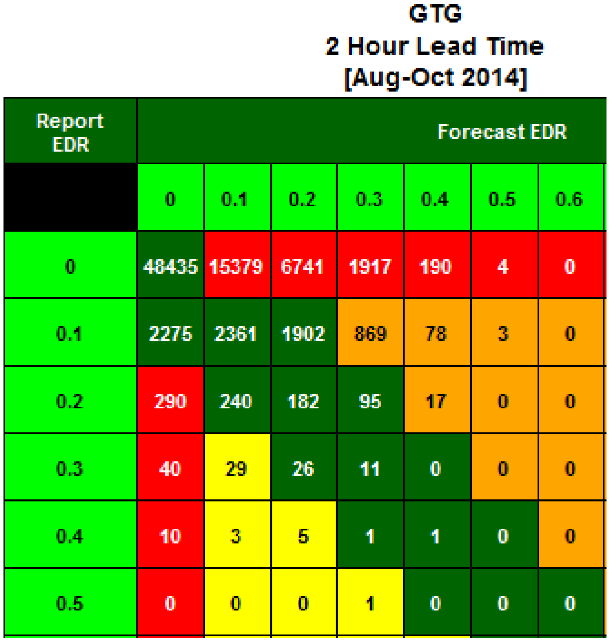
a)



Schneider Electric
2 Hour Lead Time
[Aug-Oct 2014]

Report EDR	Forecast EDR						
	0	0.1	0.2	0.3	0.4	0.5	0.6
0	57947	9166	3727	1365	370	79	11
0.1	3864	2082	1000	392	127	20	3
0.2	357	235	140	57	30	5	0
0.3	34	30	24	10	8	0	0
0.4	5	9	3	3	0	0	0
0.5	0	1	0	0	0	0	0

b)



GTG
2 Hour Lead Time
[Aug-Oct 2014]

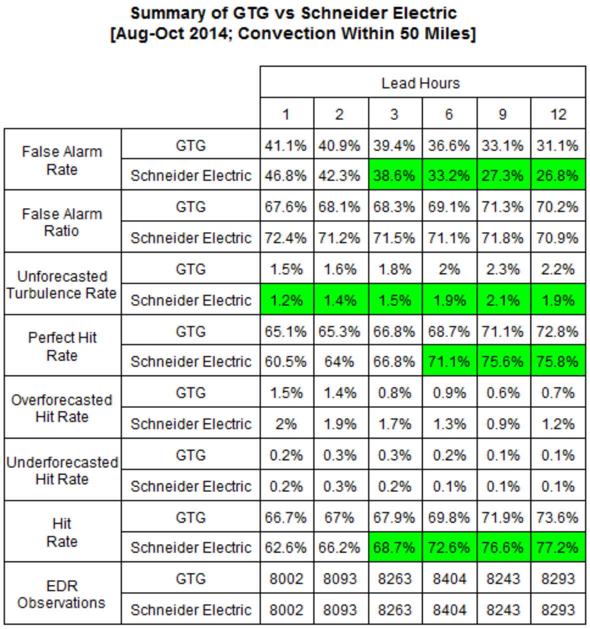
Report EDR	Forecast EDR						
	0	0.1	0.2	0.3	0.4	0.5	0.6
0	48435	15379	6741	1917	190	4	0
0.1	2275	2361	1902	869	78	3	0
0.2	290	240	182	95	17	0	0
0.3	40	29	26	11	0	0	0
0.4	10	3	5	1	1	0	0
0.5	0	0	0	1	0	0	0

Figure 5: *2Hour Lead Time Range of Forecast and Observation EDR for a) EFH and b) GTG.*

In Figures 5a and 5b, EFH has ~9,000 more Perfect Hit nulls than GTG. To put it in perspective, there were only ~8,500 observations of ≥ 0.1 EDR so forecasting EDR <0.1 has a huge impact over a typical sample set of observed EDR. The next largest number is the 0.1 EDR misses. This number can be viewed as the cost of getting a hit. From Figures 5a and 5b, for EFH to score a perfect 0.1 Forecast EDR to a 0.1 Observed EDR it took 9,166 misses from EFH to attain 2,082 hits. For GTG, it took 15,379 misses to attain 2,361 hits. Also note that when the opposite is true and there is Forecast EDR < 0.1 and Observed EDR ≥ 0.1, this number is far less than when there was Forecast EDR ≥ 0.1 and Observed EDR = 0 (i.e., a miss). That shows us both models tended to significantly over-forecast.

