Clausius-Clapeyron scaling of peak CAPE in continental severe weather environments

Vince Agard and Kerry Emanuel
Massachusetts Institute of Technology

January 24, 2017
AMS Annual Meeting
Robert A. Houze, Jr. Symposium

1 Introduction

Constraining the intensity of severe local storms in continental environments presents a major problem at the intersection of convective meteorology and climate. While continental convective storms can be among the most severe on Earth (Zipser et al 2006), it is not well understood how the distribution and severity of such storms may vary as a function of climate.

This problem can be addressed by identifying constraints on the amount of energy available for convection in such environments. Convective available potential energy (CAPE) provides a metric for which higher values theoretically correspond to more severe storms (Holton 2004) and high CAPE is required for especially severe types of continental convection such as supercells (Emanuel 1994; Rasmussen and Blanchard 1998; Bluestein 2007). Currently, a theoretical constraint on even the order of magnitude of CAPE in such environments does not exist. For instance, why should CAPE be 2,000 J/kg, and not 200 J/kg, or 20,000 J/kg?

While other studies (Parodi and Emanuel 2009; Sobel and Camargo 2011; Romps 2011; Singh and O'Gorman 2013) have derived scalings to constrain equilibrium energy scales for convection in the tropics, the highly transient nature of CAPE in continental environments precludes the use of a quasi-equilibrium framework for its study. Rather than quasi-equilibrium, the continental convection paradigm involves the time-dependent buildup and storage of potential energy in conditionally unstable profiles. It is the peak values of transient CAPE, rather than the time-averaged background levels, that are therefore relevant to the severe storm environments in which we are interested. Therefore, we develop a simple, idealized initial value problem encompassing a typical condition in which severe local storms might form in continental environments.

2 Idealized model

The scenario being modeled is one that is canonically associated with favorable severe weather environments over North America: The southwestern desert gives rise to a hot, dry air mass that is advected eastward by the mean flow aloft. Upon moving east of the Rocky Mountains, this air mass is superimposed above a cooler, moister surface in the Great Plains and Midwest (Emanuel 1994). These regions are home to the continent's most frequent occurrences of extreme peak CAPE (Brooks et al 2003), as the dry air mass acts as an inhibitive cap, allowing CAPE to rise as energy builds at the surface.

The synoptic-scale circumstances outlined above can be simply and ideally modeled by considering a one-dimensional problem in which a dry adiabatic column is placed in contact with a moist surface. As in the real-world case of dry desert air placed above a moister, vegetated surface, surface latent heat flux gives rise to a moist surface boundary layer that expands with time, given some radiative input to the surface.

The initial value problem is constructed such that the column of air initially has a dry adiabatic temperature profile and a specific humidity of zero throughout its depth. As shown in Figure 1, a surface boundary layer with a prescribed initial height is then introduced with a constant, nonzero specific humidity equal to the saturation specific humidity at the initial boundary layer top. The boundary layer is then allowed to evolve under the condition of

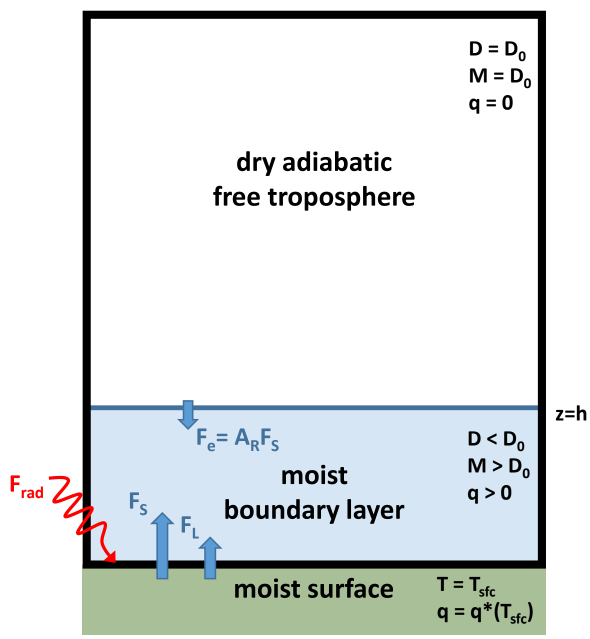


Figure 1: Schematic of the single-column model.

