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1. INTRODUCTION

Predicting the possible penetration (or injection) heights of smoke plumes from wildland forest fires is an unresolved problem. Until now, most approaches have followed those of air pollution plumes generated from tall chimneys. The relationship between the two is weak, as kinetic energy is the primary factor in chimney plumes while wildland fires are buoyancy driven.

Chimney plume models, such as the Briggs model (1972), typically use a Gaussian dispersion model to capture both the horizontal and vertical spread of the smoke plume downwind of the chimney. The vertical velocity of the emissions at the top of the chimney is converted to kinetic energy, which, when combined with ambient wind flow, is translated into vertical and horizontal spread. Stability is captured in general terms of neutral, stable or unstable.

A wildland fire does not have an initial vertical velocity. Instead the fire creates a buoyant plume, which draws in air from the surrounding surface region. Energy involved in wildland fires is typically on the scale of megajoules [MJ] released over hours.

Volcanic plumes are also dissimilar from wildland fire smoke plumes. Volcanic eruptions are dynamic events of reversed buoyancy (Kaminski *et al.* 2005), where material is ejected into the atmosphere in a negatively buoyant plume. The scale of volcanic eruptions is much greater than that of wildland fires. For example, the total energy released by the Krakatoa explosions was 200 megatons of TNT, or approximately 10^{12} MJ, released in a matter of seconds. Slower lava-releasing eruptions may be more comparable to wildland fires, though plumes from these are not typically studied.

The Atmospheric Dispersion Index (ADI), developed by Lavdas (1986), was designed specifically for prescribed burning. The ADI model applies Gaussian dispersion over the depth of the mixing layer to give a numerical index of potential smoke dispersion. The ADI does not simulate smoke plumes nor predict plume heights.

Latham (1994) developed PLUMP, a one-dimensional smoke plume model that captures cloud physics and entrainment (1994). This model was used to estimate CO emissions from biomass burning in the Amazon basin (Freitas *et al.* 2007) and is perhaps the best way to conduct smoke plume modelling at present. However, its complexity makes it a challenge to incorporate into current smoke management plans.

Cunningham *et al.* (2005) conducted numerical simulations of smoke plumes with the use of the Weather Research and Forecasting (WRF) model. Two approaches were used: solving the governing equations, and using large eddy simulation. Like the PLUMP model, this is a complex method of approaching the problem and would be a challenge to implement operationally.

The BlueSky Smoke Modeling Framework is an operational smoke management system currently running in the US and Western Canada (Larkin et al, 2009). The model uses a modified form of the Briggs approach (Pouliot 2005) for plume rise.

Most recently, Achtemeier *et al.* (2011) implemented the Daysmoke model for prescribed fires in the southern US. This is a simpler model involving turrets of smoke that expand with plume

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rise. The intended use of the model is for prescribed burns and at present does not incorporate adiabatic expansion.

This paper constructs an alternate approach to estimating the penetration height of smoke plumes from wildland fires. This initial phase focuses on the penetration height of the plume given a specific amount of heat injected into the atmosphere. Following thermodynamic principles, plume height is estimated by modifying a column of air affected by the fire, with the plume top height corresponding to the top of buoyant rise, or level of thermal equilibrium. The benefit of this approach is that it can provide an easy substitute for the Briggs method currently used in smoke management programs such as BlueSky. It can also be quickly calculated for the hundreds of satellite-detected hotspots such systems are required to handle.

2. THEORY

To solve for penetration height, one must consider two principal factors: the energy being released into the atmosphere by the fire and the ambient (environmental) lapse rate. An appropriate amount of heat is mixed through a column of air above the fire until the column reaches thermal balance as represented by the dry adiabatic lapse rate.

2.1 *Energy Released*

The energy released from a wildland fire can be determined using Byram's equation

(1)
$$I = H \, w \, r$$

where I is the intensity of a fire per unit length of fire front [typically measured as kW m^{-1}], H is the heat of combustion [kJ kg^{-1}], w is the weight of the fuel consumed per unit area [kg m^{-2}] and r is the rate of spread [m s^{-1} but normally measured as m min^{-1}] (Byram 1959). The heat of combustion H is a constant, typically 18 000 kJ kg^{-1} for dry wood. Following a similar format, the energy released by the fire, Q [kJ] becomes

(2)
$$Q = H \, w \, A$$

where A [m^2] is the area burned. A more rigorous representation would take the form

(3)
$$\frac{dQ}{dt} = I \, L = H \, w \, r \, L$$

where L is length of the fire line [m].

