Aggregation of spatially dependent forecasts

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Ensemble modeling is being used widely

Examples

- NOAA's - North American Multi-Model Ensemble [3]
- Federal Reserve Bank Survey of Professional Forecaster
- Analysts consensus forecasts available on most investor websites

Benefits [1]

- Frequently the average of multiple forecasts is more accurate than even the best individual forecast
- The variation in the forecasts indicates the overall degree of uncertainty

Issues [2]

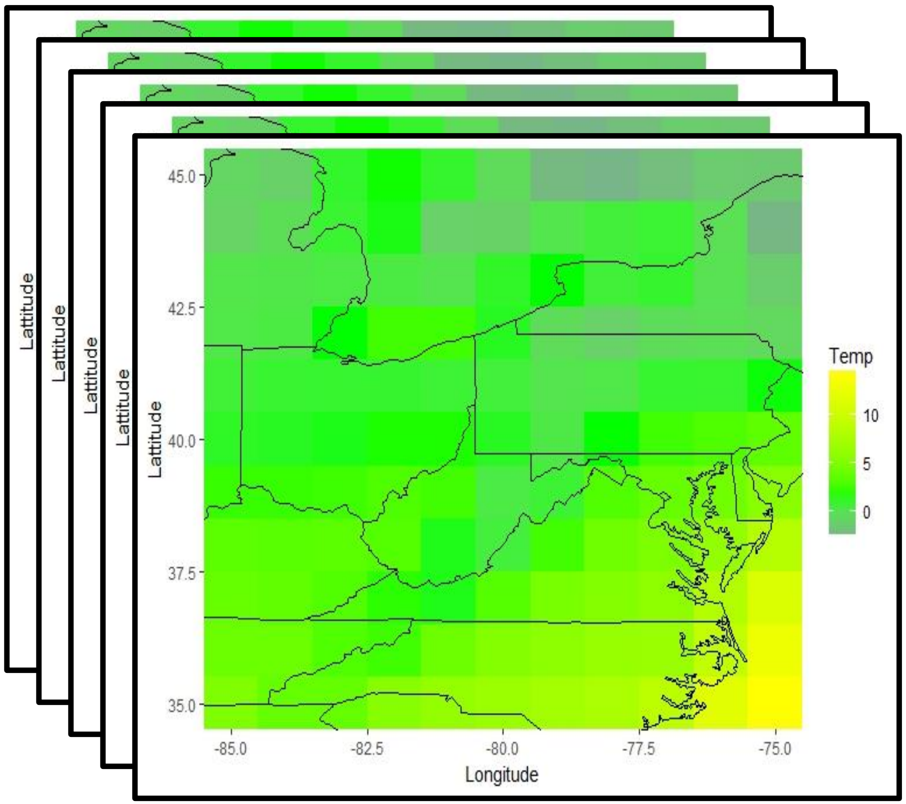
- In-sample optimization versus out of sample performance
- When is a simple average better than more complex approaches?
- Common assumptions and approaches create dependencies between forecasts.

This work will focus on understanding and mitigating the dependencies - across models, and across location simultaneously

1) Jason RW Merrick, 2008; Kenneth F. Wallis, 2011; Robert T. Clemen., 1989.
2) Hsiao 2014, Winkler 1992, Weigel 2010
3) Kirtman 2014, Palmer 2004

Today, decision makers have multiple forecasts of multiple data points

Temperature Forecasts from Multi-model ensembles



11 month ahead forecasts for Dec. 2016

North American Multi-Model Ensemble
IRI/LDEO collection of climate data

<u>Models used</u>

CANSIPS

CMC1-CanCM3 – Forecast & Hindcast

CMC2-CanCM4

COLA-RSMAS-CCSM3

COLA-RSMAS-CCSM4

GFDL-CM2p1-aer04

GFDL-CM2p5-FLOR-A06

GFDL-CM2p5-FLOR-B01

"We acknowledge the agencies that support the NMME-Phase II system, and we thank the climate modeling groups (Environment Canada, NASA, NCAR, NOAA/GFDL, NOAA/NCEP, and University of Miami) for producing and making available their model output. NOAA/NCEP, NOAA/CTB, and NOAA/CPO jointly provided coordinating support and led development of the NMME-Phase II system."