Also noteworthy is the benefit EFH has by having a convective algorithm included in its turbulence forecast suite. When categorized as non-convective versus convective (i.e., convection within 50 miles), the statistics heavily favor EFH when convection is present and the forecast EDR ≥ 0.2 (Figure 6). Proximity to convection was determined by identifying National Convective Weather Diagnostic (NCWD) VIP Level 3-or-greater cells within 50 miles of an EDR observation.

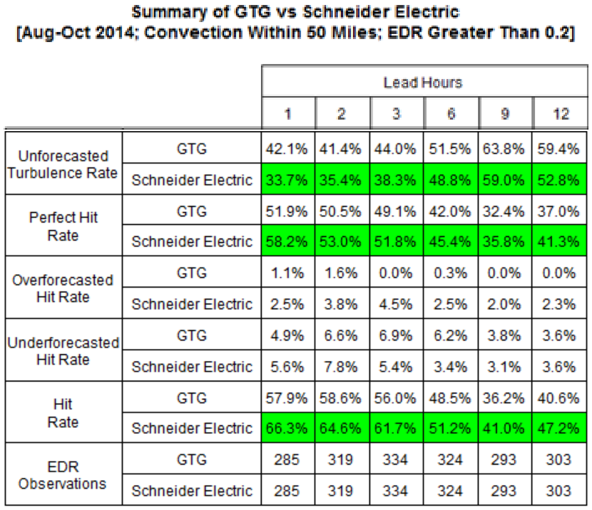
a)



Summary of GTG vs Schneider Electric
[Aug-Oct 2014; Convection Within 50 Miles]

		Lead Hours					
		1	2	3	6	9	12
False Alarm Rate	GTG	41.1%	40.9%	39.4%	36.6%	33.1%	31.1%
	Schneider Electric	46.8%	42.3%	38.6%	33.2%	27.3%	26.8%
False Alarm Ratio	GTG	67.6%	68.1%	68.3%	69.1%	71.3%	70.2%
	Schneider Electric	72.4%	71.2%	71.5%	71.1%	71.8%	70.9%
Unforecasted Turbulence Rate	GTG	1.5%	1.6%	1.8%	2%	2.3%	2.2%
	Schneider Electric	1.2%	1.4%	1.5%	1.9%	2.1%	1.9%
Perfect Hit Rate	GTG	65.1%	65.3%	66.8%	68.7%	71.1%	72.8%
	Schneider Electric	60.5%	64%	66.8%	71.1%	75.6%	75.8%
Overforecasted Hit Rate	GTG	1.5%	1.4%	0.8%	0.9%	0.6%	0.7%
	Schneider Electric	2%	1.9%	1.7%	1.3%	0.9%	1.2%
Underforecasted Hit Rate	GTG	0.2%	0.3%	0.3%	0.2%	0.1%	0.1%
	Schneider Electric	0.2%	0.3%	0.2%	0.1%	0.1%	0.1%
Hit Rate	GTG	66.7%	67%	67.9%	69.8%	71.9%	73.6%
	Schneider Electric	62.6%	66.2%	68.7%	72.6%	76.6%	77.2%
EDR Observations	GTG	8002	8093	8263	8404	8243	8293
	Schneider Electric	8002	8093	8263	8404	8243	8293

b)



