P5.20 DIRECT USE OF SATELLITE HORIZONTAL GRADIENTS IN VARIATIONAL ANALYSIS

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1. INTRODUCTION

In 1989, when the first version of the Local Analysis and Prediction System (LAPS) initially came on line (McGinley et al. 1991), the moisture analysis essentially modified a model background using Horizontal Shape Matching (HSM) (Birkenheuer 1996) to integrate satellite structure into the moisture analysis. This approach was taken primarily in response to the fact the satellite data was from the Visible Spin Scan Radiometer Atmospheric Sounder (VAS) instrument predating the three-axis stabilized geostationary operational environmental satellite (*GOES-8*) that had better on-board calibration capabilities. HSM was based on a minimization technique imposing weak constraints to merge satellite gradient structure with more accurate ground-based data. The VAS instrument was poorly calibrated compared to more modern weather satellites, and as such, bias error was routinely treated in order to make best use of the data. HSM was but one means of dealing with bias.

In the mid-1990s, the moisture module in the LAPS system became more based on one dimensional variational minimization (1DVAR) operating at each individual gridpoint (or in the interest of speed, every second or third point using interpolation to achieve a finished product) to combine the diverse data sources that were added to the assimilation in the late 1990s and early 2000s. At about this same time, *GOES-8* data products became available and it was assumed that these products would have minimal bias error due to the better onboard black-body calibration techniques available with the new satellite series. The variational system at that time dropped gradient assimilation in lieu of directly using derived product total precipitable water (TPW) values in concert with other new moisture data sources such as global positioning system (GPS), GOES direct radiance data through the use of a newer forward model (community radiative transfer model, CRTM), and the inclusion of cloud information in the solution (Birkenheuer 1991).

This approach to moisture analysis was deemed most favorable until about 2004 following an extensive reanalysis of the moisture data from the International H_2O Project (IHOP)–2002 (Birkenheuer and Gutman 2005). It was observed that the published low bias figures from satellite product moisture used in the analysis system were valid for only 0000 UTC and that asynoptic times as well as 1200 UTC GPS-GOES comparisons showed much greater GOES moisture bias than had been perceived to exist. Since modern variational methods are highly dependent on both bias and variance/covariance error statistics, this recent finding has far-reaching effects.

There were two independent strategies to cope with a GOES bias problem; one was directly addressed by product developers (i.e., make a better product), and in addition, a second was to modify the analysis system to ignore bias and focus on structure in a similar way to HSM. Both approaches were exclusive so the Environmental System Research Laboratory (ESRL) established a web page that compares the GOES product with both RAOB and GPS data in real-time. This page has already established beyond a doubt that the GOES product continues to contain unexplained bias on the order observed during IHOP 2002. It is being used by GOES product developers to

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study and address the bias problem. The following figure compares a daily difference of GOES and GPS data for a 12-day time series. A periodicity is readily seen that reinforces the idea that the GOES bias is best at 0000 UTC exactly in line with IHOP results (refer to Fig. 1), in addition there appears to be a regular synoptic variability.

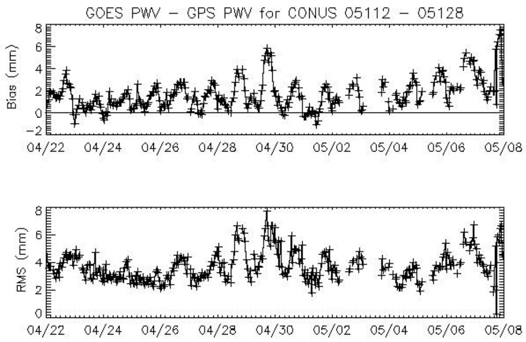

Fig. 1. GOES-GPS difference (mm "bias") top and the RMS differences (mm) below from late 22 April–8 May 2005. The periodic nature of the derived moisture is evidenced in the top figure where "bias" values drop to near zero at evenly spaced intervals. This was the identical behavior noted during IHOP–2002. The computation here, however, is derived from GOES-12 and all of the GPS sites in the eastern half of the continental U.S., a much larger region than IHOP, but for a shorter time period.