There can be correlations between forecasters as well as locations

Various forms of forecast averaging are typically used

Simple Average
(Genre 2013)

$$w_i = 1/n_m$$

Weighted Average
(Hsiao 2013)

$$w_i = \sigma_i^{-2} / \sum_{j=1}^{n_m} \sigma_j^{-2}$$

Bayesian Model Averaging
(Smith 2009)

$$P(Y \mid f_1, \ldots, f_{n_m}) = \sum_{j=1}^{n_m} \mathcal{L}(Y \mid f_j) P_0(f_j)$$

These approaches do not consider inter-model or intra-model correlations

Approaches have been developed for error correlations in models

Intra-model spatial error correlations
(Cressie 1991)

$$f_i(s_j) = Y_{s=j} + \phi_i \sum_{k \neq j} w_{j,k} \left(f_i(s_k) - Y_{s=k} \right) + \epsilon_i$$

$$e_i(s_j) = \sum_{k=1}^{n_r} \phi_i w_{j,k} \left(e_i(s_k) \right) + \epsilon_i$$

$$\vec{\epsilon}_i = (\mathbf{I}_{\mathbf{r}} - \phi_i \mathbf{W}_{rxr}) \overrightarrow{e_i}$$

Error adjusted for spatial correlations

Inter-model error correlations
(Winkler 1981)

$$e_i = f_i - Y$$

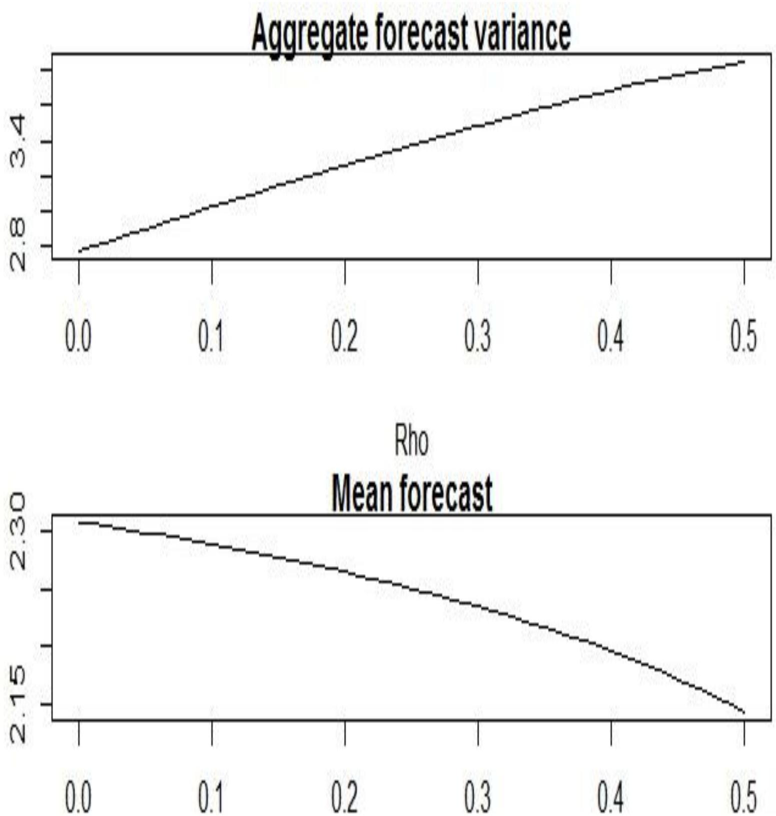
$$(e_1, \ldots, e_{n_m})^t \propto MVN(\vec{0}_{n_m}, \Sigma_m)$$

$$\hat{\sigma} = 1 / (\vec{1}_{n_m}^t \, \mathbf{\Sigma}_m^{-1} \vec{1}_{n_m})$$

$$\hat{Y} = \vec{1}_{n_m}^t \, \mathbf{\Sigma}_m^{-1} \vec{f} \, / \, \hat{\sigma}$$

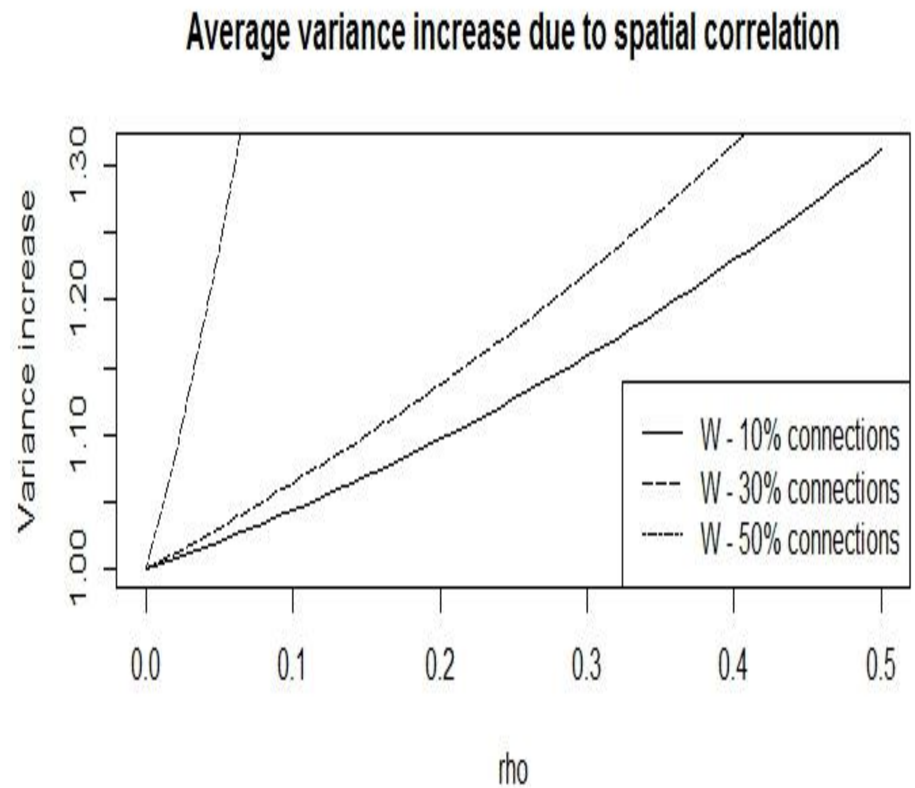
Correlations between models or locations impact the aggregation