constant surface radiative flux (F_{rad}), which is balanced by turbulent surface sensible and latent heat fluxes (F_S and F_L, respectively). Additionally, as the boundary layer expands upward and entrains hot, dry air from the free troposphere, there is an entrainment heat flux (F_e) into the boundary layer that is assumed to be proportional by a constant to the surface sensible heat flux as in Lilly (1968). The model thereby consists of a sysefatm of ordinary differential equations describing the time evolution of the dry static energy (D), moist static energy (M), and height (h) of the boundary layer, and the temperature of the land surface (T_{sfc}). Appropriately nondimensionalized[1], these equations are as follows:

$$\frac{d\delta}{d\tau} = -\frac{1+A_R}{\eta}(\delta_s + \delta - 1) \tag{8}$$

$$\frac{d\mu}{d\tau} = \frac{\Phi}{\eta} - \frac{\mu A_R}{\eta\delta}(\delta_s + \delta - 1) \tag{9}$$

$$\frac{d\eta}{d\tau} = \frac{A_R}{\delta}(\delta_s + \delta - 1) \tag{10}$$

$$(1-\alpha)(\delta + \delta_s) + \alpha(\mu_s - \mu) - 1 = \Phi, \tag{11}$$

where δ and μ are the dry static energy deficit and moist static energy surplus, respectively, of the boundary layer with respect to the free troposphere; η is the boundary layer height; Φ is the total net radiative forcing to the system; A_R is the coefficient for the Lilly entrainment flux closure (herein taken to be 0.2); α is a coefficient denoting the fraction of surface moisture available for evaporation; and the subscript $_s$ denotes a value pertaining to the surface rather than to the boundary layer.

3 Scaling of peak CAPE

By considering a parcel lifted from the boundary layer with dimensional moist static energy M into the free troposphere, whose environmental moist static energy is a constant M_0 in height and time, we can calculate an approximate CAPE as a function of the boundary layer moist static energy surplus (ΔM, a function of μ):

$$CAPE \approx \Delta M \ln\left(\frac{T_h}{T_{LNB}}\right), \tag{12}$$

[1] Nondimensionalizations are made according to

$$\delta \equiv \frac{D_0 - D}{D_0} \tag{1}$$

$$\mu \equiv \frac{M - D_0}{D_0} \tag{2}$$

$$\eta \equiv \frac{h}{h_0} \tag{3}$$

$$\tau \equiv \frac{C_T v_{sfc}}{h_0} t \tag{4}$$

$$\delta_s \equiv \frac{c_p T_{sfc}}{D_0} \tag{5}$$

$$\mu_s \equiv \frac{c_p T_{sfc} + L_v q^*(T_{sfc})}{D_0} \tag{6}$$

$$\Phi \equiv \frac{F_{rad}}{\rho C_T v_{sfc} D_0}, \tag{7}$$

where h_0 is the initial boundary layer height, $M_0 = D_0$ is the initial moist (and dry) static energy of the dry adiabatic column (and therefore of the free troposphere), and V_{sfc} is the assumed surface wind speed.