Current fire danger rating systems provide a means of calculating the rate of spread and the fuel consumed in forest fires. Two systems, the Canadian Forest Fire Danger Rating System (CFFDRS) (Stocks *et al.* 1989) and the USDA Forest Service's National Fire-Danger Rating System (NFDRS) (Burgan 1988), provide these values. The Canadian Forest Fire Behaviour Prediction (FBP) System (Forestry Canada Fire Danger Group 1992), a component of the CFFDRS, predicts the rate of spread (ROS) [m min^{-1}] and total fuel consumption (TFC) [kg m^{-2}]. The TFC can be used as the w term in equation 2 to provide an estimate of the energy released into the atmosphere per square metre. Likewise, the TFC and the ROS can be used in equation 3 to calculate the energy released over time, which could be used to model plume development.

At this stage, it must be recognized that not all of this heat is released into the atmosphere. Byram points out that heat is lost to radiation, evaporation of fuel moisture, and incomplete combustion (Byram 1959). One must consider that a certain amount of heat is also injected downward into the ground. Further investigation into these factors is in order and will be conducted in future development of this model.

2.2 *Tephigram*

Thermodynamics can be used to solve the problem of the penetration height of smoke plumes. This approach uses the tephigram, a thermodynamic diagram where energy per unit mass is proportional to area on the diagram. The tephigram (see Figure 1) captures isothermal and adiabatic processes by using temperature and the natural logarithm of the potential temperature (the temperature of a parcel if adiabatically brought to a standard reference pressure, in this case 1000 hpa) as the coordinates. The diagram itself is rotated 45° to show pressure levels [hpa] as nearly horizontal lines on the diagram.
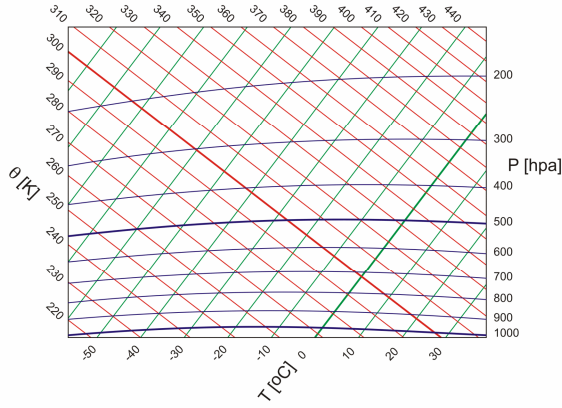
Figure 1. The tephigram. Isobars are indicated in blue, isotherms are indicated in green and dry adiabats are indicated in red. Area on the tephigram is proportional to energy per unit mass.

Using the tephigram, the penetration height of smoke can be determined based on an average environmental lapse and the energy released by the fire. Figure 2 illustrates how the energy of the fire heats the column of air above it until this column reaches the dry adiabatic lapse rate. The height of this modified column is thus dependent on the energy of the fire and the environmental lapse rate. If the environmental lapse rate is near the dry adiabat, little energy will be needed to lift the plume and corresponding plume tops will be higher.
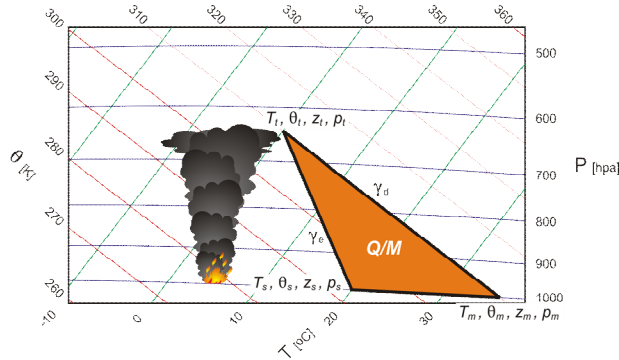

Figure 2. Representation of a smoke plume on a tephigram. The area, Q/M, indicates the energy released by the fire, which is constrained by the environmental lapse rate γ_e and the dry adiabat γ_d. Values for the temperature, potential temperature, height and pressure (T, θ, z and p) can be determined at the surface, the top and within the modified zone (subscripts s, t, and m).

