Mixing of a Passive Scalar in an Urban-Street Canyon

Guangdong DUAN (g.duan@my.cityu.edu.hk), Keith NGAN (keith.ngan@cityu.edu.hk), James George JACKSON
School of Energy and Environment, City University of Hong Kong, Hong Kong SAR

American Meteorological Society 97th ANNUAL MEETING | SEATTLE
13th Symposium on the Urban Environment

1. Introduction

- Mixing refers to the homogenization of a scalar field. Although it is an important subject in fluid dynamics, mixing has not attracted much attention in the urban air quality literature, where processes such as dispersion and ventilation have been highlighted. For certain applications (e.g. the accidental release of toxic chemicals), however, knowledge of the mixing rate is extremely useful.

- This work investigates the mixing of a passive scalar in a unit-aspect-ratio street canyon. Using large-eddy simulation, the mixing rate is characterised using the decay rate of the variance. The mixing is inhomogeneous and strongly influenced by the mean circulation; the evolution is qualitatively distinct within (i) the central vortex; (ii) corners excepting the upper downwind one; (iii) remaining regions. For each region, separate regimes related to advection and mixing can be discerned.

- Following established theoretical predictions, the mixing rates are related to the divergence of Lagrangian trajectories (Lyapunov exponent) and Péclet number. On account of the open boundary and inhomogeneous mixing, sensitivity of the scalar field to the initial conditions persists in the long-time limit.

2. Stirring and Mixing

- Pure stirring (advection)

$$\frac{\partial\theta}{\partial t}+\boxed{\mathbf{u}\cdot\nabla\theta}=\cancel{\kappa\nabla^2\theta}^{\,0}$$

- Stirring and mixing

$$\frac{\partial\theta}{\partial t}+\boxed{\mathbf{u}\cdot\nabla\theta=\kappa\nabla^2\theta}$$

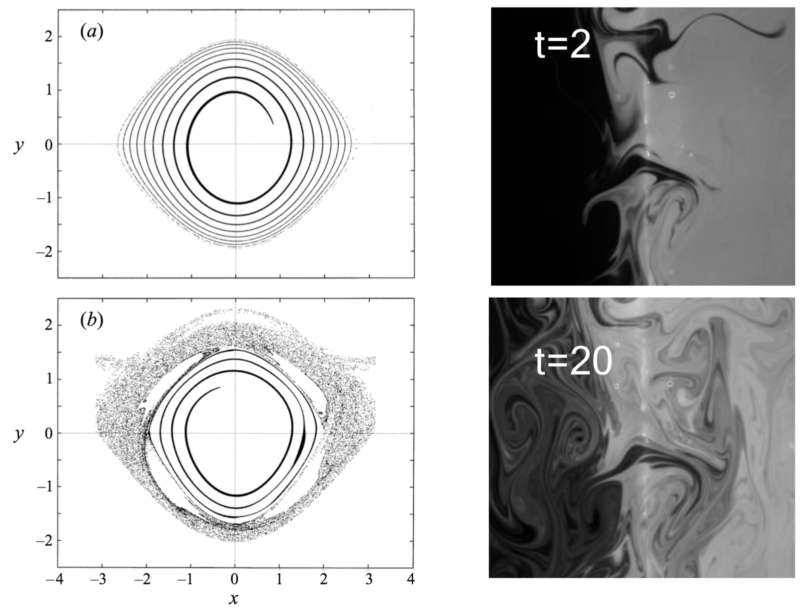

Fig.1 Creation of small-scale structures and associated homogenisation (left panel [1]; right panel [2]).

3. Theoretical Background: whether theories on mixing carry over to the literature of urban environment?

For a spatially smooth, large-scale velocity field, e.g.

$$x_{n+1}=x_n+a\sin(y_n+\phi_n)$$
$$y_{n+1}=y_n+\cos(x_{n+1}+\phi_n)$$

the variance decay rate is predicted by the Lyapunov exponents in the absence of solid boundaries [3]. With no-slip boundary conditions, the long-time behaviour differs [4].

Theoretical predictions for the variance decay rate depend on the existence of a well-defined large-scale velocity field.

