Extreme Values in Atmospheric Science: Progress and Current Challenges

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Outline

Notes:

- Extremes from a *statistician's* POV.
- Much of current extremes work focuses on *dependence*.

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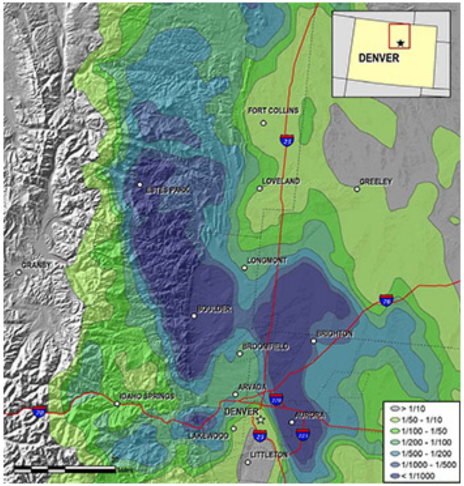
1. *Quick* review of "classical" univariate extremes–via illustration.
2. Multivariate extremes and tail dependence.
3. Snapshots of research and challenges.
   (a) Evaluating RCM's ability to produce extreme events.
   (b) Spatial extremes.
   (c) Dimension reduction.

Colorado Flood of 2013

- Widespread heavy precip Sept 9 - 15, 2013.
- 8 killed, > \$1B damage



NOAAs NWS, HDSC

Big Thompson River Canyon

Boulder:

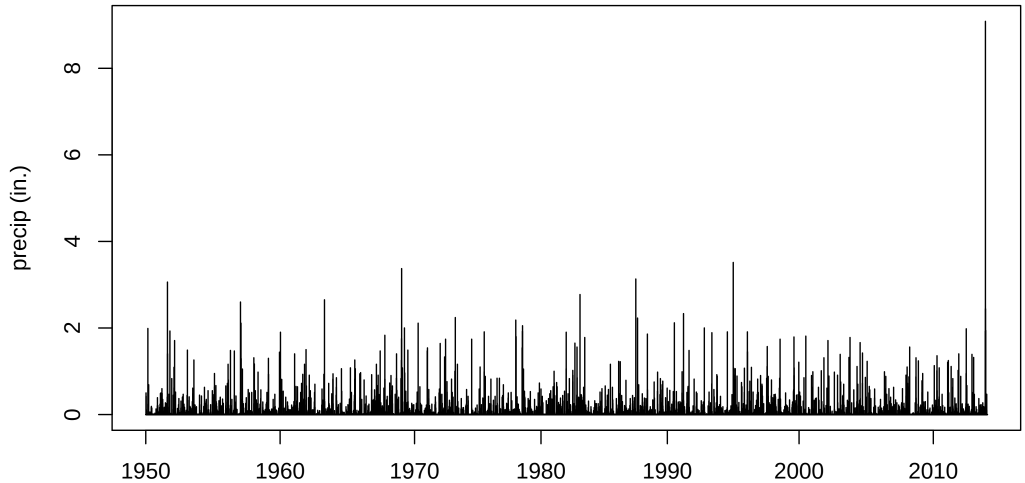
- Flash flood event Sept 12: 9.08 in.
- NOAA HDSC 1000-year rtn level est for 24 hr precip: 8.16 in; 90% CI: (5.46-10.9).

Univariate Extreme Value Analysis

EVA has a relatively long history of answering questions like:

- very high quantile: e.g., return level of '100-year flood'.
- return frequency of observed event.

Illustration: Boulder precipitation record (May-Sept)



Analyze the data two ways:

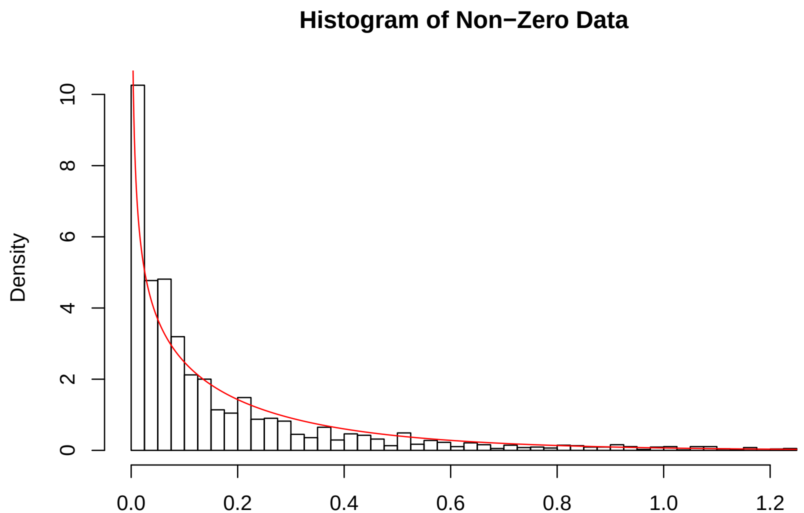
1. Model *all* of the data.
2. Model only the tail. (Classical EVA)

Modeling all precipitation data

Let X_t be the daily "summer" precipitation amount for Boulder. (Summer = May-Sept)

Assume: $\begin{cases} X_t > 0 & \text{w.p. } p \\ X_t = 0 & \text{w.p. } 1-p. \end{cases}$ $\quad \hat{p} = 0.32.$

Further, assume that $[X_t \mid X_t > 0] \sim \text{Gamma}(k, \theta)$.
ML estimates: $\hat{k} = 0.653$, $\hat{\theta} = .322$.