3. SOLUTION

A thermodynamic solution to the penetration height of the smoke plume follows the approach captured in Figure 2. Heat produced from the fire is equated to the energy term, Q [J], which then modifies the column of air above the fire. The mass of the air column, M [kg], is required to solve the problem

3.1 *Energy per Unit Mass*

The energy per unit mass required to modify the atmosphere can be found using the equation

$$q = -c_p \oint T \, d \ln \theta$$

(4)

where q is the energy per unit mass [J kg^{-1}], c_p is the heat capacity of dry air [1005 J kg^{-1} K^{-1}], T is the temperature [K] and θ is the potential temperature [K]. The area within the closed curve, as shown in Figure 2, equals

$$q = -\frac{1}{2} c_p (T_t - T_m) \left[\ln(\theta_m) - \ln(\theta_s) \right]$$

(5)

where subscripts s and t represent unmodified conditions at the surface and at the top of the plume while m represents the surface conditions modified by the fire. Note that the modified conditions do not represent the temperature of the air within the combustion zone *per se* but the heating effect through the depth of the column. This captures the effect of air drawn in to the combustion zone.

Following adiabatic ascent,

$$T_t - T_m = \gamma_d \Delta z$$

(6)

where γ_d is the dry adiabatic lapse rate (negative with height) and Δz is the change in height. At the surface (~1000 hpa)

$$\ln(\theta_m) - \ln(\theta_s) = \ln(T_m) - \ln(T_s)$$

(7)

$$= \ln(T_m / T_s)$$

$$= \ln \left\{ \left[T_s + \Delta z(\gamma_e - \gamma_d) \right] / T_s \right\}$$

allowing equation 5 to be reformulated as

$$q = -\frac{1}{2} c_p \gamma_d \Delta z \ln \left[1 + \frac{\Delta z(\gamma_e - \gamma_d)}{T_s} \right]$$

(8)

where γ_e is the environmental lapse rate.

3.2 *Mass of the Affected Column*

The mass of the atmosphere per unit area below a height can be solved from the hydrostatic equation

$$\frac{dp}{dz} = -\rho g = -\frac{pg}{R_d T}$$

(9)

where p is the pressure [pa], ρ is the atmospheric density, g is gravitational acceleration [9.8 m s^{-1}], and R_d is the gas constant [287.05 J kg^{-1} K^{-1}]. Rearranging the terms and integrating with height,

$$\int_{p_1}^{p_2} \frac{1}{p} dp = -\int_{z_1}^{z_2} \frac{g}{R_d T} dz$$

(10)

$$= -\int_{z_1}^{z_2} \frac{g}{R_d (T_1 + \gamma_e z)} dz$$

$$\ln(p)\big|_{p_1}^{p_2} = -\frac{g}{R_d \gamma_e} \ln(T_1 + \gamma_e z)\big|_{z_1}^{z_2}$$

(11)

$$\ln(p_2 / p_1) = -\frac{g}{R_d \gamma_e} \ln\left(\frac{T_1 + \gamma_e z_2}{T_1 + \gamma_e z_1}\right)$$

(12)

If we define $z_1 = 0$ at the surface, T_1 and p_1 as the surface temperature and pressure, and the plume top height as $z_2 = \Delta z$, equation 12 becomes

$$\ln(p_t / p_s) = -\frac{g}{R_d \gamma_e} \ln\left(\frac{T_s + \gamma_e \Delta z}{T_s}\right)$$

(13)

$$= -\frac{g}{R_d \gamma_e} \ln\left(1 + \frac{\gamma_e \Delta z}{T_s}\right)$$

then

$$p_t = p_s \left(1 + \frac{\gamma_e \Delta z}{T_s}\right)^{-g/\gamma_e R_d}$$

(14)