Model correlation impact



Impact of correlation on the aggregation of a N(2,4) and a N(3,9) component forecasts using Winkler's(1981) model.

Spatial correlation impact



Impact of spatial auto-correlation on overall forecast variance. Assumes a 10x10 grid and a 1st order Markov Random Field and an SAR model.

Both inter model correlations and intra model spatial correlations are linked by the forecast errors

An integrated approach for model and spatial dependencies is proposed

Likelihood model

$$\begin{pmatrix} \vec{e}_{1,r} \\ \vec{e}_{2,r} \\ \vec{e}_{3,r} \end{pmatrix} = MVN\left[\begin{pmatrix} \vec{0}_r \\ \vec{0}_r \\ \vec{0}_r \end{pmatrix}, \begin{pmatrix} \mathbf{A_1}\sigma_{11}\mathbf{A_1}^t & \mathbf{A_1}\sigma_{12}\mathbf{A_2}^t & \mathbf{A_1}\sigma_{13}\mathbf{A_3}^t \\ \mathbf{A_2}\sigma_{12}\mathbf{A_1}^t & \ddots & \vdots \\ \mathbf{A_3}\sigma_{13}\mathbf{A_1}^t & \cdots & \mathbf{A_3}\sigma_{33}\mathbf{A_3}^t \end{pmatrix} \right]$$

Where: $\mathbf{A_i} = (\mathbf{I}_r - \phi_i \mathbf{W}_{rxr})$ is the spatial correlation matrix for model i

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

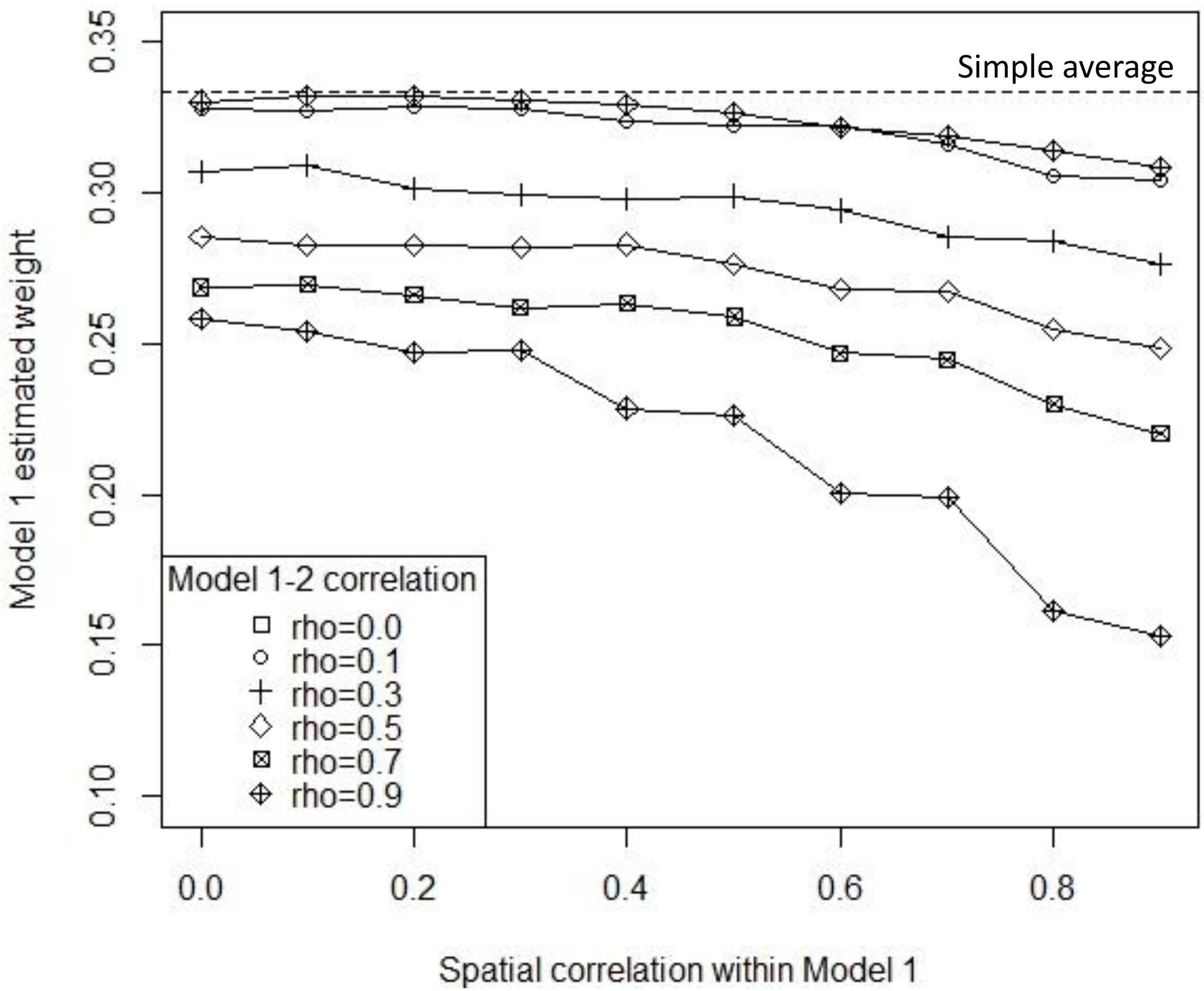
is the inter model error correlation matrix