Tail Estimates (Modeling *all* data)

100-year rtn level estimate: 2.395 (2.29, 2.50)
NOAA: 5.52 (4.20, 6.93)

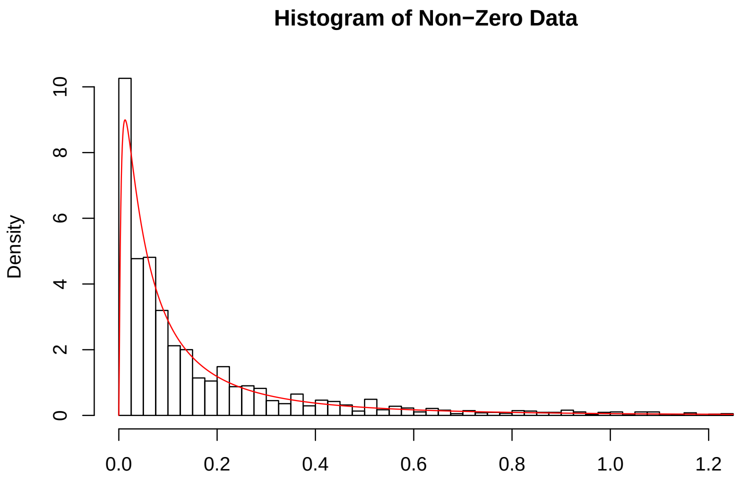
Rtn pd of 2013 event est: 161 Billion years (42B, 727B)

Note: < 1% of all data and 2.7% of non-zero data are > 1.25.

Tail Estimates (Modeling *all* data)

Q: Is the model to blame?
A: Only partly.

Lognormal: $\widehat{\mu} =$ -2.49, $\widehat{\sigma} = 1.37$



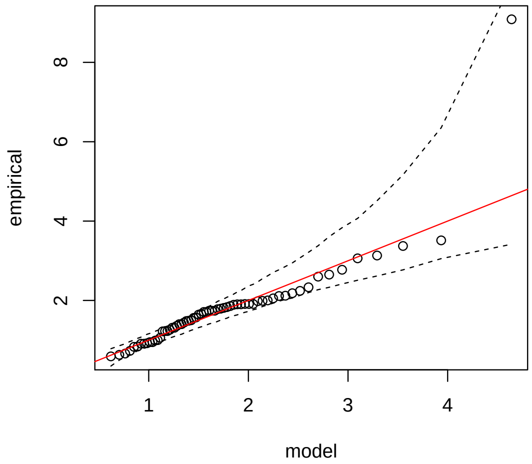
100-year rtn level estimate: 10.42 (9.31, 11.60)
NOAA: 5.52 (4.20, 6.93)
Rtn period of 2013 event est: 68.8 years (52.2, 93.2)

Classical Extremes Approach

Select subset of 'extreme' data, fit a model from EVT.

- block maxima $\rightarrow$ GEV
- threshold exceedances $\rightarrow$ GPD

GEV analysis: (annual max – only 64 data points!)
$\widehat{\mu} = 1.35$; $\widehat{\sigma} = 0.57$; $\widehat{\xi} = 0.15$.



100 year RL est: 5.12 (4.06, 7.32); NOAA: 5.52 (4.20, 6.93)
Rtn pd of 2013 event est: 1654 years (188.3, 65K)

Summary of Classical Univariate Extremes

Mantra: "Let the tail speak for itself."

Fit only a subset of extreme data because . . .

- any single distribution is wrong.
- non-extreme data overwhelm the fit $\rightarrow$ tail poorly fit.
- large amount of data results in small uncertainties in parameter estimates, $\rightarrow$ underestimates uncertainty in tail (model uncertainty not accounted for).

Use a distribution from extreme value theory because . . .

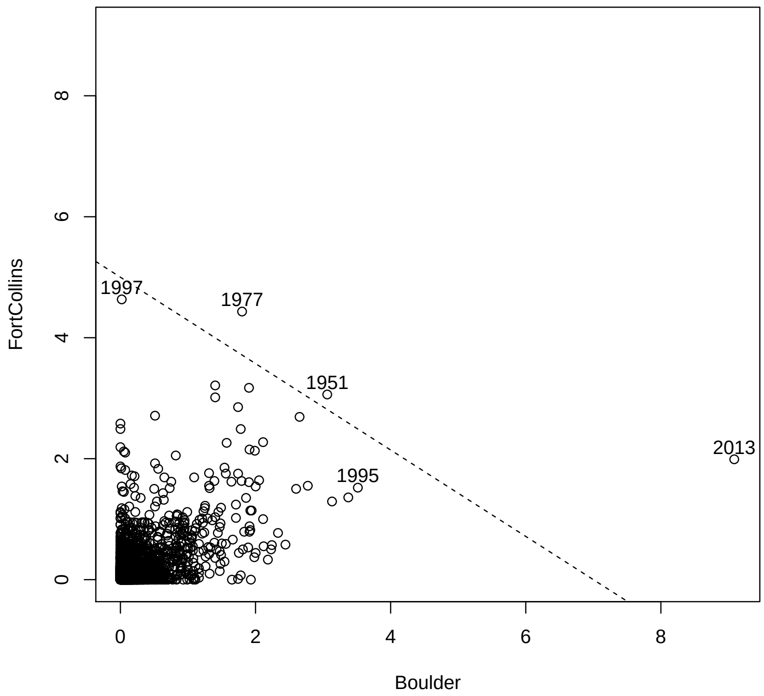
- asymptotically justified (probability theory).
- it doesn't matter what the underlying distribution is.
- justification for extrapolation into tail.