Large-scale nature of the canyon flow:

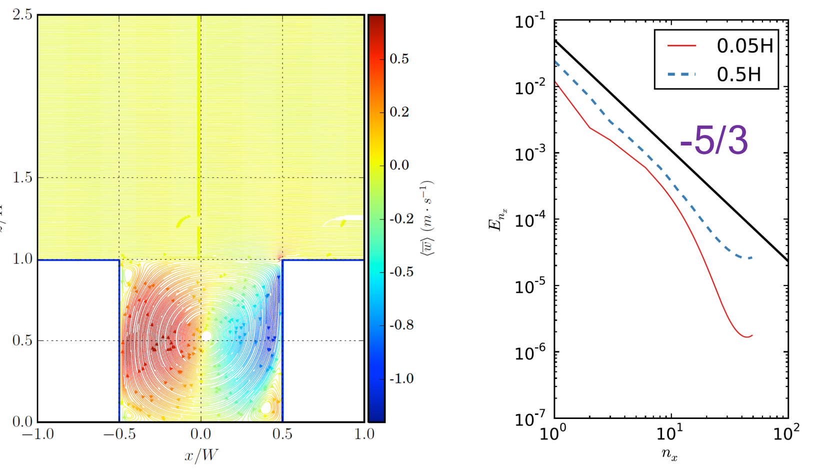

Fig.2 (Left) Streamline; (right) streamwise energy spectra inside the canyon at z=0.05H and 0.5H [5].

4. Model Configuration

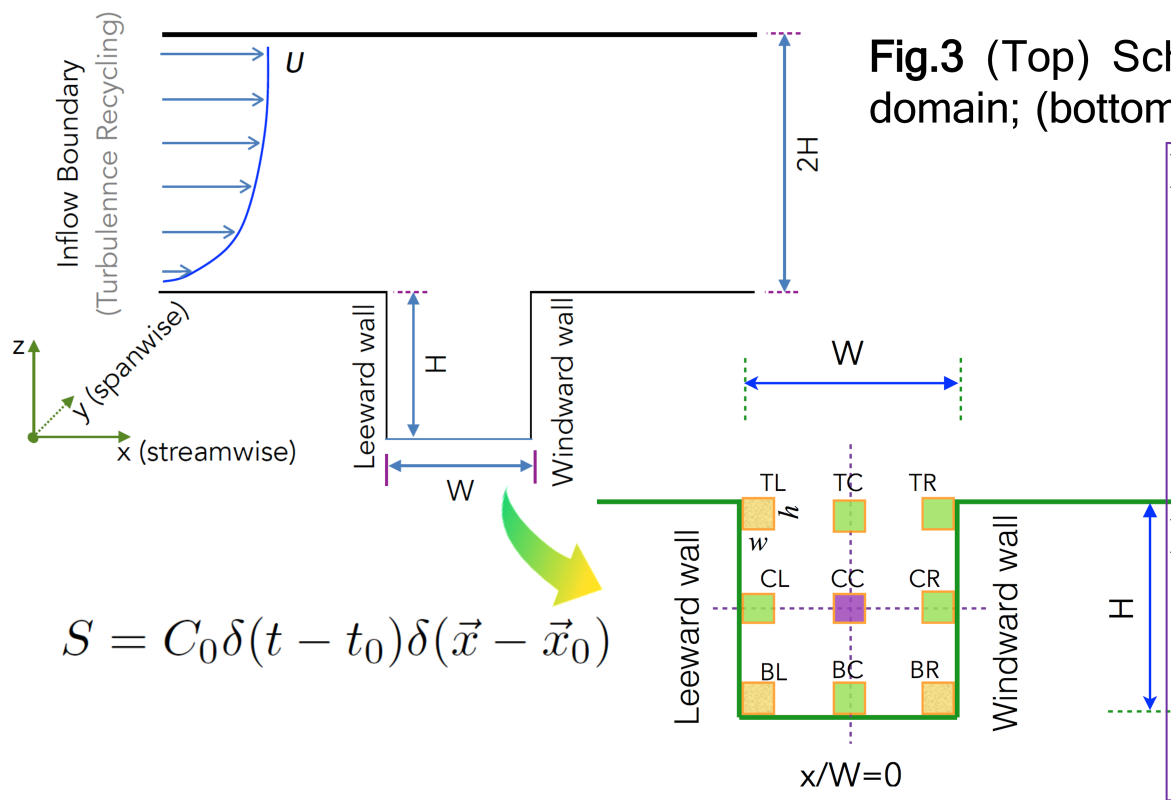

Fig.3 (Top) Schematic diagram of the computational domain; (bottom) setup of initial conditions.