The second approach is the theme of this paper. Here is documented just how the variational algorithm that has served the moisture product, for many years has been adjusted from a 1DVAR system to one that contains data from surrounding gridpoints to assimilate horizontal gradient information that is now automatically incorporated in the minimization processing. By minimizing partial derivatives in orthogonal directions, the assimilation system both incorporates the gradient structure inherent in high-resolution satellite product data while becoming immune to bias problems.

2. THEORY

The first modification of the functional based on the current LAPS 1DVAR minimization (Birkenheuer 2001) consists of replacing the earlier satellite product moisture term for 3-layer precipitable water (subscripted as GVAP) with two new gradient terms for the partial derivatives in the satellite data to eliminate problems with satellite bias highlighted (terms 5 and 6 in the following equation). The premise is that adding gradient data from satellite *image* products or single field-of-view (FOV) sounding channels will be superior to supplying the measurement to the minimization since the bias in the product data (especially the single FOV data) has been shown to be greater than anticipated at asynoptic times (Birkenheuer and Gutman 2005). However, the single FOV data have the advantage of supplying greater horizontal structure information. By using satellite gradients, the bias problem is eliminated, and the analysis relies on other more accurate (but sparser) data such as GPS integrated moisture for establishing the proper moisture value. Insertion of gradients in the system will offer the potential for more-detailed analysis between the sparser, more accurate measurements, in theory, but we have no experience using gradient data in this current context (currently GPS data, for example, have roughly 200 km separation over CONUS with better spatial density in other areas). Therefore, a series of tests using a simple synthetic dataset was used to get an idea of what could be expected.

The functional below is what was envisioned to be the new local scale 1DVAR system that would include features to eliminate gradient use in cloudy or data-missing areas. The terms in blue are partial derivatives in *x* and *y* that relay gradient information rather than the direct observation from GOES to the solution, thereby avoiding any bias from GOES. This has the advantage of utilizing the structural information from GOES, while at the same time avoiding the moist bias problem inherent in that data source.

$$
\begin{aligned}
J = {} & S_{SAT}\sum_{k=1}^{7}\frac{\mathrm{GT}(g_i)[\mathrm{R}(T,cq,o_3)_i - R_i^o]^2}{E_{SAT}^2} + \sum_{i=1}^{N}\frac{(1-c_i)^2}{E_{BACK}^2} \\
& + S_{GPS}\frac{\left(\sum_{i=1}^{N} c_i q_i - Q^{GPS}\right)^2}{E_{GPS}^2 L_{GPS}} + S_{sonde}\frac{\sum_{i=1}^{N}\left[\mathrm{RH}(T,p,cq)_i - RH_i^o\right]^2}{E_{sonde}^2 L_{sonde}} \\
& + S_d S_{GVAP}\sum_{j=1}^{3}\frac{\mathrm{G}(g)[\sum_{i=1}^{N}\frac{\Delta}{\Delta x}\mathrm{P}_{ji}(c_i q_i) - \frac{\Delta}{\Delta x}Q_j^{GVAP}]^2}{E_{xGVAP_j}^2 L_{GVAP}} \\
& + S_d S_{GVAP}\sum_{j=1}^{3}\frac{\mathrm{G}(g)[\sum_{i=1}^{N}\frac{\Delta}{\Delta y}\mathrm{P}_{ji}(c_i q_i) - \frac{\Delta}{\Delta y}Q_j^{GVAP}]^2}{E_{yGVAP_j}^2 L_{GVAP}} \\
& + S_{CLD}\sum_{i=1}^{N}\frac{\mathrm{g}_i[c_i q_i - q_s(t_i)]^2}{E_{CLD}^2}
\end{aligned}
\qquad (1)
$$