Q: How do these ideas translate to multidimensional case?

Tail Dependence

Much of current extremes work focuses on describing dependence in the tail.

Settings: Multivariate, Time Series, Spatial



Q: What is probability of event in risk region?

How do we describe tail dependence?

Extremes Mantra: Let (joint) tail speak for itself.

- Use only extreme data.
- Use a model suggested by EVT.
- DON'T use correlation to describe dependence.

A Start: Asymptotic Dependence/Independence:

Rand. vec. (X, Y) *with common marginals* is asy. indep. if

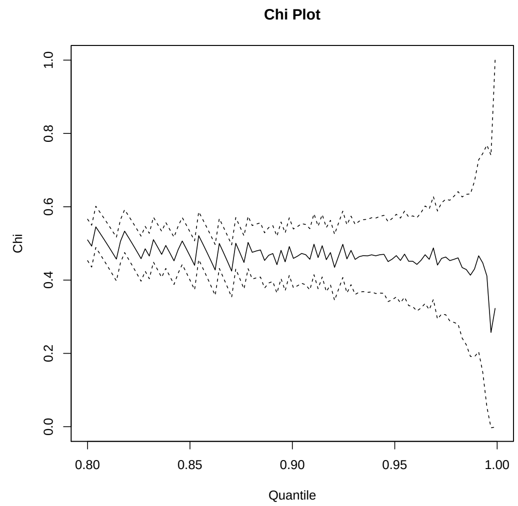
$$\lim_{u \to x^+} P(X > u \mid Y > u) = 0.$$

Important: To talk about tail dependence, we need to know what it means to be in the tail of each component:

- have a common marginal,
- or account for different marginals.

Asymptotic dependence/independence is a way to *begin* to talk about tail dependence, but doesn't yield whole picture.

Boulder and Fort Collins Tail Dependence



Data strongly exhibits asymptotic dependence.

Notes:

- Asymptotic dependence implies a special (and strong) type of dependence.
- Few *models* exhibit asymptotic dependence.

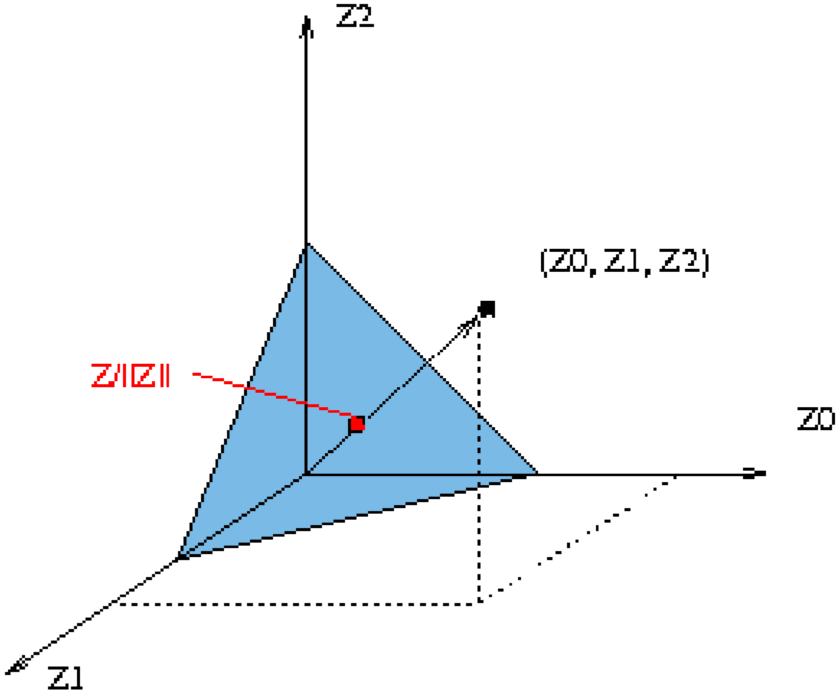
Modeling Framework: MV Regular Variation

A Definition: Let $R = \|\boldsymbol{Z}\|$ and $\boldsymbol{W} = \|\boldsymbol{Z}\|^{-1}\boldsymbol{Z}$. $\boldsymbol{Z}$ is regular varying if there exists a normalizing sequence $\{b_n\}$ where $P(b_n^{-1}\|\boldsymbol{Z}\| > r) \sim n^{-1}$, such that

$$nP\left(b_n^{-1}R > r, \boldsymbol{W} \in A\right) \xrightarrow{v} r^{-\alpha}H(A)$$

where d is the dimension of $\boldsymbol{Z}$, and where H is some probability measure on the unit 'ball' $S_d = \{\boldsymbol{z} \in \mathbb{R}^d \mid \|\boldsymbol{z}\| = 1\}$.

Modeling Framework: MV Regular Variation



Idea: Distribution of *large* points described by:

1. radial component which decays as a power function
2. angular component (which has a probability distribution H on the unit simplex).