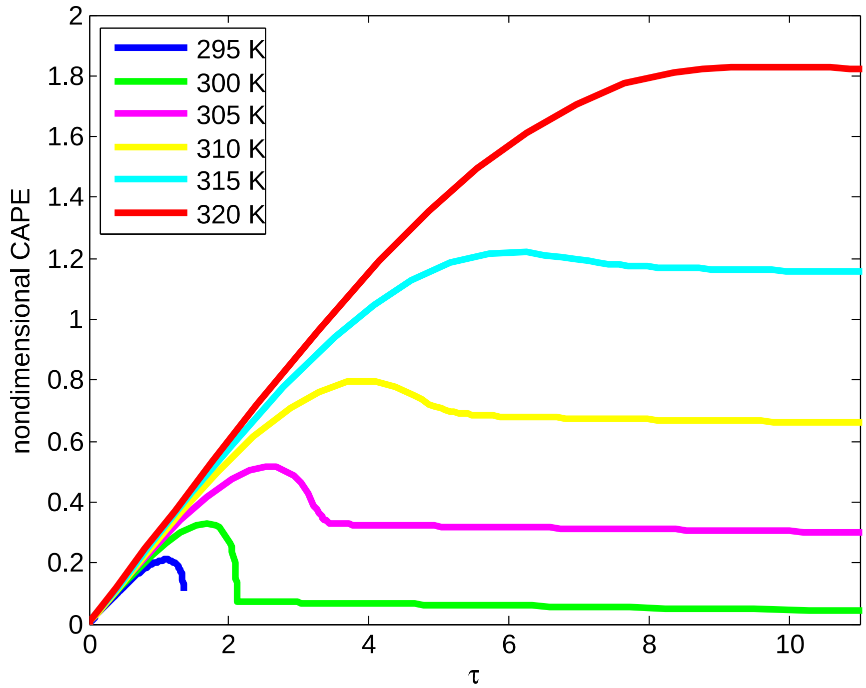


Figure 2: Evolution of CAPE with time for several different values of T_0, given $\alpha = 0.5$. Here, CAPE is nondimensionalized by a factor of $\frac{\rho C_T v_{sfc}}{F_{rad}}$, while time is nondimensionalized according to (4).

where T_h is the temperature at the top of the boundary layer (a function of δ and η), and T_{LNB} is the temperature at the parcel's level of neutral buoyancy, herein assumed to be a constant value of 220 K, representative of a typical tropopause temperature in relevant regions (Hoinka 1999; Holton 1995; US Standard Atmosphere 1976).

Figure 2 shows the time evolution of CAPE for a particular choice of the surface moisture parameter, but for several values of T_0, the initial near-surface air temperature of dry adiabatic column. CAPE in this model is transient, initially increasing from an initial value of zero, peaking at some point in time, and decreasing thenceforth. This transience is due to the competing effects of two different mechanisms: Surface fluxes heat and moisten the boundary layer, thereby increasing its moist static energy (ΔM), and by extension increasing the CAPE in the column. Meanwhile, the upward growth of the boundary layer engenders entrainment of dry air from the free troposphere above. This acts to decrease ΔM, and by extension decreases the CAPE in the column. CAPE is further diminished as the increasing altitude of the boundary layer top lessens the difference between T_h and T_{LNB}. Near the beginning of this initial value problem, the surface fluxes dominate as the boundary layer grows slowly, but they are eventually overwhelmed by the boundary layer growth process–this is the time at which peak CAPE is reached.

The value of peak CAPE increases monotonically with increasing temperature when other parameters are held fixed.

The functional relationship between peak CAPE and initial near-surface air temperature is shown in Figure 3. For any given fixed set of parameters (α, h_0, V_{sfc}), peak CAPE is found to increase approximately exponentially with increasing temperature T_0. Note that