Bayesian Posterior model

$$\pi(\overrightarrow{Y_r} \mid f's, \phi_{m's}, \mathbf{\Sigma}_m) \propto \mathcal{L}(f's \mid \overrightarrow{Y_r}, \phi_{m's}, \mathbf{\Sigma}_m) P(Y_r) P(\phi_{m's}) P(\mathbf{\Sigma}_m)$$

The interaction of spatial and model correlations can be important – but not always

Optimal weight of Model 1 in ensemble

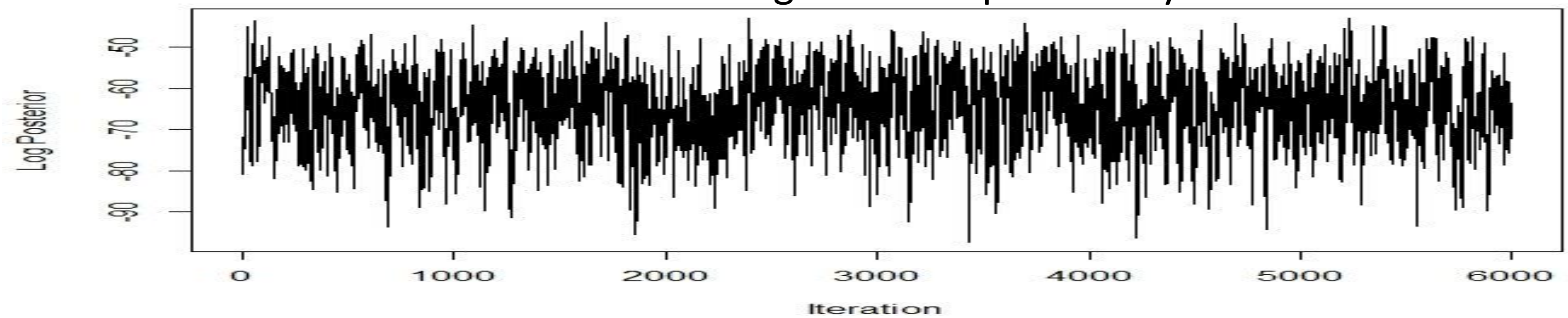


<u>Simulation of model</u>

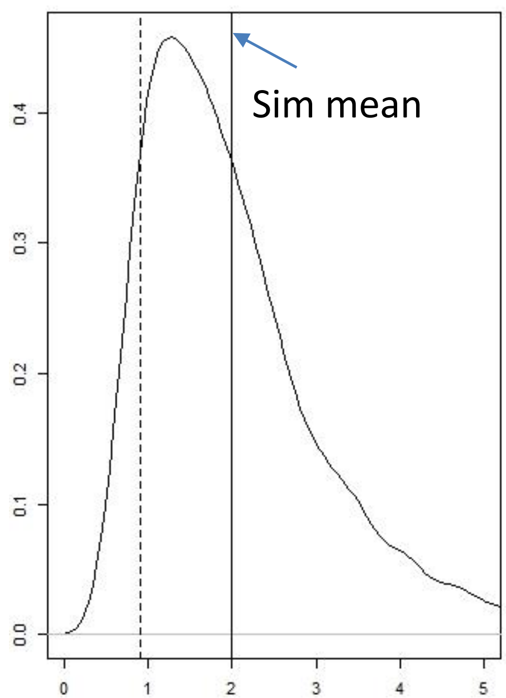
- **3 model ensemble**
- **Unit variances prior to spatial correlation**
- **Correlation between models 1 & 2**
- **Spatial correlation model 1 only**