From first principles, the mass per unit area, M / A, is the difference in pressure between two levels divided by g

$$M / A = (p_s - p_t) / g$$

(15)

thus, equation 14 becomes

$$M = \frac{p_s}{g} A \left[1 - \left(1 + \frac{\gamma_e \Delta z}{T_s}\right)^{-g/\gamma_e R_d}\right]$$

(16)

3.3 Plume Height

Combining equations 8 and 16, we get

$$Q = q\, M$$

(17)

$$= -\frac{1}{2} c_p \gamma_d \Delta z \ln\left[1 + \frac{\Delta z(\gamma_e - \gamma_d)}{T_s}\right] \times$$

$$\frac{p_s}{g} A \left[1 - \left(1 + \frac{\gamma_e \Delta z}{T_s}\right)^{-g/\gamma_e R_d}\right]$$

$$= -\frac{1}{2} p_s A \Delta z \ln\left[1 + \frac{\Delta z(\gamma_e - \gamma_d)}{T_s}\right] \times$$

$$\left[1 - \left(1 + \frac{\gamma_e \Delta z}{T_s}\right)^{-g/\gamma_e R_d}\right]$$

In its current form, Δz cannot be isolated to provide a solution from equation 17 for the plume height, rather one must use an iterative procedure which converges on the answer by entering an estimate of the plume height, Δz, into equation 17 and the resulting value of Q is compared to the value of Q from the fire (equation 2). The plume height is then adjusted and the calculations are redone.

Note that from this equation, as γ_e approaches γ_d, the term $\gamma_e - \gamma_d$ approaches zero causing Q to approach zero, and the equation becomes unsolvable for Δz (normally, dry adiabatic lapse

rates do not extend beyond the mixing layer, typically a few kilometers at most).

3.4 Entrainment

Entrainment can be captured within the model by modifying the plume shape as illustrated in Figure 3. The first case is the conical shape, where the plume width increases with height. For a circular fire,

$$r_s = \sqrt{A_s / \pi}$$

(18)

$$r_t = r_s + \Delta z_e \tan \alpha$$

(19)

where r_s and r_t are the radii of the column at the surface and at the top, Δz_e is the adjusted entrainment height and α is the half-angle capturing the volume increase with height and thus the entrainment (typically 6-15°).

Keeping the mass of the affected volume constant (as determined in equation 16), an average density $\overline{\rho}$ is calculated for the cylindrical column. This average density is used to calculate the mass of the expanding column as

$$M = \frac{1}{3} \pi \overline{\rho} \Delta z_e \left(r_s^2 + r_s r_t + r_t^2 \right)$$

(20)

and this term is then incorporated into the convergence scheme.

Another approach to capture entrainment would be to assume the fire exists as a line of combustion with negligible width compared to plume height. The plume takes the shape of a wedge above this line, as shown in Figure 3. Using this approach, and the average density $\overline{\rho}$ as discussed above, the solution for the mass within the entrained zone would be

$$M = \overline{\rho} L \Delta z_e^{\,2} \tan \alpha$$

(21)

where L is the length of the fire perimeter.

It is worth noting that the area burned A appears in both equations 2 and 17, thus cancelling out for the vertical cylinder solution shown in Figure 3. In the case of the conical and wedge shape described, the fire size reappears as factor represented by the

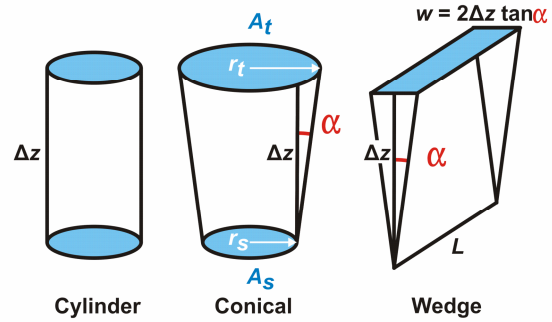radii in equation 20 and the length of the fire perimeter in equation 21.