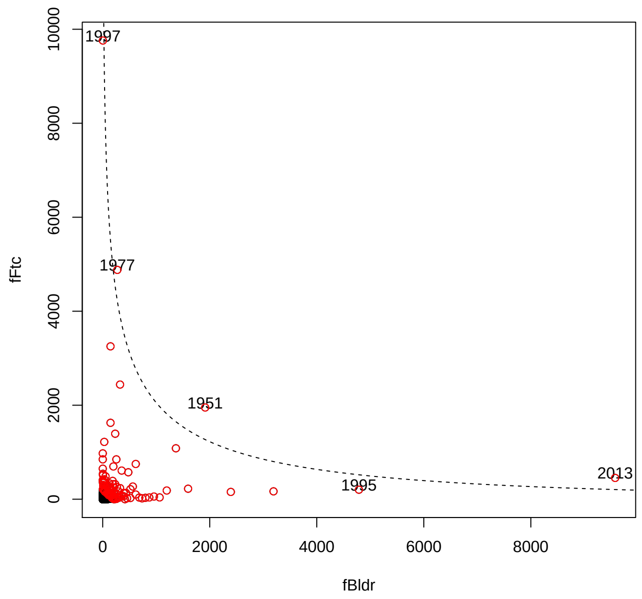
Why is reg. var. ~~right~~ useful for modeling tail dependence?

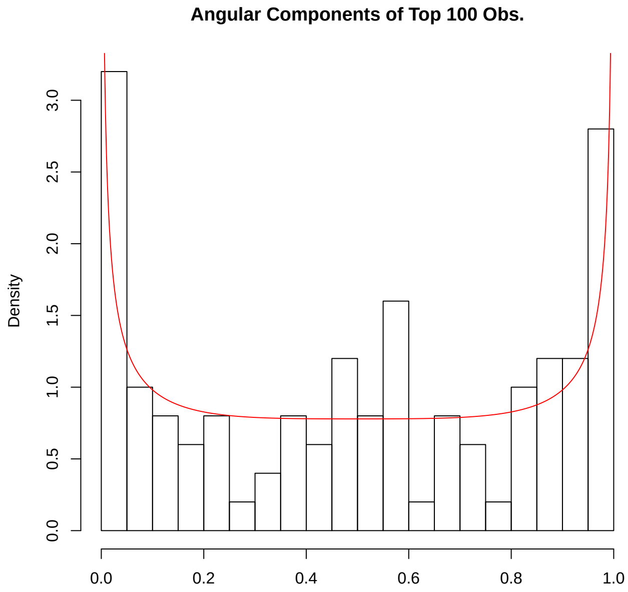
- theoretical justification; fundamentally tied to MVEVDs.
- defined in terms of tail, says nothing about distn's 'bulk'.
- allows for extrapolating further into the tail.
- a multivariate model for asymptotic dependence.

Statistical practice:

- Transform marginals to convenient heavy-tailed dist'n.
- Similar to copula approaches, but models differ, and we only use extreme observations.
- We choose one where $\alpha = 1$ and use L_1 norm.
- After marginal transformation, radial behavior is known.
- Procedure (after transformation):
  1. Retain large points (in terms of radial component).
  2. Model the angular (or spectral) measure H.
  3. Make inference on quantity of interest.