Bayesian techniques can be used to estimate the model

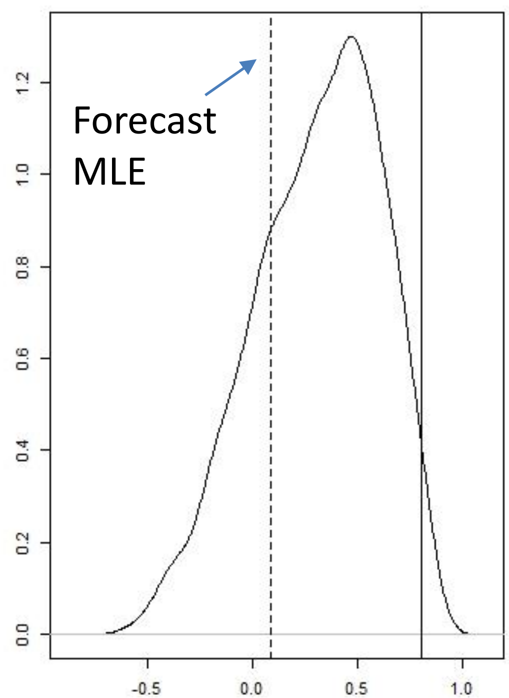
MCMC trace of Log Posterior probability



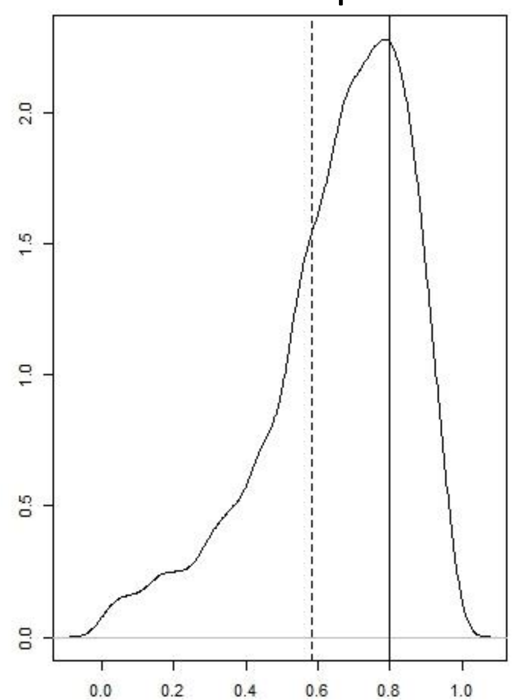
Model 1 Err Variance



Rho12 Err correlation



Model 3 Err spatial corr.



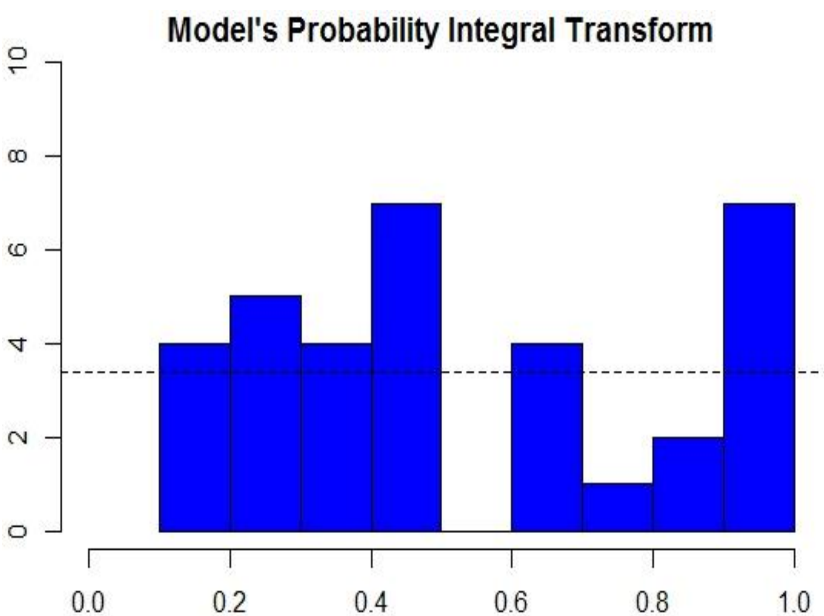
The proposed model fitted to temperature forecast data shows marginal improvements