Figure 3. Possible plume shapes. The cylinder represents no entrainment as captured by equation 17. The conical shape assumes increasing plume width with height (equation 20). The wedge assumes entrained expansion from a line of combustion (equation 21).

4. PAPER EXERCISE

As a means of testing the model while cross-checking the calculations, an exercise was conducted manually on a tephigram and following the conditions of the International Civil Aviation Organization (ICAO) Standard Atmosphere (International Civil Aviation Organization 1993).

In this exercise, a plume is assumed to rise from 1000 hpa (111 m ASL) to 500 hpa (5574 m ASL). At sea level, the ICAO Standard Atmosphere is defined as 15°C with an environmental lapse rate of -6.5°C/km. The resulting surface temperature, T_s, would be 14.3°C and plume top temperature at 500 hpa, T_t, would be -21.2°C. Following the dry adiabat to 1000 hpa, the modified temperature, T_m, would be 32.3°C. The energy required to heat this layer to the dry adiabat would be 1636.5 J kg^{-1} using equation 5. The mass per unit area of the atmosphere from 1000 hpa to 500 hpa would be 5102 kg m^{-2}. using equation 15. Thus for a 1 ha fire (10^4 m^2), the required energy to heat the column above the fire would be 8.35 x 10^{10} J. Using this value as the energy of the fire Q and solving for w in equation 2, the required fuel consumption to heat the column would be 0.464 kg m^{-2}. The same data used in equation 17 yields a plume height of 5463.9 m, practically matching a height of 5463 m based on the Standard Atmosphere.

By fire behaviour standards, a fuel consumption of 0.464 kg m^{-2} is a low value. On the other hand,

this value assumes that all of the energy of combustion is converted into buoyant plume rise, and overlooks the energy that goes into radiation, evaporation of fuel moisture, and incomplete combustion. It also does not consider entrainment. When all these factors are taken into account, it is likely that the values arrived at by this method will approximate the effective fuel consumption typical of normal forest fires.

5. DISCUSSION

The thermodynamic solution presented in this paper is for the simple case of a column expanding vertically in an environment with a constant lapse rate and little wind. There is no exchange of energy between the column and the air outside the smoke column.

It is assumed there is a combustion zone at the base of the column. A circulation is present where external air is brought in at the surface into the combustion zone and air exits at the top of the column. No energy is exchanged in these parts of the circulation.

There is no temporal component to this model. The model measures the plume height after thermal equilibrium within the column is achieved.

The model does not address the following issues.

- *Development of the boundary layer*. Over the course of the day, solar heating will lead to the development of a convective boundary layer. This layer will usually have close to a dry adiabatic lapse rate resulting from the convective mixing. This could be addressed in the calculations by including a solar heating term.

- *Variable environmental lapse rate*. The solution provided considers a constant environmental lapse rate and any irregularities are overlooked. This is acceptable unless the plume reaches a capping inversion. In such a case, one could conclude that the penetration height would be limited to this level (assuming there is insufficient energy to penetrate the inversion).

- *Condensation*. The solution provided assumes plume rise is a dry adiabatic

process and that no condensation occurs. This could be addressed by comparing penetration height to the lifted condensation level (LCL) based upon the surface humidity. Once the plume rises above the LCL, the solution is no longer valid and the plume could develop as a pyrocumulous cloud.

- *Vertical wind shear*. Presently, the model is structured to calculate the vertical growth of a smoke plume. It does not account for the effects of vertical wind shear, which would increase entrainment within the plume and further reduce vertical growth. The plume height predicted by this model could be viewed as the maximum possible plume height as determined by thermodynamics.

- *Energy from the fire.* At this point the model does not capture the energy balance of the fire. Heat is lost to radiation, evaporation of fuel moisture, and incomplete combustion (Brown and Davis 1973). One must consider that a certain amount of heat is also injected downward into the ground.

6. CONCLUSIONS

A method was developed to predict smoke plume heights using a thermodynamic approach. Equations were developed with and without the effect of entrainment on plume development. The degree of entrainment is dependent on the fire size, which may or may not be readily available.

This is a work in progress. Future efforts will concentrate on factors affecting the energy released into the atmosphere including radiation, fuel moisture, and completeness of combustion.

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