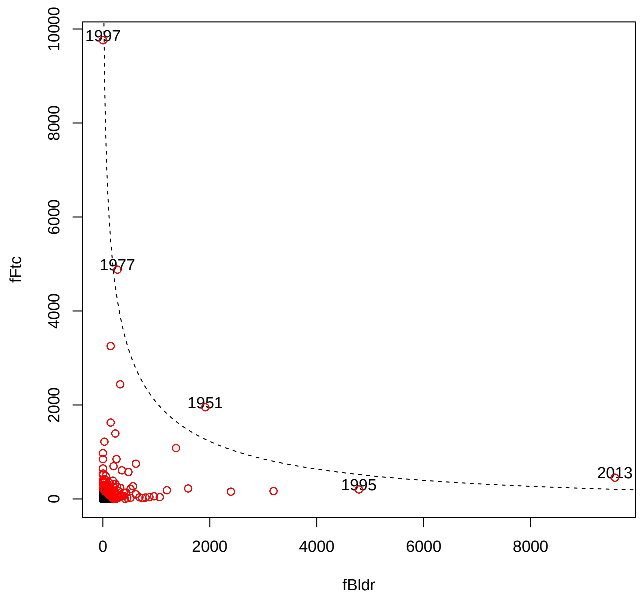
MV Reg Var Estimation of Risk Region

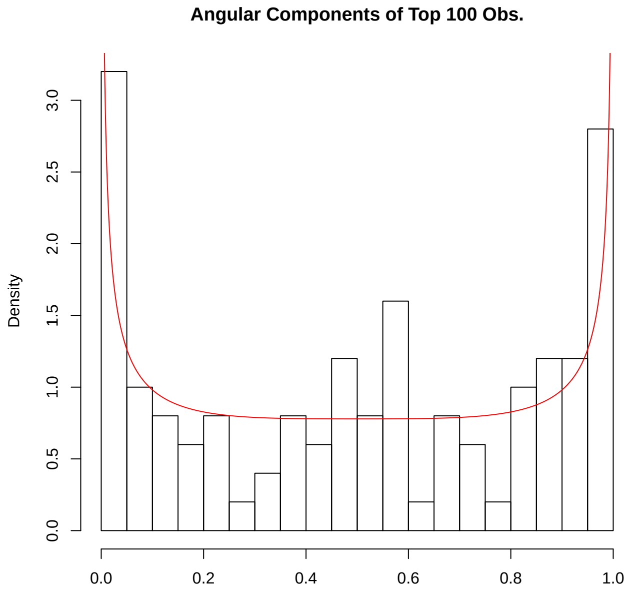




1. Transform marginal (Fréchet – very heavy tailed!).
2. Set threshold, estimate H.
3. Integrate to find probability.
   $P(\boldsymbol{X} \in R) \stackrel{\text{est.}}{=} 0.00048 \rightarrow$Rtn Pd$\stackrel{\text{est.}}{=}$ 14.2 years.
   CI: Takes some work.

MV Reg Var Estimation of Risk Region

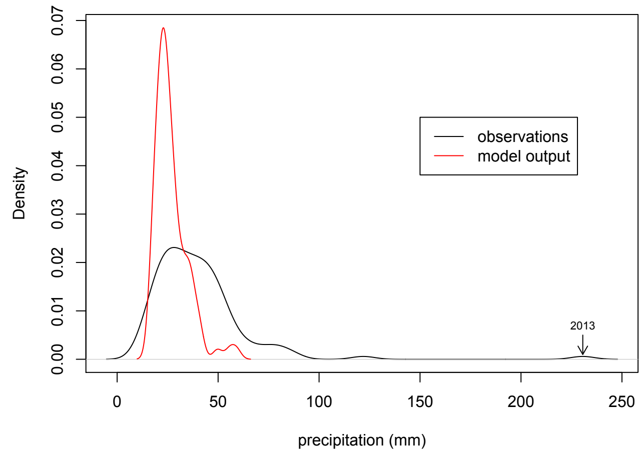




Mantra:

- Used only large observations to characterize marginal tails *and* tail.
- Used a framework suggested by EVT.
- Framework captures asymptotic dependence.

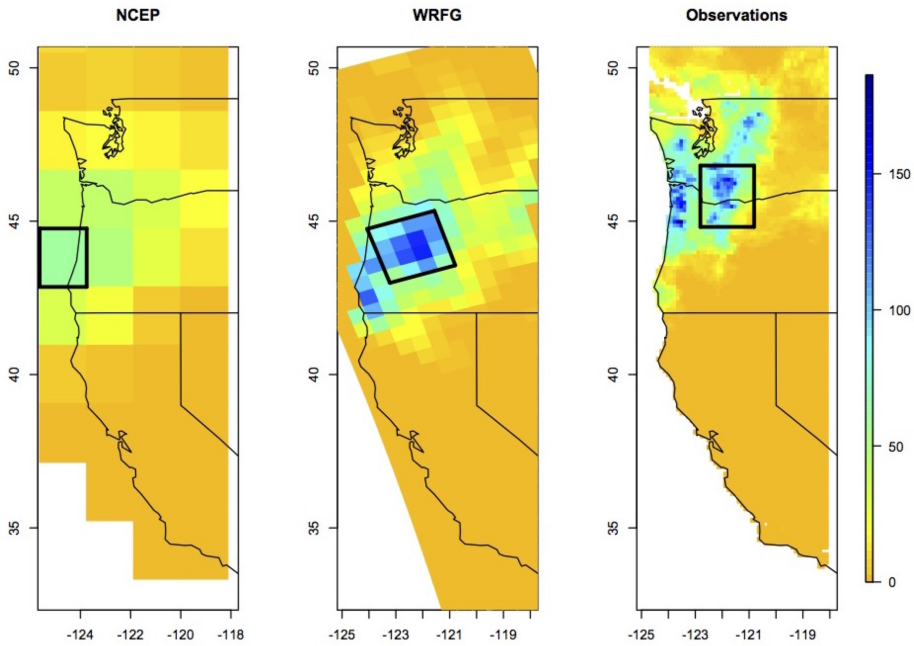
Do RCM's get extreme precip right?



- Do RCM's get marginal distributions right?
  - Even if marginal isn't right, could downscale.
- Do RCM's produce extreme behavior when they should?
  - When (large scale) conditions are right for extremes, do the RCM's produce extremes?
  - Marginal unimportant, correspondence is important.
  - Perhaps answering: Does downscaling make sense?

For second question, we describe the *tail dependence* between NCEP-driven RCM output (NARCCAP) and observations.