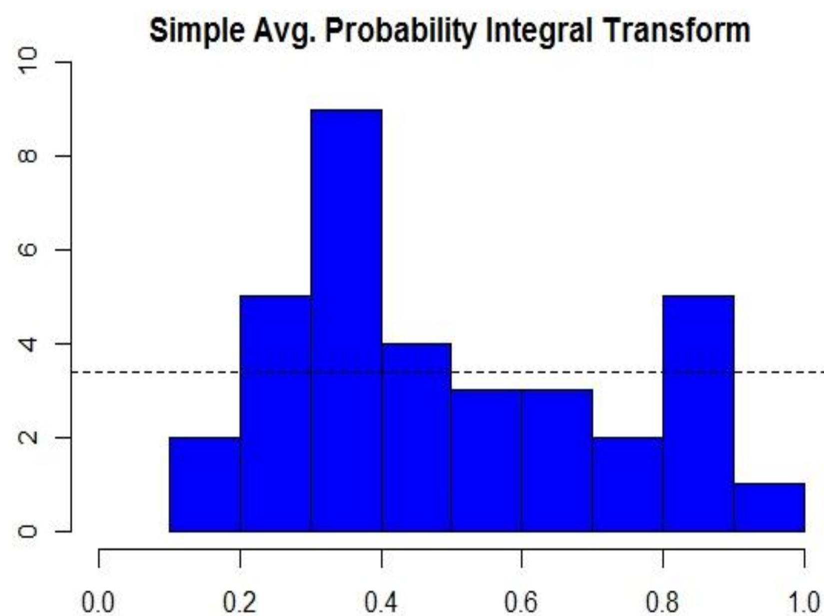
Proposed ensemble aggregation technique

RMSE	1.51
Max Abs Error	4.19
Points in mid distribution	50%

Simple ensemble average

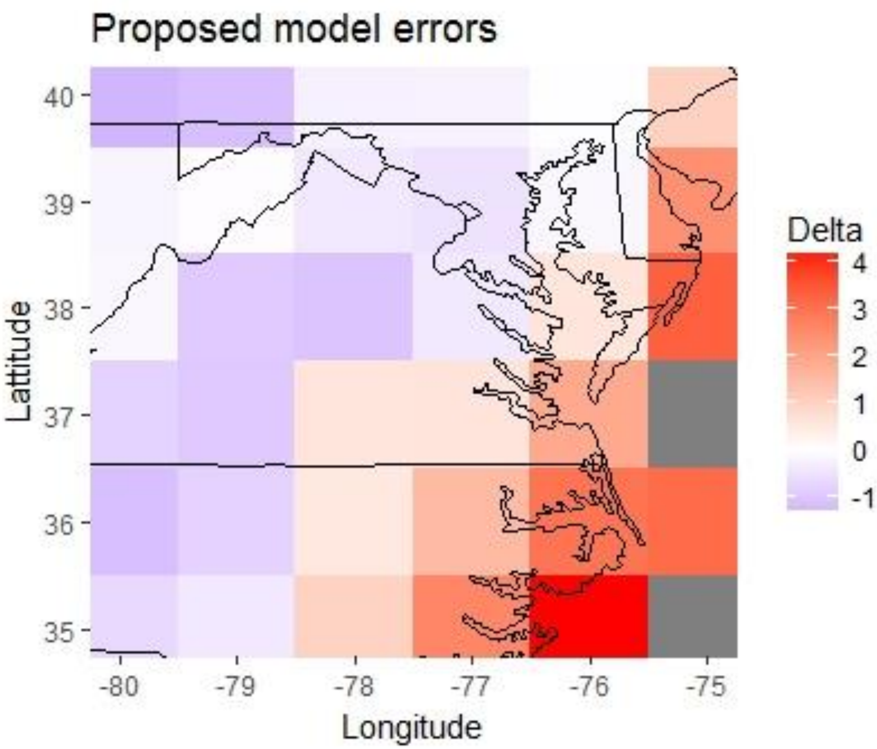
RMSE	1.56
Max Abs Error	4.40
Points in mid distribution	47%

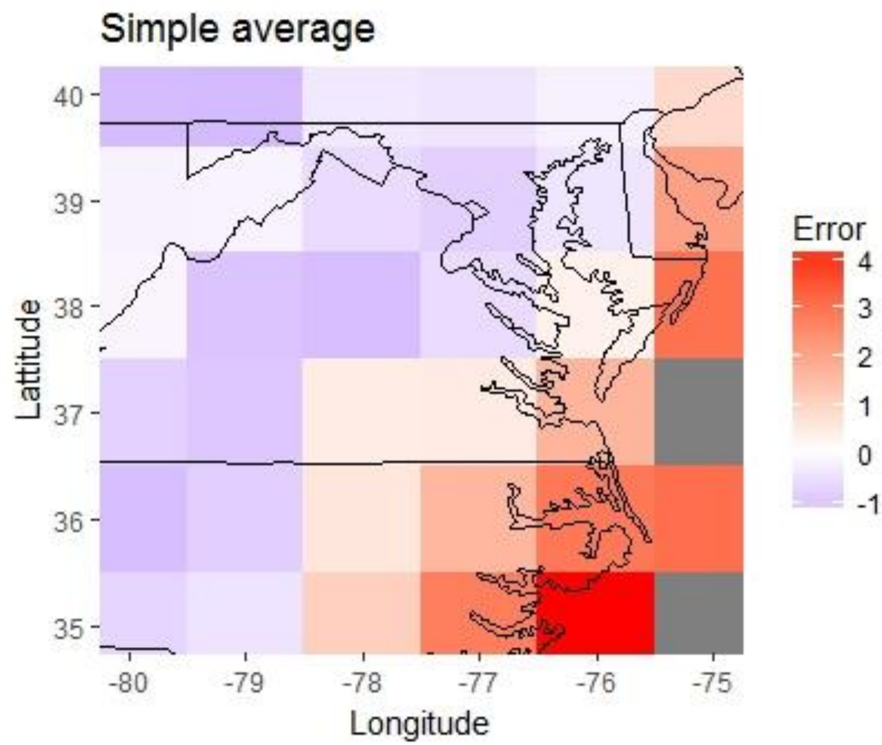




Forecasts from Jan 2016, 11 month ahead, NMME forecast of monthly temp; Observations are from the NMME .CMC1-CanCM3 .HINDCAST model.