The non-gradient terms in the above equation are defined in Birkenheuer (2001). The new gradient terms (in blue) in (1) contain variables:

S_d = 1 or 0, (on or off), gradient existence check
S_{GVAP} = 1 or 0, (on or off) data presence check
G(g) = cloud influence (degradation fraction, 0.0 to 1.0)
$\frac{\Delta}{\Delta x, y}$ P(c,q) = variationally modified gradients,
and $\frac{\Delta}{\Delta x, y}$ Q = background gradients where c is the minimized variational scaling factor, and q is the background specific humidity
E_{GVAP}^2 = Product gradient error squared
L_{GVAP} = Spatial dependence, reducing influence at large distances (exponential decay)

2.1 *Testing the Formulation*

There are various ways to test the above formulation; by drawing on past experience it was decided the easiest approach was to isolate the terms of interest and perform a minimization analysis on an analytic function for which one could easily compute both "truth" and numeric derivatives, thus simulating and ideal setup. The following equation was used to simulate the GOES and GPS product terms in this simplified functional for testing; a series of numerical tests were conducted using the following model.

$$Truth\ \ data = T = T(x,y) \qquad (2)$$

$$T(x,y) = 12.5\ [\sin(x\frac{\pi}{4}) + \sin(y\frac{\pi}{4})] + x^2 + y^2 \qquad (3)$$

For this example, the sine terms were included to simulate variability in the field that was about 1% of the value mid-way in the interval studied (1 – 50). The testing was performed in two parts; first was to examine only the last two squared terms as the truth function, followed by a more complex test using the full equation that included the sine terms.

The satellite gradient data was derived from the truth data with the hypothetical (best case) initial approximation that the satellite data was a perfect measurement, and that the gradient information were simple derivatives.

$$
\begin{aligned}
\frac{\partial}{\partial x}T(x,y) &= 12.5\frac{\pi}{4}\cos(x\frac{\pi}{4}) + 2x \\
\frac{\partial}{\partial y}T(x,y) &= 12.5\frac{\pi}{4}\cos(y\frac{\pi}{4}) + 2y
\end{aligned}
\qquad (4)
$$

Numerically for both the truth data and its partial derivatives, evaluation was performed using x and y as grid coordinates and placing GPS observations at grid locations:

$$
\begin{aligned}
x &= i \\
y &= j
\end{aligned}
\qquad (5)
$$

The background field for the study was initially set as an inferior match to the truth function:

$$Background = B(x,y) = x^{1.8} + y^{1.8} \qquad (6)$$

Similarly, the background function was evaluated using the same method above (5).

Thus, the background is not a perfect fit to the squared power law followed by the truth data nor does it reflect the sinusoidal characteristics of the added "structure" incorporated in the "truth" field.

Furthermore, the boundary regions were ignored in the study of error. In actual application, this same approach could be taken (i.e., not to include satellite gradient effects at the boundary) or a simple approach of replicating the gradient adjacent to the boundary could be applied.

The minimized functional for evaluation was then formulated by a relationship using the above terms:

$$J = c_1[p(x,y)B(x,y) - B(x,y)]^2 + c_2[p(x,y)B_x^{'}(x,y) - T_x^{'}(x,y)]^2 + c_2[p(x,y)B_y^{'}(x,y) - T_y^{'}(x,y)]^2 \quad (7)$$

where the primed functions are partial derivatives, either numerically or analytically generated. The "*c*" terms are empirical weights that control their effect relative to each other. It should be pointed out that this functional differs from (1) in that (1) is used to minimize the best fit to water vapor for the operational moisture analysis problem. Equation 7 on the other hand is only used here to examine specific terms of the functional in this analytic test, in particular the new gradient terms, to gain understanding of how the new derivative terms "act" with respect to the direct data assimilation terms such as are in (1). The goal is to ascertain if the gradient terms should carry more or less weight in (1) than the conventional data terms. The function *p(i,j)* is computed at each gridpoint to minimize *J*. The above functional does not include GPS data which would appear as a fourth term (left out here but added in later testing). Thus, when the minimization of (7) is complete, the analyzed field (*A*) is computed as a modified background modulated by the "*p*" function:

$$A(x,y) = p(x,y)B(x,y) \quad (8)$$

Numerical derivatives were not computed at the boundaries, they were excluded from the computation of error. Error was defined as the squared difference summed between the truth function and (8) over all gridpoint locations.