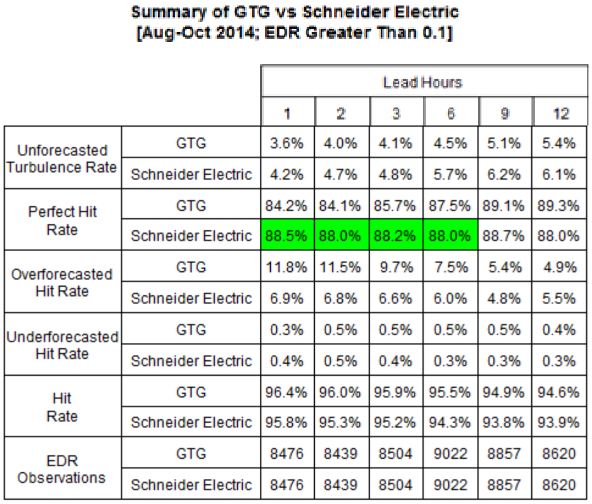
Summary of GTG vs Schneider Electric
[Aug-Oct 2014; Convection Within 50 Miles; EDR Greater Than 0.2]

		Lead Hours					
		1	2	3	6	9	12
Unforecasted Turbulence Rate	GTG	42.1%	41.4%	44.0%	51.5%	63.8%	59.4%
	Schneider Electric	33.7%	35.4%	38.3%	48.8%	59.0%	52.8%
Perfect Hit Rate	GTG	51.9%	50.5%	49.1%	42.0%	32.4%	37.0%
	Schneider Electric	58.2%	53.0%	51.8%	45.4%	35.8%	41.3%
Overforecasted Hit Rate	GTG	1.1%	1.6%	0.0%	0.3%	0.0%	0.0%
	Schneider Electric	2.5%	3.8%	4.5%	2.5%	2.0%	2.3%
Underforecasted Hit Rate	GTG	4.9%	6.6%	6.9%	6.2%	3.8%	3.6%
	Schneider Electric	5.6%	7.8%	5.4%	3.4%	3.1%	3.6%
Hit Rate	GTG	57.9%	58.6%	56.0%	48.5%	36.2%	40.6%
	Schneider Electric	66.3%	64.6%	61.7%	51.2%	41.0%	47.2%
EDR Observations	GTG	285	319	334	324	293	303
	Schneider Electric	285	319	334	324	293	303

Figure 6: *Summary Statistics Table showing all relevant statistics where forecast EDR > 0.2. Rows in green show where EFH has the advantage a) All Days near Convection and b) All Days near Convection with Forecast EDR ≥0.2.*

As mentioned previously in this section, ~8,500 have Observed EDR ≥ 0.1. The overall scores for that subset show the Perfect Hits that GTG accumulates is directly a result of over-forecasting because there was a greater coverage of Forecast EDR ≥ 0.1 than EFH. The net effect makes the GTG Perfect Hit scores higher but they were higher at the cost of a higher False Alarm Rate. When analyzing the observed EDR ≥ 0.3 EDR hits, EFH higher statistical numbers show its higher precision with elevated turbulence values (Figure 7a-c) at this threshold. Note sample sizes are relatively large for EDR ≥ .1 and .2, however decrease significantly for EDR ≥.3.

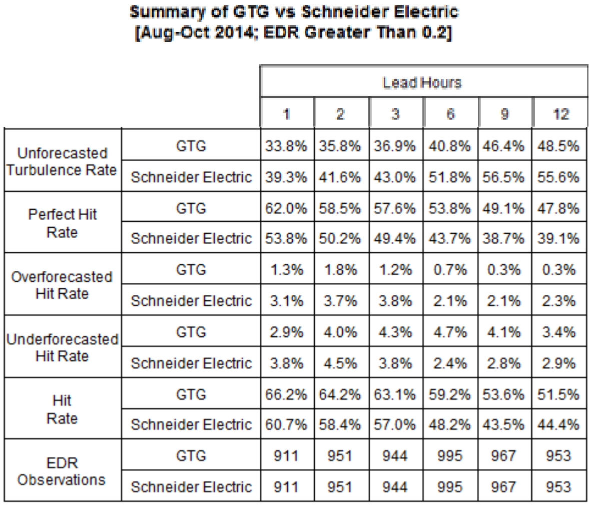
a)