Each approach has a similar spatial error distribution





The proposed model is better in 26 out of 34 (76%) of the locations

Further work is required

Model Improvements

- Explore / mitigate impact of heteroskedasticity on estimations
- Estimate variance for each point versus for each model
- Explore other parameterizations to improve aggregation performance
- Explore use of Bayesian point estimates to simplify assessments
- Use climate history to create informed priors

Model Assessment

- Use simulated data to understand circumstances where the extra effort would be valuable
- Explore Brier score and other forecast scoring rules

Heuristic approach

- Model behavior may suggest a simpler heuristic approach
- Develop less calculation intensive methods to aggregate ensembles in the presence of spatial autocorrelations and model correlations

Thank you

Questions or Discussion?

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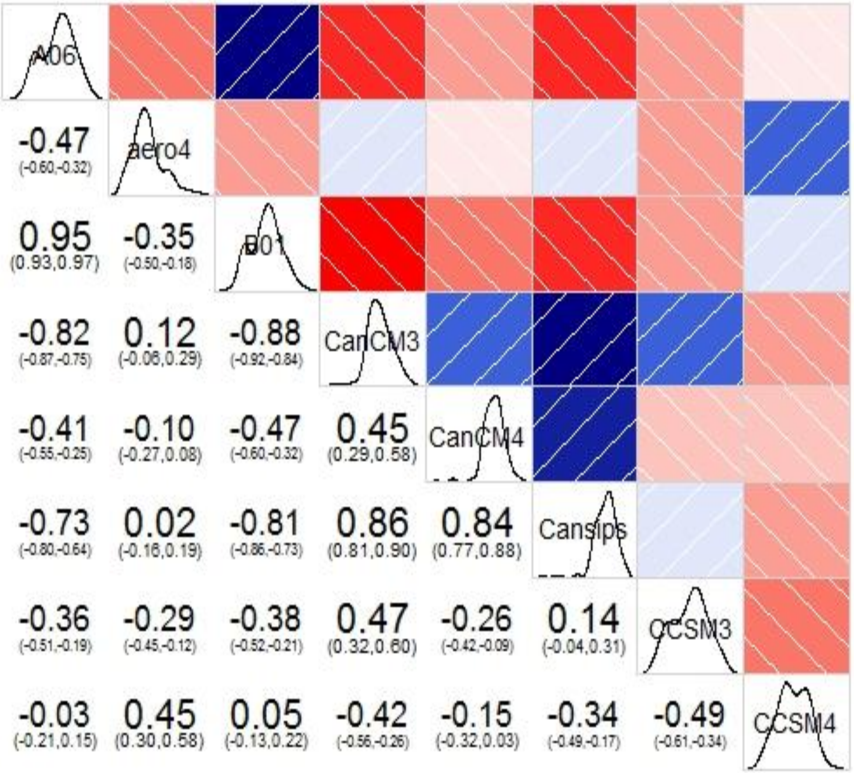
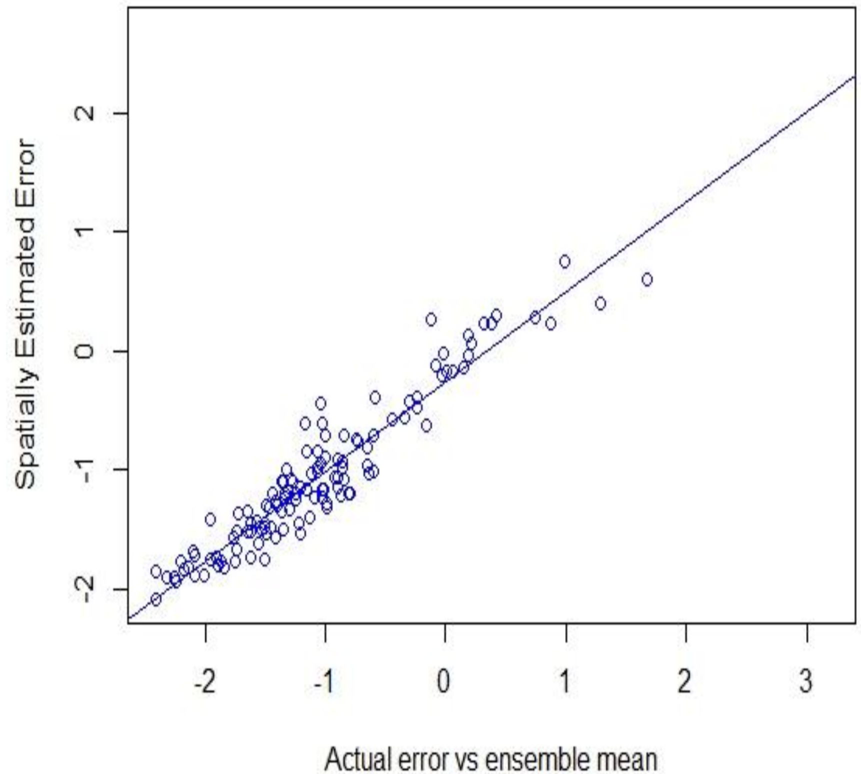
Appendix