2.2 *Experiment 1 (ignore sine terms):*

The first experiment (and all subsequent tests) kept the gradient term (C_2) at a value of 1.0 while varying the weight of the background term (C_1). Relative improvement was computed from the background-only term that was run one time where C_1 equaled unity and all other coefficients were assigned zero, so there were no other effects on the solution. A maximum 87% error reduction was observed when C_1 equaled 0.0001 or ten thousandth the value of the gradient term. Giving the gradient a high weight with respect to the non-derivative terms produced the best results.

2.3 *Experiment 2: (include sine structure)*

The next test included more structure. It was unexpected that addition of the gradient term to the fairly smooth analysis field would have substantial impact. A more realistic test would be to include higher frequency structure, and the objective here would be to ascertain whether the gradient approach would further contribute to error reduction with more complex structure, or would be unable to achieve much improvement, since there was no sinusoidal structure in the synthesized background field.

The set of measurement simulations added sinusoidal "variability" in the truth field shown earlier. This amplitude was taken to be about 1% mid-way up the squared function curve, so it is really about 100% variability at the low end and very small variability at the high end (large *x*, *y* values). Here we apply equations (2), (3), and (4) in full form.

The quarter wavelength in (3) was selected so a complete wave could be represented by four points or 40km in this simulation (assuming 10km grid spacing). This is less than current GPS spacing, and is nearly what one would get from the current GOES with a ~10 km sounder resolution.

Again the results were very similar to the initial case in that there was a minimum error (max error reduction) of about 19%, with the same order of magnitude for C_1 as in the first experiment.

Additional testing included synthetic GPS data at every 100km (every 10^{th} gridpoint) with a 50% influence at 20 km and less beyond. Using the GPS data in combination with the gradient data improved the error reduction by about another 2%; similar to what was seen in the less complex field (see Fig. 2).

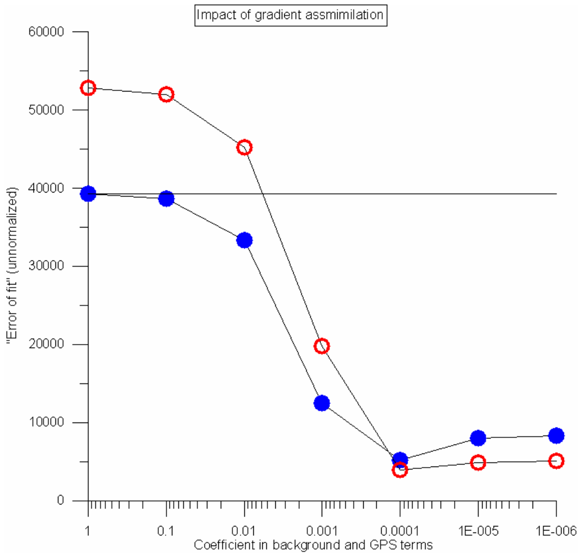

Fig. 2. Impact of simulated gradient data (blue); open red circles also contain untuned GPS data from experiment 1. (i.e., c_3 , the GPS coefficient, was unity in the related minimization equation [not shown]).