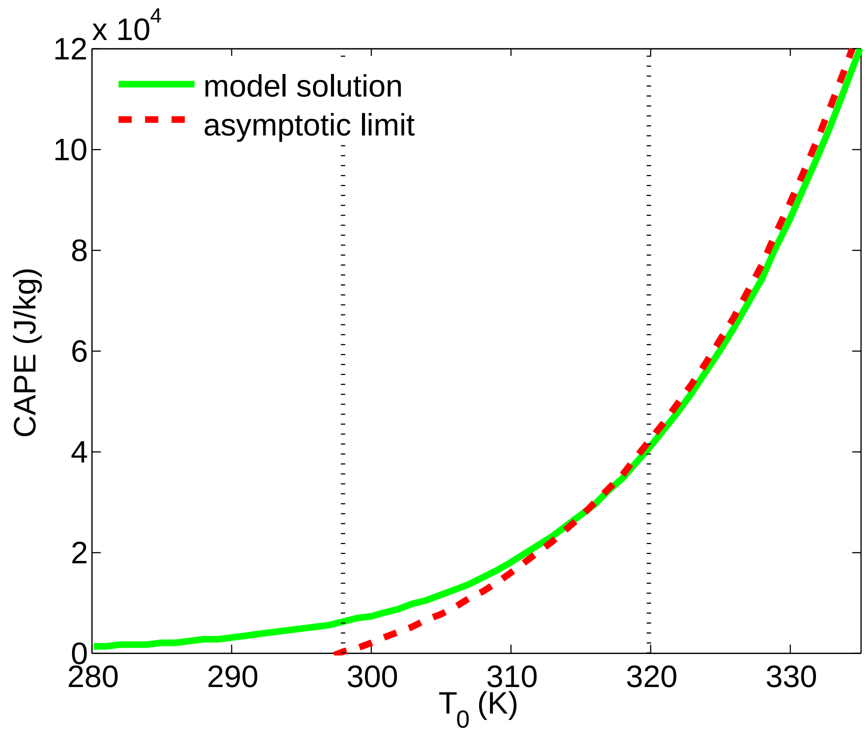


Figure 3: Modeled dimensional peak CAPE as a function of T_0 (solid green line) for $\alpha = 0.5$, $h_0 = 100$ m, $v_{sfc} = 5$ m/s, and $F_{rad} = 200$ W/m^2; and a theoretical curve (dashed red line) corresponding to the asymptotic limit of CAPE that arises as $t \to \infty$, or $h_0 \to 0$. This curve is an exact solution to the Clausius-Clapeyron relation, as described by (13). The dotted black lines indicate the boundaries between three regimes of model solutions.

dimensional CAPE in this model is considerably higher than that which is observed in nature; this is due to the idealizations made in constructing the model, in particular the assumption that the boundary layer is topped by a perfectly dry adiabatic layer with unlimited depth.

In all but the coldest model regime, the dimensional moist static energy surplus approaches a finite positive value as time goes to infinity. This long-time limit is given by

$$\lim_{t\to\infty} M - D_0 = L_v q^*(T_0) - \frac{F_{rad}}{\alpha\rho C_T v_{sfc}}. \tag{13}$$

Combining this limit with (12) provides an exact solution to the Clausius-Clapeyron equation relating saturation specific humidity to temperature. This solution defines the response of the long-time peak of CAPE to changes in temperature. As T_0 increases, the transient peak of CAPE gets closer to its long-time limit, so this asymptotic limit becomes a very good approximation for peak CAPE at warm temperatures. This solution also corresponds exactly to peak CAPE for the limit in which $h_0 \to 0$. The solution is linearly dependent on $q^*(T_0)$, and peak CAPE therefore scales exactly according to the Clausius-Clapeyron relation in the temperature space for which it applies.

As peak CAPE increases with increasing temperature, so too does the time required for peak CAPE to be achieved. This presents an issue, since in nature the boundary layer does not grow continuously for infinite time. Instead, energetic input to the system stops as the sun sets and the net radiative input to the surface goes to zero. This issue could be addressed by imposing a diurnal cycle on the surface radiative input; for simplicity, we merely introduce a limiting dimensional time scale after which the constant radiation is to be cut off. We assume a cut off time scale of 12 hours (or nondimensionally, $\tau_{diurnal} = \frac{C_T v_{sfc}}{h_0} \cdot 12$ hours) to be a duration representative of diurnal radiative input. The maximum value of CAPE achieved within time $\tau_{diurnal}$ is shown in Figure 4 as a function of T_0 for a particular choice of h_0, α and v_{sfc}.