With multiple forecasts and data points the model can be estimated apriori

Cross Model error correlation estimate

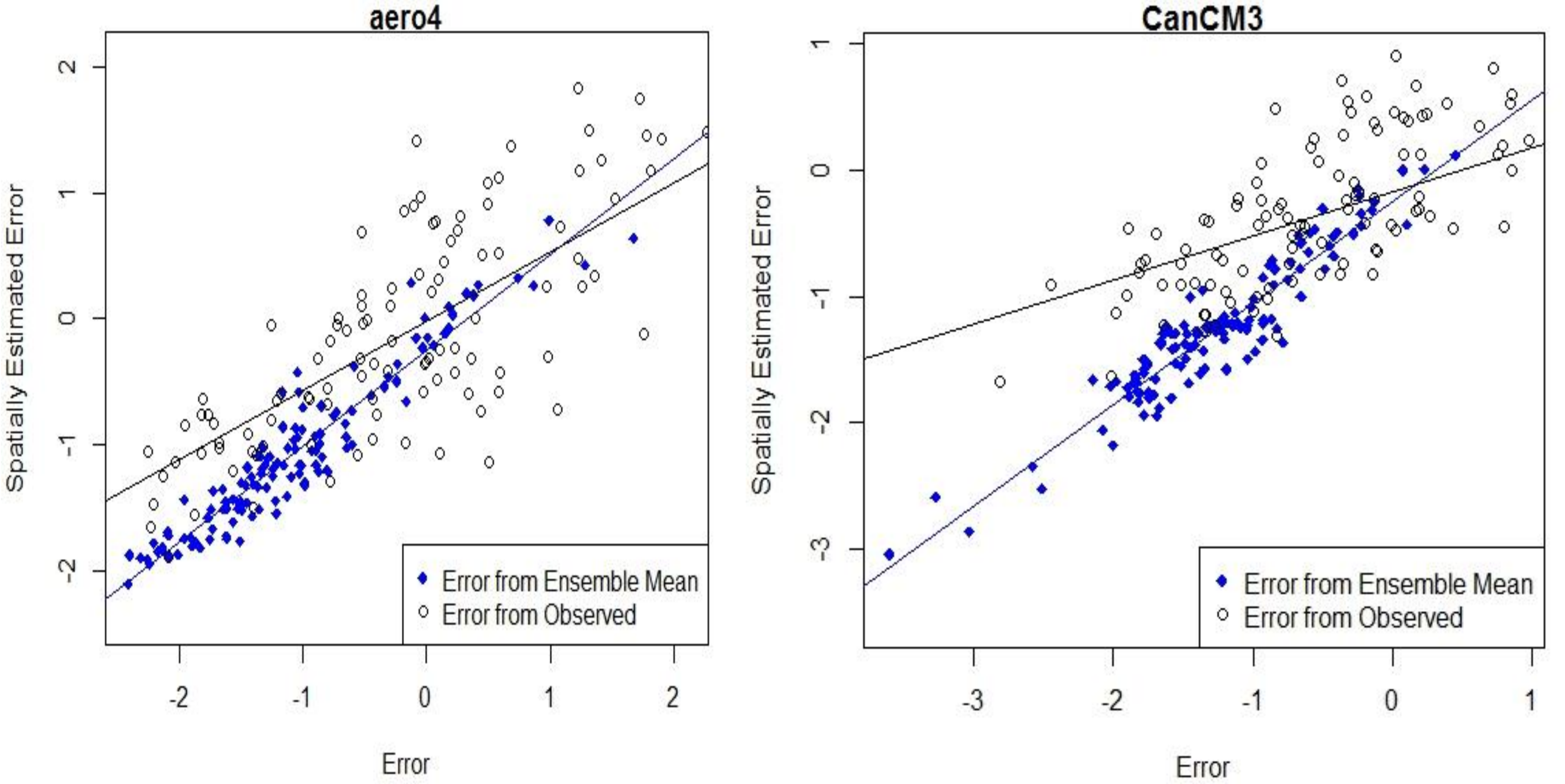


Within Model Spatial error correlation estimate – model aer04



Simultaneous estimation is required due to interactions between parameters

Comparison of spatial correlation estimation vrs. observed



Apriori, the ensemble mean can be used to estimate the degree of spatial correlation in each model. The integrated model should improve upon this estimate