2.4 *Summary of Analytic Tests*

In summary, the numeric/analytical tests demonstrate that the gradient approach in a minimization configuration is tractable and requires little modification to existing code (an easy modification of the functional plus some preliminary computations of gradient fields are all that is required for implementation), solves the bias problem, and adds little to computational overhead with the benefit of substantial reduction in error. It compliments the addition of GPS data in that non-gradient PW data is seen to not detract from the solution but adds to it. Any theoretical system is not perfect and these new ideas were applied to real cases to gain insight and experience. Several problems for gradient application are anticipated to be missing data, bad pixels, and clouds (as well as other data void areas or product artifacts that lead to false edges or unrepresentative gradients). Furthermore, partly cloudy regions may or may not have significant impact; this remains a subject for additional study.

3. APPLICATION OF GRADIENT METHOD TO THE LOCAL ANALYSIS AND PREDICTION SYSTEM (LAPS)

The prescribed changes using a gradient approach to the LAPS variational moisture system were incorporated in March 2005. The following observations were encountered during this process, some of which were not envisioned in the initial implementation plan.

The computation of the background layer gradient ran into problems when surrounding points were underground. Then when the layer gradient was computed (and averaged), it might again be influenced by nearby missing data flags. This was coded around by making sure the gradient was under 1000. Generally accepted gradients were on the order of 1.0e-5, so values under 1000 were protected from missing data flag contamination. This problem would disappear if a terrain-following coordinate system was used, but the local analysis traditionally uses an isobaric vertical coordinate.

Another aspect of real-time operation that has yet to be investigated will be the relationship to the sparseness of other data such as GPS in the final solution. It would be logical to assume that the gradient technique will lose more and more effectiveness as ground-based data density with superior error figures becomes more and more dense.

The first set of comparison products were made between the local analysis parallel and operational runs. The parallel runs contained the gradient method, and the operational runs did not. The gradient method ran without problem during the entire moist season in 2005 and produced reliable results.

Figs. 3 and 4 illustrate the new method's impact on a single case time.