Summary of GTG vs Schneider Electric
[Aug-Oct 2014; EDR Greater Than 0.1]

		Lead Hours					
		1	2	3	6	9	12
Unforecasted Turbulence Rate	GTG	3.6%	4.0%	4.1%	4.5%	5.1%	5.4%
	Schneider Electric	4.2%	4.7%	4.8%	5.7%	6.2%	6.1%
Perfect Hit Rate	GTG	84.2%	84.1%	85.7%	87.5%	89.1%	89.3%
	Schneider Electric	88.5%	88.0%	88.2%	88.0%	88.7%	88.0%
Overforecasted Hit Rate	GTG	11.8%	11.5%	9.7%	7.5%	5.4%	4.9%
	Schneider Electric	6.9%	6.8%	6.6%	6.0%	4.8%	5.5%
Underforecasted Hit Rate	GTG	0.3%	0.5%	0.5%	0.5%	0.5%	0.4%
	Schneider Electric	0.4%	0.5%	0.4%	0.3%	0.3%	0.3%
Hit Rate	GTG	96.4%	96.0%	95.9%	95.5%	94.9%	94.6%
	Schneider Electric	95.8%	95.3%	95.2%	94.3%	93.8%	93.9%
EDR Observations	GTG	8476	8439	8504	9022	8857	8620
	Schneider Electric	8476	8439	8504	9022	8857	8620

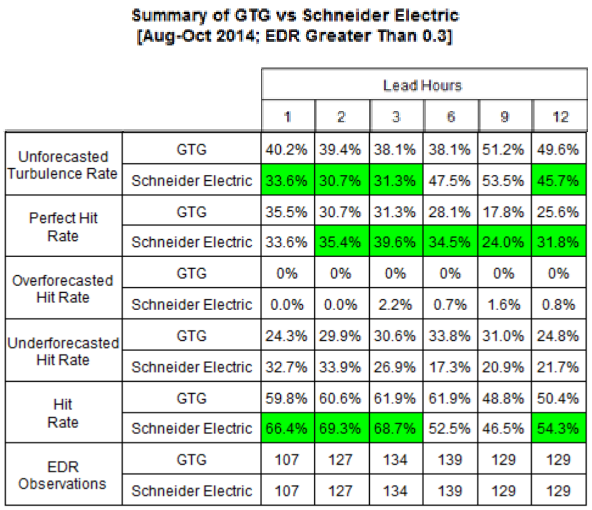
b)



Summary of GTG vs Schneider Electric
[Aug-Oct 2014; EDR Greater Than 0.2]

		Lead Hours					
		1	2	3	6	9	12
Unforecasted Turbulence Rate	GTG	33.8%	35.8%	36.9%	40.8%	46.4%	48.5%
	Schneider Electric	39.3%	41.6%	43.0%	51.8%	56.5%	55.6%
Perfect Hit Rate	GTG	62.0%	58.5%	57.6%	53.8%	49.1%	47.8%
	Schneider Electric	53.8%	50.2%	49.4%	43.7%	38.7%	39.1%
Overforecasted Hit Rate	GTG	1.3%	1.8%	1.2%	0.7%	0.3%	0.3%
	Schneider Electric	3.1%	3.7%	3.8%	2.1%	2.1%	2.3%
Underforecasted Hit Rate	GTG	2.9%	4.0%	4.3%	4.7%	4.1%	3.4%
	Schneider Electric	3.8%	4.5%	3.8%	2.4%	2.8%	2.9%
Hit Rate	GTG	66.2%	64.2%	63.1%	59.2%	53.6%	51.5%
	Schneider Electric	60.7%	58.4%	57.0%	48.2%	43.5%	44.4%
EDR Observations	GTG	911	951	944	995	967	953
	Schneider Electric	911	951	944	995	967	953

c)



Summary of GTG vs Schneider Electric
[Aug-Oct 2014; EDR Greater Than 0.3]