Pacific Coast Winter Extreme Precipitation



February 7, 1996

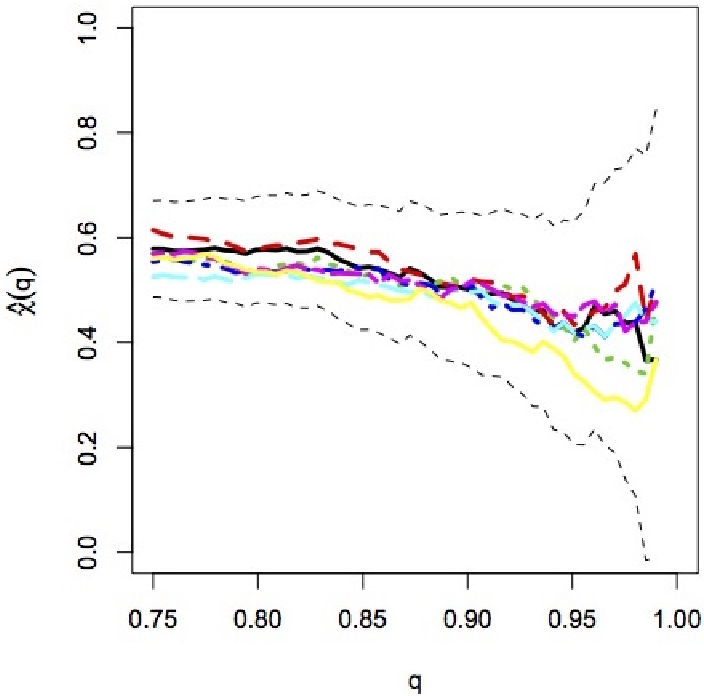
- Data: max of daily precipitation 'footprints' $\sim$(200km)^2.
- Bivariate pairs (X_{jt}, Y_t) of output from model j and obs.
- Do not require location of footprints to coincide.
- Note different spatial resolutions.
- RCM and NCEP show evidence for extreme precip above.

Marginal Behavior

Model	u_j	$\widehat{\psi}_j$ (se)	$\widehat{\xi}_j$ (se)	$\widehat{x}_{j,20}$ (CI)	$\widehat{x}_{j,50}$ (CI)
CRCM	863	172.5 (21.6)	−0.02 (0.09)	102.3 (93.0, 125.7)	111.3 (98.6, 148.0)
ECP2	1129	325.9 (43.8)	−0.04 (0.10)	157.4 (140.5, 203.5)	172.5 (149.4, 245.3)
HRM3	1032	273.9 (32.3)	−0.13 (0.08)	124.5 (115.6, 145.8)	132.5 (114.2, 161.6)
MM5I	1026	246.7 (33.3)	0.11 (0.10)	159.0 (135.0, 222.5)	184.0 (148.3, 293.9)
RegCM	1093	325.2 (42.4)	−0.06 (0.10)	151.6 (136.4, 192.4)	165.4 (144.9, 228.7)
WRFG	1086	339.8 (43.2)	−0.06 (0.09)	153.8 (138.4, 193.1)	167.7 (147.2, 228.0)
NCEP	46	10.4 (1.2)	−0.07 (0.08)	88.3 (81.3, 105.0)	95.1 (86.1, 120.1)
(Obs)	14969	3938.5 (554.6)	0.00 (0.11)	116.1 (102.4, 154.8)	128.8 (109.5, 192.1)

- RCM's have relatively consistent estimates (CRCM lower).
- Difference between RCM, NCEP, and obs.
- Mostly negative point estimates for ξ, obs 0.0.
- My conclusion: downscaling still needed.

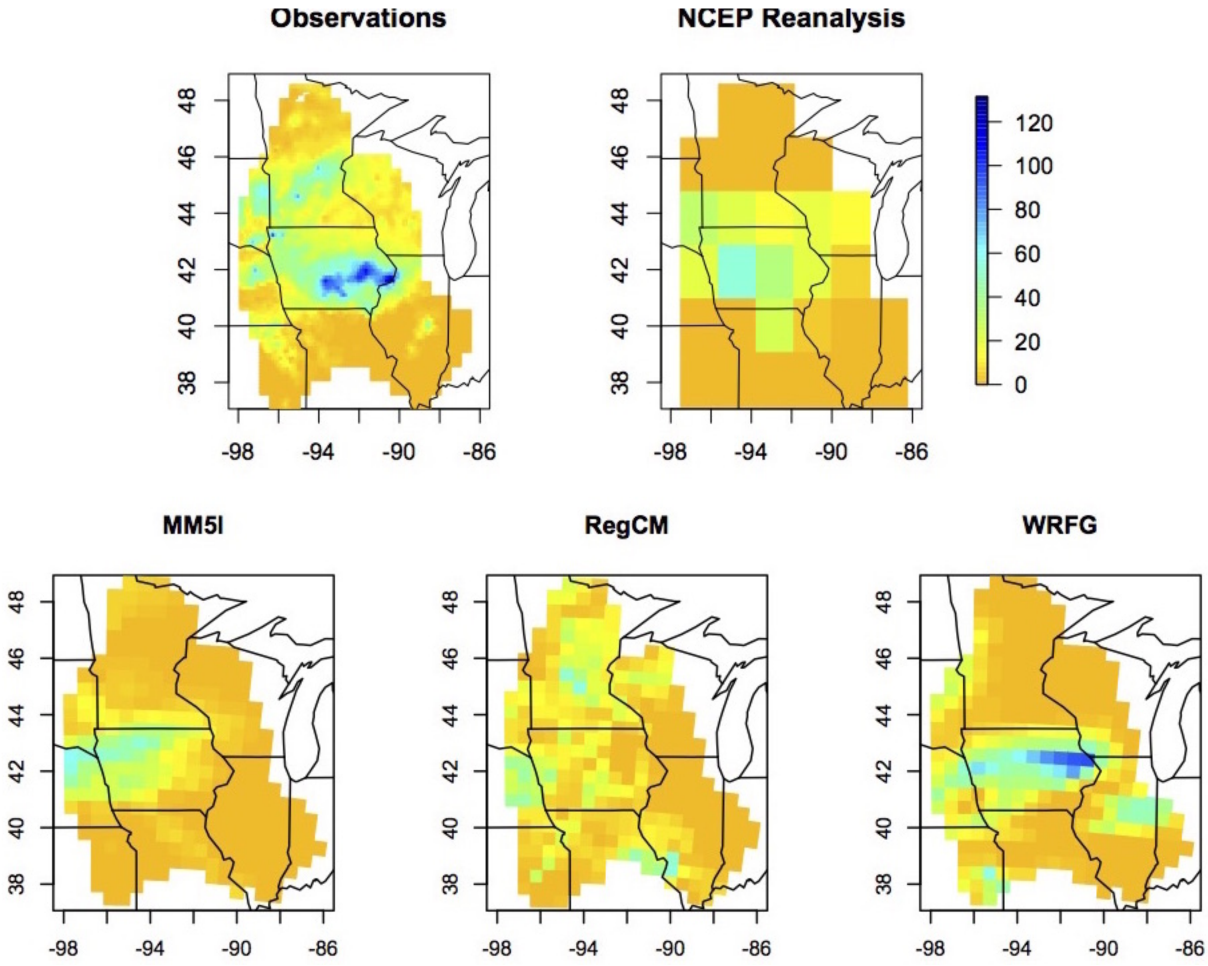
Assessing Correspondence of Extreme Precip