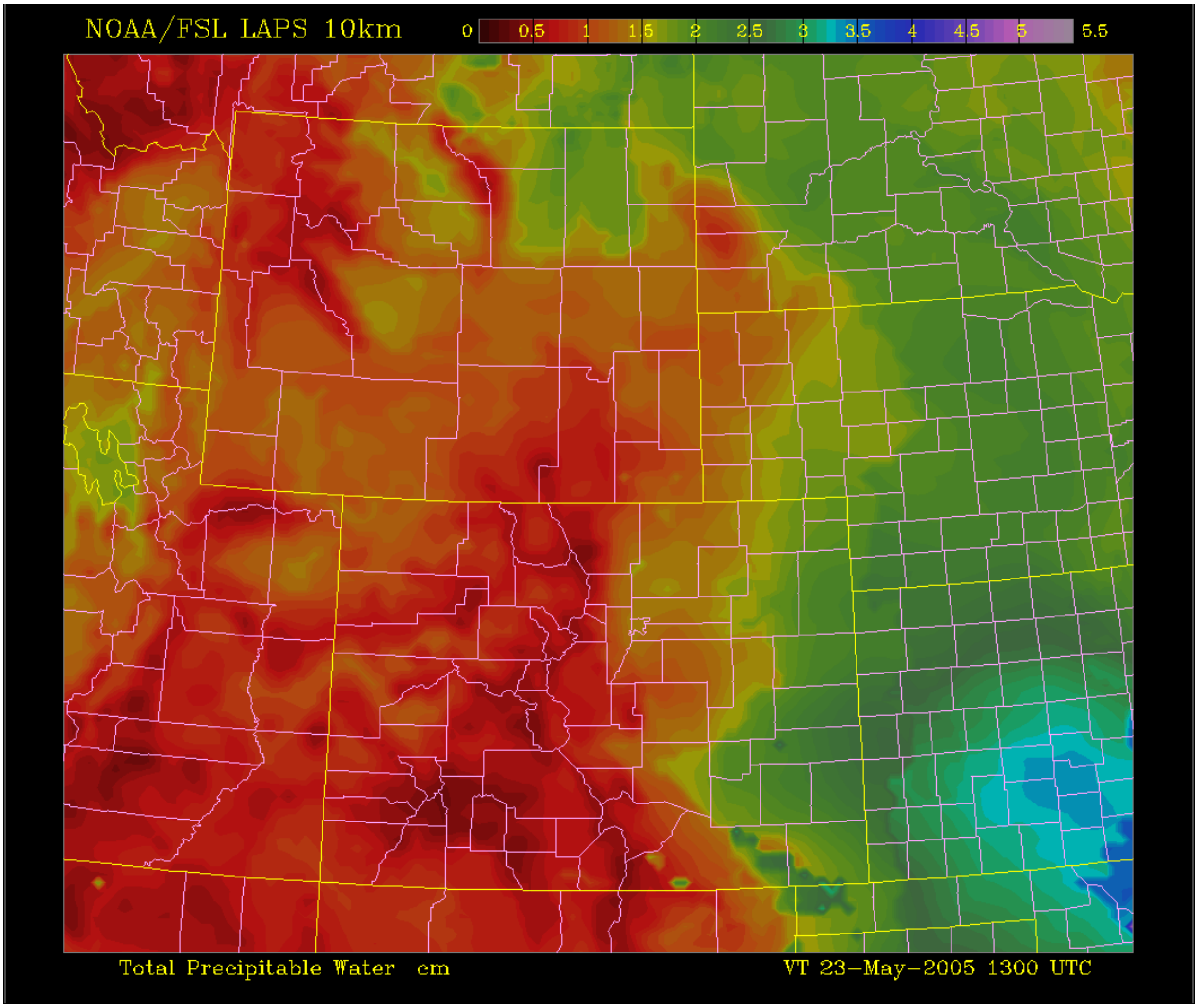

Fig. 3. Conventional analysis of total precipitable water (cm). Satellite data product used – direct use of satellite data potentially contaminated with moist bias for 23 May 2005 1300 UTC over Colorado and surrounding states. Note that prior studies have demonstrated that the maximum bias in the GOES product data occurs at 1800 UTC. This 1300 UTC analysis is actually in a timeframe of a secondary bias minimum. However, moist bias is observed to be reduced in a number of circled areas, and increased structure is observed in "boxed regions" in the following figure.