		Lead Hours					
		1	2	3	6	9	12
Unforecasted Turbulence Rate	GTG	40.2%	39.4%	38.1%	38.1%	51.2%	49.6%
	Schneider Electric	33.6%	30.7%	31.3%	47.5%	53.5%	45.7%
Perfect Hit Rate	GTG	35.5%	30.7%	31.3%	28.1%	17.8%	25.6%
	Schneider Electric	33.6%	35.4%	39.6%	34.5%	24.0%	31.8%
Overforecasted Hit Rate	GTG	0%	0%	0%	0%	0%	0%
	Schneider Electric	0.0%	0.0%	2.2%	0.7%	1.6%	0.8%
Underforecasted Hit Rate	GTG	24.3%	29.9%	30.6%	33.8%	31.0%	24.8%
	Schneider Electric	32.7%	33.9%	26.9%	17.3%	20.9%	21.7%
Hit Rate	GTG	59.8%	60.6%	61.9%	61.9%	48.8%	50.4%
	Schneider Electric	66.4%	69.3%	68.7%	52.5%	46.5%	54.3%
EDR Observations	GTG	107	127	134	139	129	129
	Schneider Electric	107	127	134	139	129	129

Figure 7: *Summary Statistics Table showing all relevant statistics where forecast EDR a)≥ 0.1, b)≥0.2, and c)≥0.3. Rows in green show where EFH has the advantage.*

When evaluating the results from the entire study period, the Schneider Electric EFH forecasts had higher Hit Rates, averaging 11% higher, over all forecast periods compared to the GTG forecasts. The False Alarm Rates for GTG were on average 14% higher than the EFH forecasts (Figure 8).

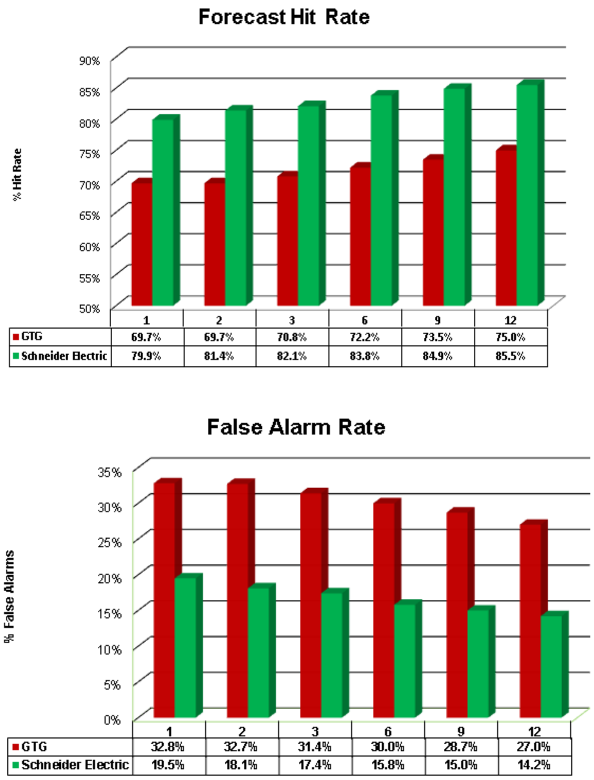


Figure 8: *Top graph shows the comparison of Hit Rate between EFH (green) and GTG (red). The bottom graph shows the False Alarm Rate between EFH (green) and GTG (red). Both graphs represent data from all lead times.*

Analyses of daily summary statistics over the evaluation period shows Schneider Electric EFH to be more consistent (i.e., less deviation from average) compared to GTG with the RMSE error consistently less than GTG through the evaluation period (Figure 9). This shows the EFH forecast technique to be a positive step forward in day-to-day consistency for the numerical prediction of turbulence.