yellow = NCEP, RCMs are other colors, 95% CI for CRCM model (black, dashed)

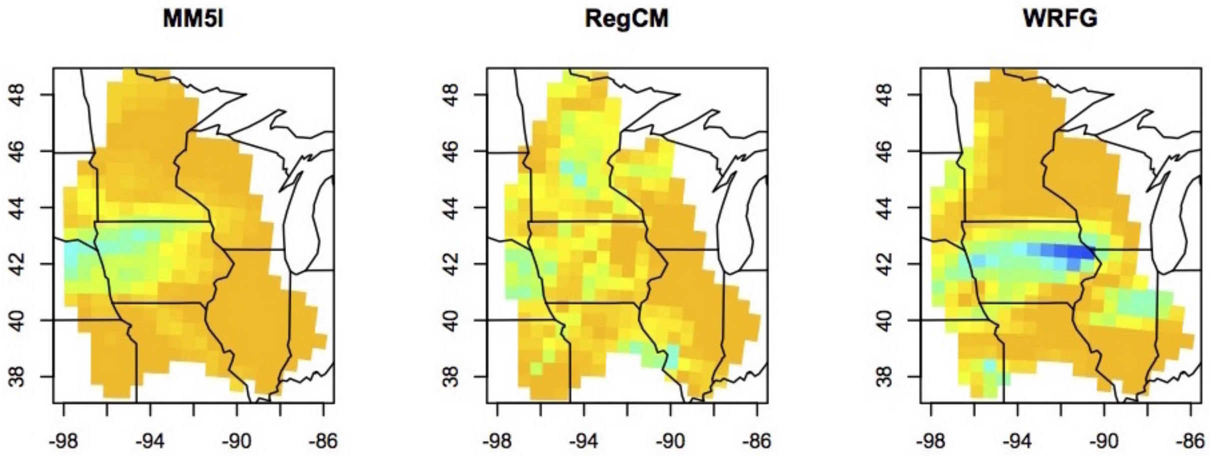
- RCMs exhibit quite strong tail dependence ($\hat{\chi} \approx 0.5$).
- RCMs an improvement over NCEP.
- *Also:* Little spatial discrepancy between RCM and obs.

Corn Belt Summer Precipitation



June 16, 1990

Corn Belt Summer Precipitation



Findings:

- RCMs and obs are asymptotically *independent.*
- "Models do not produce their most extreme behavior when conditions are such that we see largest obs."
- *Also:*
  - High variabilty in RCM marginal parameter estimates.
  - Large spatial discrepancies in footprints.

Overall Conclusions

- Pacific Coast winter precipitation: NCEP-driven RCMs produce extreme precip when and where they should.
- Corn Belt summer precip: Do not produce their most extreme precip on days when obs are most extreme.
- Not a huge surprise.

Method:

- Allows one to analyze *correspondance* of extreme events.
- Used sensible framework for tail dependence.
- Simple. χ and $\bar{\chi}$ in existing R packages.
- Does not depend on marginal behavior.

More to the Pacific Coast study:

- BV ext framework used to link ext precip to SLP fields.
- Produced simulations of future ext precip events from GCM-driven future RCM runs.

Spatial Extremes

Status:

- Theoretically justified structure: max-stable processes.
- Developed models: Brown-Resnick process, others.
- Dependence described after a marginal assumption.

Important question: Is aim to describe dependence at data level or marginal behavior or both?

Data dependence:

- Assess aggregate effect of ext event across locations.
- Ex: Boulder/Ft. Collins area high at same time.
- Requires max-stable process; simple marginal structure?

Marginal behavior:

- How does ext behavior change with location?
- Ex: Return level map of Western US.
- Does not necessarily require MS process.
- Often: hierarchical model, spatial model on GEV params.