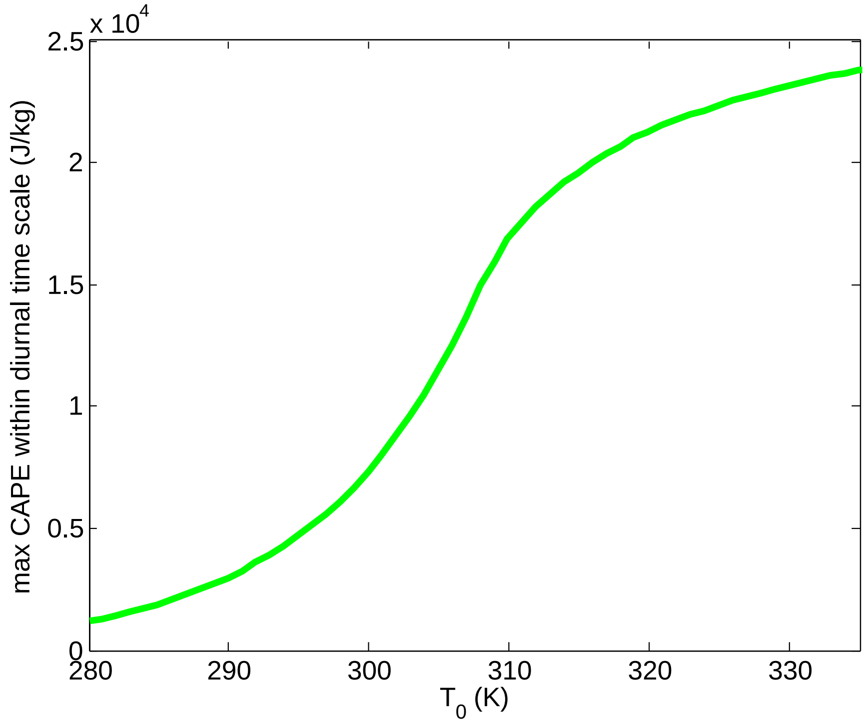
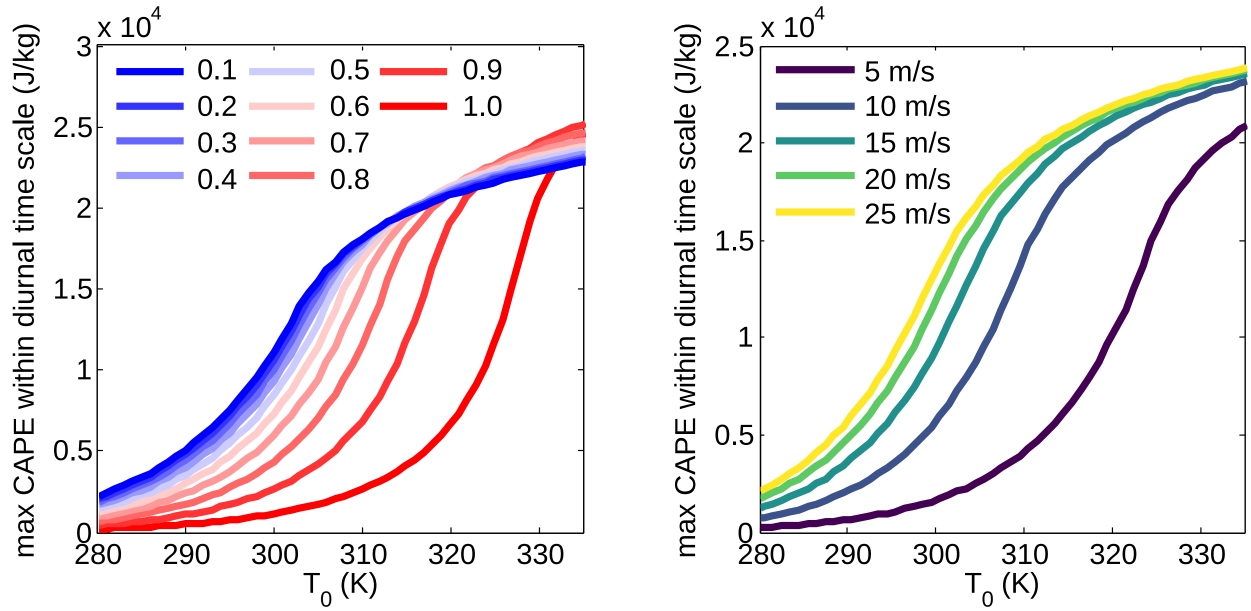


Figure 4: Modeled maximum CAPE within a diurnal time scale, as a function of T_0 for $\alpha = 0.5$, $h_0 = 100$ m, $v_{sfc} = 5$ m/s, and $F_{rad} = 200$ W/m^2.

The imposition of the diurnal time limit introduces a new behavior for peak CAPE values at high temperature. This model predicts that, for a given choice of constant parameters, increasing T_0 will cause the peak in CAPE to occur later in the day. At low temperatures (in this case, $T_0 \lesssim 305$ K), the CAPE peak occurs prior to the end of the diurnal time scale, so the curve of peak CAPE with T_0 retains its exponential nature. At higher temperatures, CAPE is still increasing when $\tau = \tau_{diurnal}$, so the maximum in CAPE occurs at the diurnal time limit. In this regime, the maximum in CAPE still increases with increasing T_0, but its rate of increase is lessened. A consequence of this is the existence of a maximum in the sensitivity of peak CAPE with respect to temperature. In Figure 4, this sensitivity maximum occurs at $T_0 \approx 305$ K.