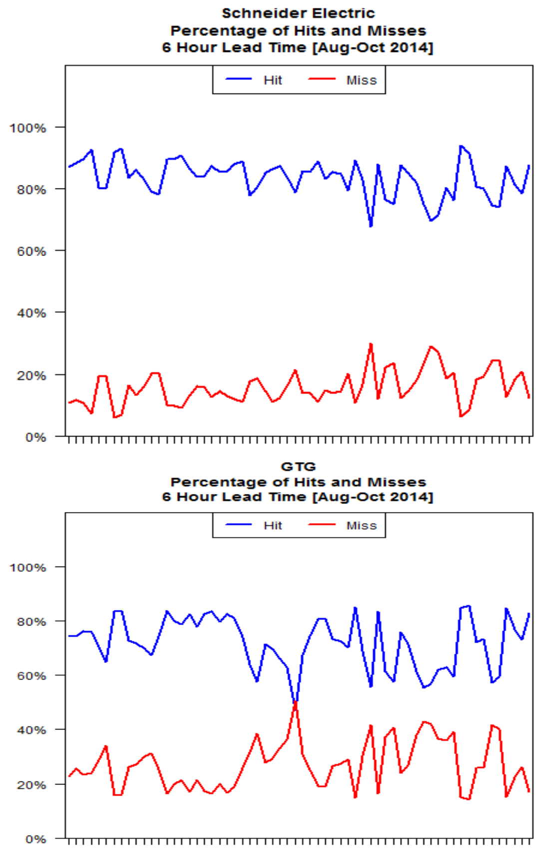


Figure 9: *Daily Summary of Results showing hits (blue) and misses (red) between EFH (top) and GTG (bottom) illustrating GTG's larger deviation from day to day than EFH.*

6 Conclusions and Future Work

Schneider Electric has recently deployed a numerical deterministic turbulence forecast as a part of an Enhanced Flight Hazard product suite. To assess its potential operational merit, AvMet was tasked to conduct an independent third party evaluation of the EFH turbulence forecast. This paper presents results from the three-month evaluation period of turbulence forecast information provided by EFH and GTG (v2.5) products as compared to EDR in-situ observations. Evaluations included identifying the accuracy of the products via contingency tables showing relationships between forecast and observed EDR values, graphs showing daily statistics, and summary tables presenting relevant statistics from both forecast models. Results from the evaluation showed:

- For all data, EFH has a consistently lower False Alarm Rate compared to GTG for all lead times evaluated (EFH = 18.5%, GTG = 30.9%)
- For all data, EFH scored higher for the 'perfect Hit Rate' and overall Hit Rate with ~11% improvement over GTG noted at all lead times for both statistics
    - EFH was also observed to have a more consistent validation (i.e., lower RMSE comparing to Hit Rate on average) for all forecast periods for all days evaluated compared to GTG
- EFH validated higher than GTG when there were larger turbulence observations (i.e., ≥.1, .2, and .3)
- EFH validated higher than GTG within vicinity (50 miles) of convection for all thresholds ≥ 0.1 EDR

The validation effort has shown the potential merit in EFH turbulence forecast data which may offer opportunities for its end users to better optimize their routes thus conserving fuel, reducing emissions, and help to reduce air traffic congestion. End users will also have the benefit of more actionable information with higher accuracy where dangerous conditions exist. Finally, the user of the EFH forecasts will have better confidence employing the forecast into operations with its smaller variations in quality from day to day.

The difference in forecasting approaches between the deterministic EFH and ensemble-of-diagnostics GTG seems to have a lot to do with the deviations in results. For one, ensembles tend to spread variably from one weather pattern to the next since the ensemble members will agree and disagree variably. The result is more False Alarms from the broader coverage, dampening of highest values where divergent solutions can potentially cancel out each another, and more variability in day to day forecast confidence depending on daily member convergence or divergence. Also, the application of Lighthill-Ford theory as it is implemented in EFH turbulence model has good skill. As McCann et al. (2012) alluded to, advances in gravity wave initiation theories are an area that can be improved and could yield positive advances in quality on future numerical turbulence forecasting.

Future work includes an extended evaluation over a year or more to capture several complete cold- and warm seasons. That will enable evaluation over a far larger sample size and will help highlight seasonal performance. It is also desirable to attain additional sources of observations to make the sample size in future evaluations more robust and perhaps expand to areas other than the United States. Future comparisons of new versions of GTG or other accessible numerical turbulence forecasts with the latest version of EFH will continue to be desirable.

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