Current Challenge for Spatial Extremes: Model Fitting

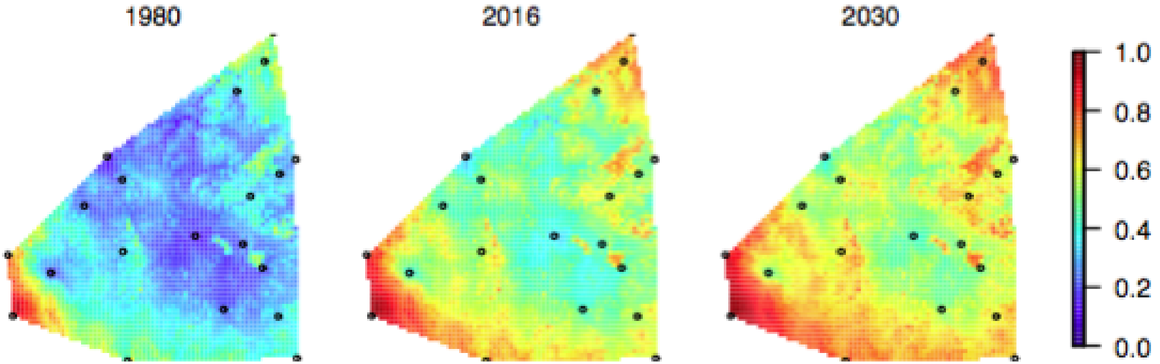
- *Bivariate* dist of MS processes tractable.
  BR/Fréchet:
  $$F(z_1, z_2) = \exp\left\{ -\frac{1}{z_1}\Phi\left(\frac{\sqrt{\gamma(h)}}{2} + \frac{1}{\sqrt{\gamma(h)}}\log\frac{z_2}{z_1}\right) - \frac{1}{z_2}\Phi\left(\frac{\sqrt{\gamma(h)}}{2} + \frac{1}{\sqrt{\gamma(h)}}\log\frac{z_1}{z_2}\right)\right\}$$
- Recent work with higher-dimensional joint distributions.
  BR/Fréchet: can be written in terms of increments/lags.

Fitting:

- Use bivariate distributions only via composite likelihood.
  + point estimates are unbiased.
  − lose information by using only pairs of points.
  − accurate accounting of uncertainty takes work.
  − not a true l'hood: hierarchical modeling challenging.
- Use higher dimensional representations.
  − Computationally challenging. Limited # of locations.

Example: Thibaud, et. al. 2015

- Study of annual minimum temperatures in Finland.
- Forestry motivation: moth eggs cannot survive $< -36^o$ C.
- Mostly interested in *marginal behavior*: How are extreme low temps changing?
- However, strong data dependence, need to account for it.
- *Study/Model:*
  - Full l'hood of BR process for data dependence.
  - Bayesian hierarchical model on GEV parameters.
  - 20 locations, $\sim$ 2 days of processing to fit.

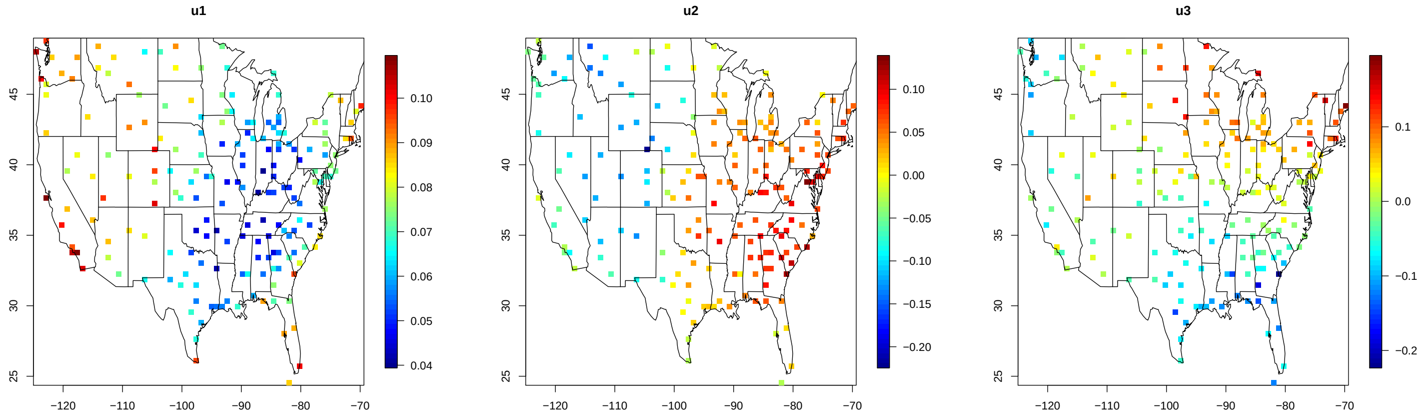


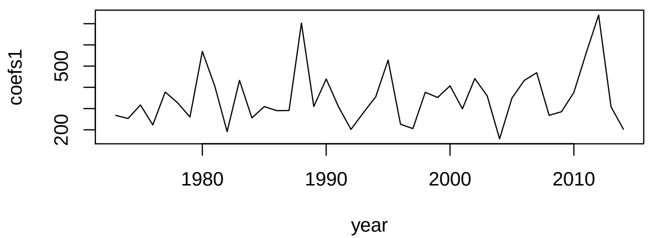
Prob ann min $> -36^o$ C

EOF/PCAs for Extremes (Work in Progress)

Goal: Dimension reduction. Find modes of extreme behavior.

- Summarize *multivariate* dependence in terms of *bivariate* relationships.
- Get a pairwise tail dependence matrix.
- Perform an eigen-like decomposition of the matrix.





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