- Initial conditions:
  B: Bottom
  C: Centre
  T: Top
  L: Left
  R: Right
- e.g.
  CL refers to the centre-left pollutant block;
  TR denotes the top-right one;
  CC located at the canyon centre.

5. Diagnostics

- Scalar variance is calculated with each pollutant block R_i. λ_i is the variance decay rate.

$$\langle\sigma^2\rangle_{\mathcal{R}_i}=\frac{1}{V_i}\int_{\mathcal{R}_i}\frac{\left(C(t,\vec{x})-\langle C\rangle_{\mathcal{D}}\right)^2}{C_0^2}d\vec{x}\sim e^{-\lambda_i t} \quad (1)$$

- Divergence of Lagrangian trajectories. λ_0 is the largest Lyapunov exponent.

$$\|\delta\mathbf{x}(t)\|\approx e^{\lambda_0 t}\|\delta\mathbf{x}(t_0)\| \quad (2)$$

6. Results

On account of the open boundary and inhomogeneous mixing, sensitivity of the scalar field to the initial conditions persists in the long-time limit.

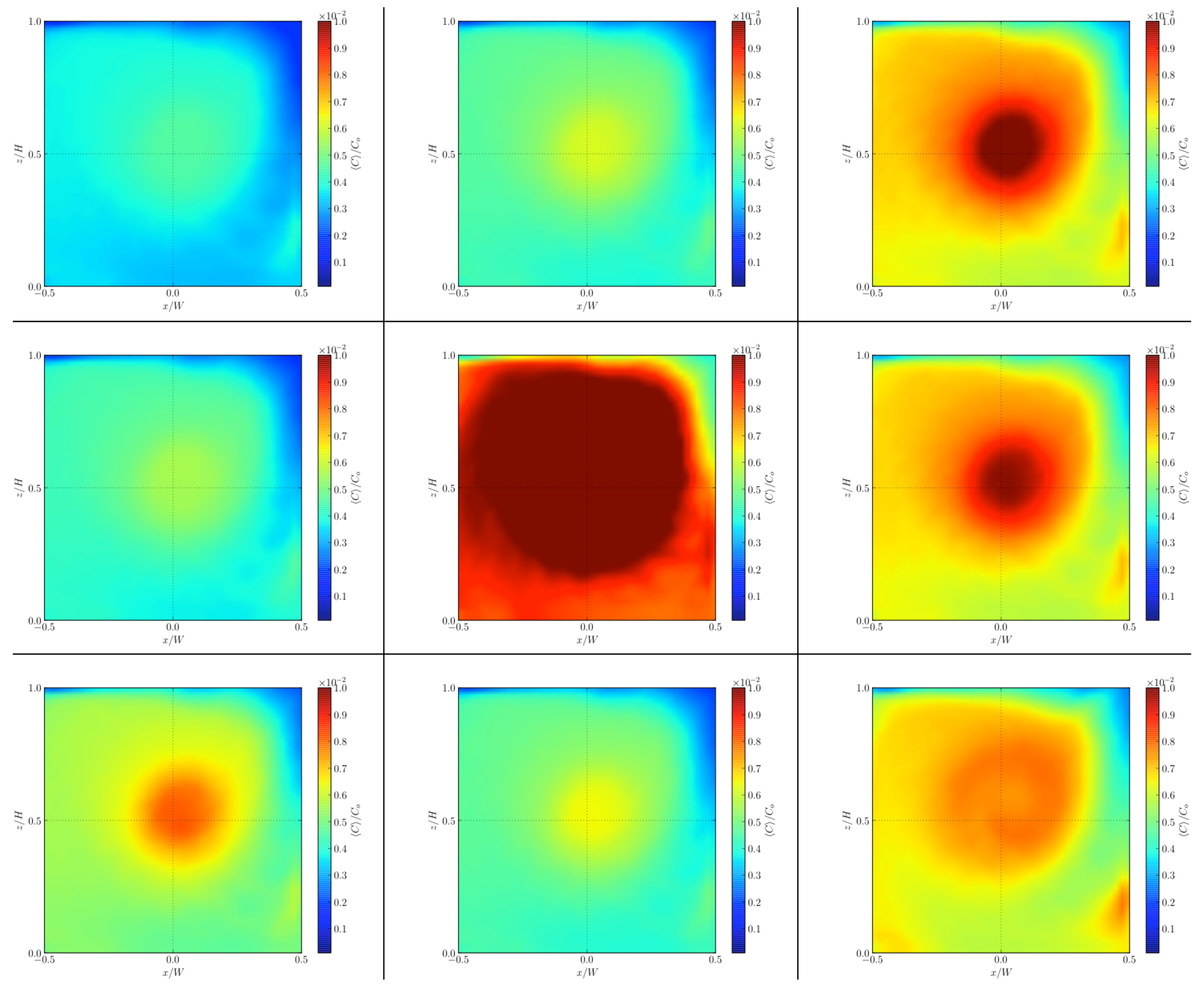

Fig.4 Long-time concentration fields (t=1000s) for different initial conditions.

The mean concentration decays exponentially for all initial conditions.

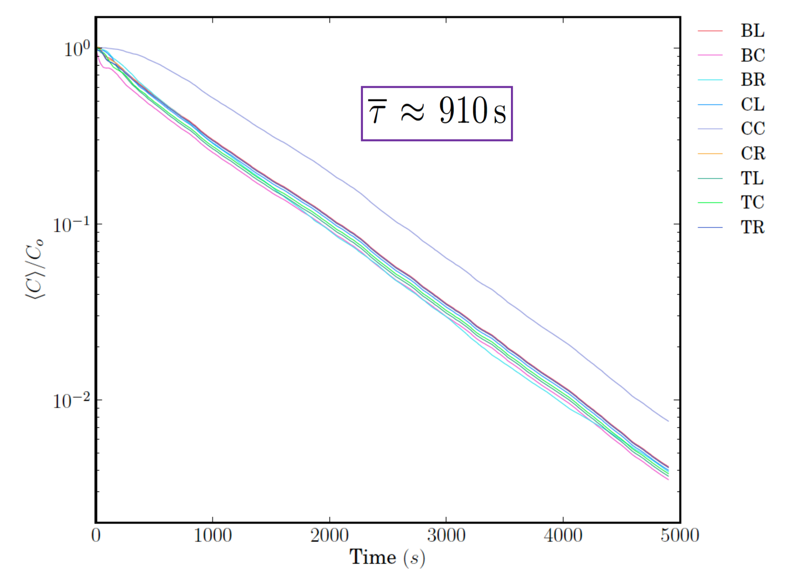

Fig.5 Decay of the mean concentration within the canyon.

Initial variance decay depends on R_i. For large t (≳800s), the variance decay rate is essentially independent of R_i.

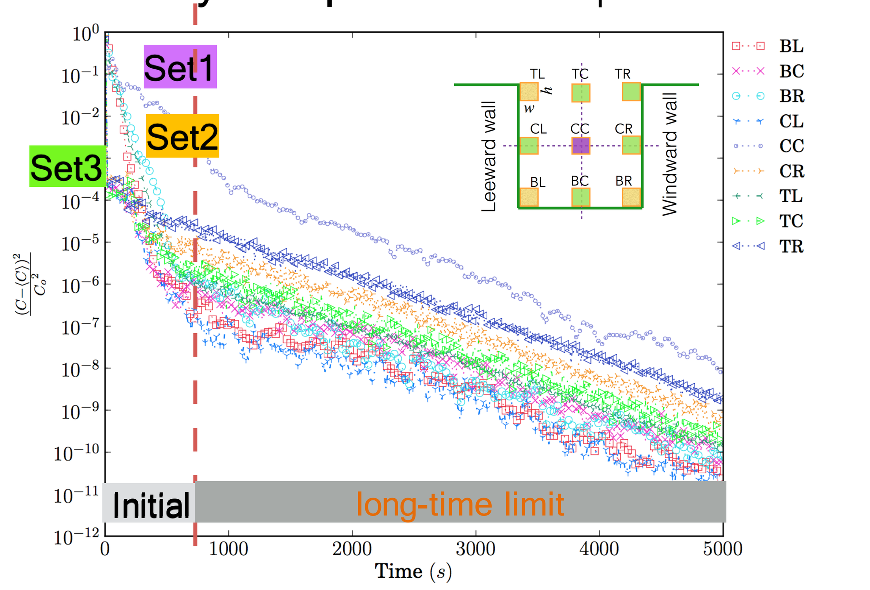

Fig.6 Variance decay of the passive scalar for different initial conditions.