Finally, we vary the surface moisture (α) and wind speed (V_{sfc}) parameters to determine their effects on peak CAPE. These results are shown in Figure 5. The flux of moist static energy from the surface to the boundary layer that fuels CAPE buildup is a function of both surface moisture and wind speed. Increasing either the surface moisture parameter or the surface wind speed parameter has the effect of modifying the Bowen ratio such that a greater



(a) varying surface moisture parameter α; constant $v_{sfc} = 5$ m/s

(b) varying surface wind speed parameter V_{sfc}; constant $\alpha = 0.5$

Figure 5: Modeled maximum CAPE within a diurnal time scale, as a function of T_0 with varying α and v_{sfc}; $F_{rad} = 200$ W/m^2 and $h_0 = 100$ m.

portion of the surface moist static energy flux is partitioned to latent heat flux. Both peak CAPE and the time taken to achieve it are thereby increased. As a consequence, CAPE increases more quickly with increasing temperature, but the regime of maximum sensitivity is shifted to lower temperatures. Nevertheless, for a fixed T_0, increasing either α or V_{sfc} has the monotonic effect of increasing peak CAPE.

Both in the presence and in the absence of a limiting time scale, this model predicts increasing CAPE in midlatitude continental severe weather environments as near-surface air temperature increases. In the case of no limiting time scale, that relationship scales according to the Clausius-Clapeyron relation. Furthermore, we conclude that high temperature, high surface moisture, and high surface winds are each conducive to high CAPE, and therefore to intense thunderstorms, in such environments.

References

Bluestein, H. B. (2007), Advances in applications of the physics of fluids to severe weather systems, *Reports on Progress in Physics*, *70*(8), 1259.

Brooks, H. E., J. W. Lee, and J. P. Craven (2003), The spatial distribution of severe thunderstorm and tornado environments from global reanalysis data, *Atmospheric Research*, *67*, 73–94.

Emanuel, K. A. (1994), *Atmospheric Convection*, Oxford University Press.

Hoinka, K. P. (1999), Temperature, humidity, and wind at the global tropopause, *Mon. Weather Rev.*, *127*(10), 2248–2265, doi:10.1175/1520-0493(1999)127⟨2248:THAWAT⟩2.0.CO;2.

Holton, J. R. (2004), *An introduction to dynamic meteorology*, 4 ed., 535 pp., Elsevier.

Holton, J. R., P. H. Haynes, M. E. McIntyre, A. R. Douglass, R. B. Rood, and L. Pfister (1995), Stratosphere-troposphere exchange, *Rev. Geophys.*, *33*(4), 403–439, doi:10.1029/95RG02097.

Lilly, D. K. (1968), Models of cloud-topped mixed layers under a strong inversion, *Quarterly Journal of the Royal Meteorological Society*, *94*(401), 292–309.

NOAA, NASA, and US Air Force (1976), U.S. Standard Atmosphere, 1976, *Tech. rep.*, U.S. Government Printing Office, Washington, D.C.

Parodi, A., and K. Emanuel (2009), A theory for buoyancy and velocity scales in deep moist convection, *Journal of the Atmospheric Sciences*, *66*(11), 3449–3463.

Rasmussen, E. N., and D. O. Blanchard (1998), A baseline climatology of sounding-derived supercell andtornado forecast parameters, *Weather and Forecasting*, *13*(4), 1148–1164.

Romps, D. M. (2011), Response of tropical precipitation to global warming, *Journal of the Atmospheric Sciences*, *68*(1), 123–138.

Singh, M. S., and P. A. O'Gorman (2013), Influence of entrainment on the thermal stratification in simulations of radiative-convective equilibrium, *Geophysical Research Letters*, *40*(16), 4398–4403.

Sobel, A. H., and S. J. Camargo (2011), Projected future seasonal changes in tropical summer climate, *Journal of Climate*, *24*(2), 473–487.

Zipser, E. J., D. J. Cecil, C. Liu, S. W. Nesbitt, and D. P. Yorty (2006), Where are the most intense thunderstorms on Earth?, *Bulletin of the American Meteorological Society*, *87*(8).