7. Key Timescales

Initial variance decay rate for Set1 (central vortex) appears to be controlled by diffusion across streamlines. The mixing rate can be estimated from analytical predictions for cellular flow [6].

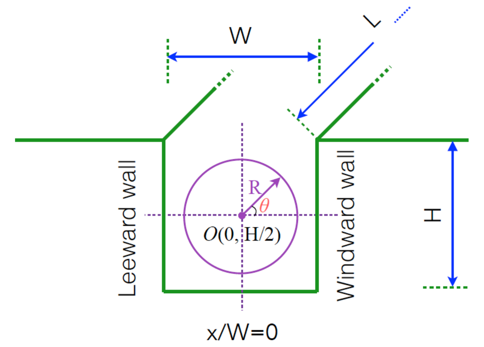

Fig.7 Illustration of a nominal vortex of radius R. The tangential velocity *V* over R is used on calculating the Péclet number: Pe = (2R)*V*/κ.

$$\kappa^*=\langle\kappa\rangle\mathrm{Pe}^{1/2}$$

$$\tau^*=R^2/\kappa^*\approx 177\mathrm{s}$$

Following established theories, mixing rates of Set2 and Set3 appear to be controlled by the Lyapunov exponents.

Table 1 Variance-decay timescales after Eq. (1)

Categories	CC	BL	BR	TL	BC	CL	CR	TC	TR
Set3					66	68	93	87	129
Set2		32	51	42					
Set1	147								
	Set1								

Table 2 Particle divergence timescales after Eq. (2)

Categories	CC	BL	BR	TL	BC	CL	CR	TC	TR
Set3					35	27	31	21	12
Set2		26	26	25					
Set1	–								
			Set2				Set3		

Variance decay in the long-time limit is controlled by the escape of scalar across the open boundary at the roof level.

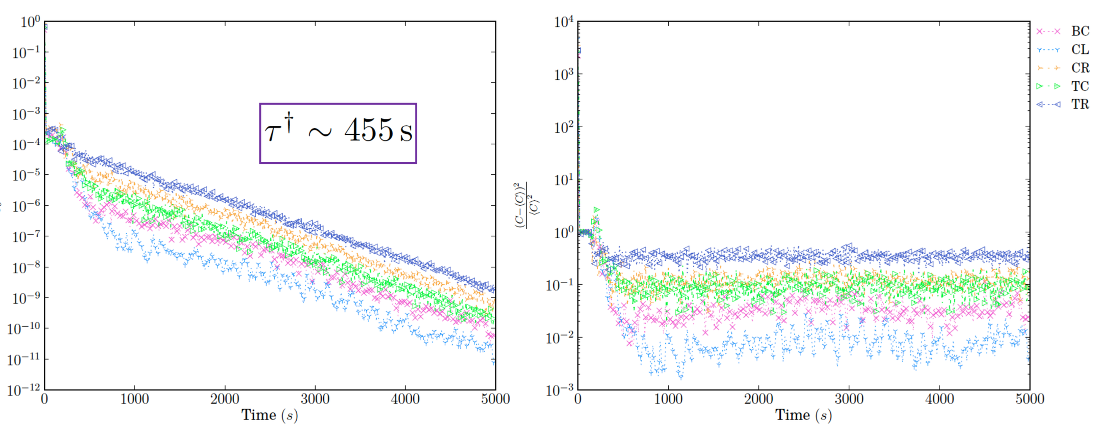

Fig.8 Variance decay in the long-time limit. (Top) Normalisation is against the mean <C>; (bottom) normalisation is over the initial value C_0.

References

[1] Ngan, K. & Shepherd, T.G., 1997. J. Fluid Mech. [2] Rothstein, D., et al., 1999. Nature. [3] Haynes, P. H. & Vanneste, J., 2005. Phys. Fluids. [4] Salman, H. & Haynes, P.H., 2007. Phys. Fluids. [5] Lo, K.W. & Ngan, K., 2015. Boundary-Layer Meteoro. [6] Shraiman, B.I., 1987. Phys. Rev. A.