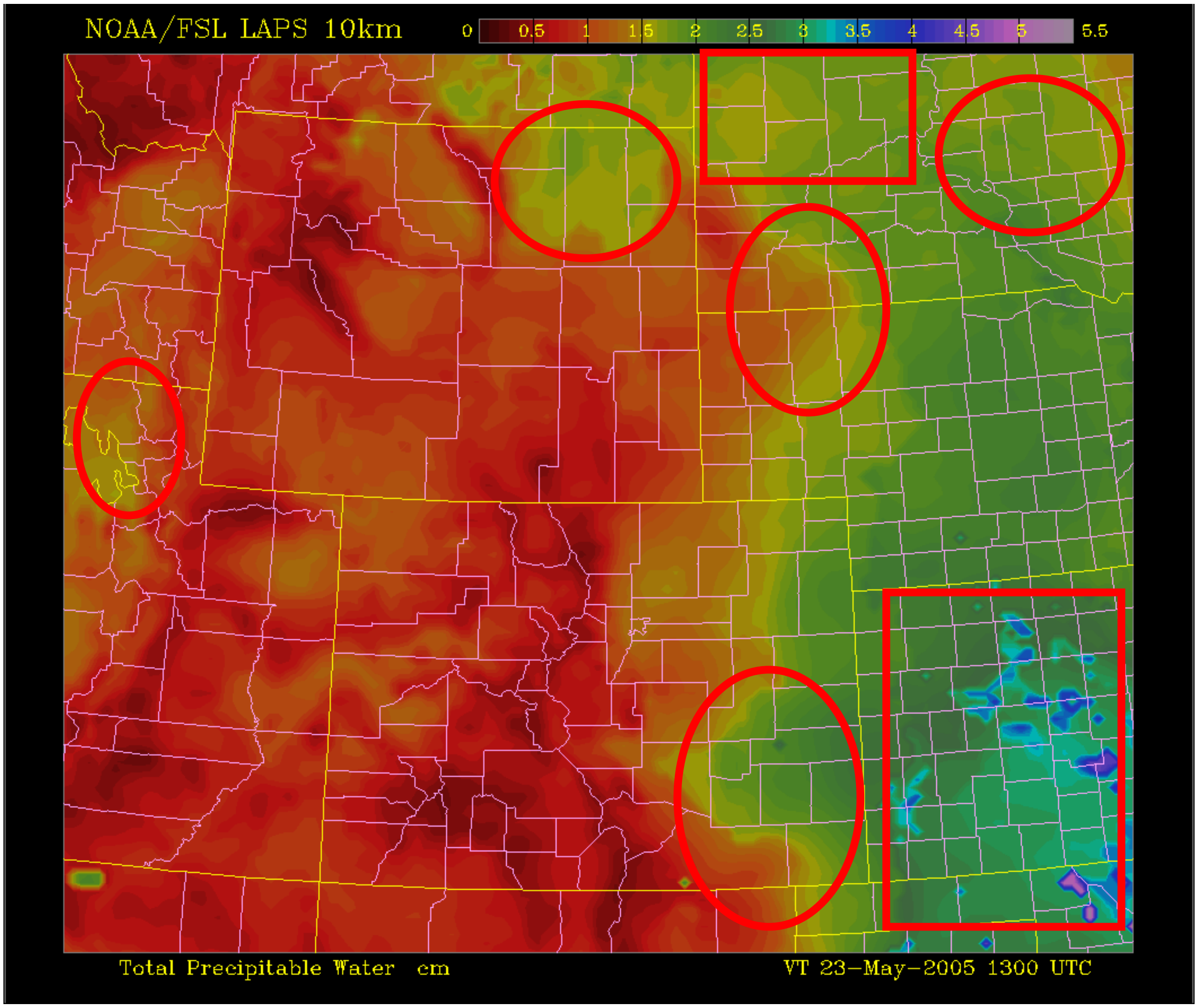

Fig. 4. Same as in Fig.3 except using the new analysis relying on gradient information from satellite data disregarding satellite bias if it exists. The lower moist bias is evident in circled areas, and better structural detail is evident in boxed regions.

Examining the two figures, it is evident that in the moist areas on the Kansas plains, there is more detail in the water vapor analysis with the gradient method. There also appears to be of higher water vapor amounts in eastern Colorado in the conventional image (Fig. 3) indicative of a high moist bias coming from the satellite data, whereas Fig. 4 lacks this higher level and appears to render improved moisture field structure (more closely resembling spatial cloud distribution), especially in the most southern boxed area.

Also it should be noted that the dry areas between both analyses (high terrain areas) show the least difference. One would expect this result since the GOES bias problem decreases with lower moisture amounts.

4. RESULTS AND CONCLUSIONS

A numerical technique to include gradient information from satellite fields was successfully devised, first analytically, and then cast into digital form for computer application. The analytical solution helped derive the weights to apply to the gradient terms in the functional used operationally in LAPS. The system was then run on real data comparing analyses that both applied and did not apply the new technique, all other elements being equal.

Subjectively, it appears that the new methodology works both to reduce the moist bias inherent in the satellite data while improving the structure in the resulting analyzed field. Figures 3 and 4 are highly representative of the routine effects that are observed when the new algorithm operates. The new method is now being run quasi-operationally in tests directly comparing the two methods. As of this writing, consistent, subjective, positive results have resulted from the new technique and it is now incorporated into the "operational" LAPS analysis.

All of the changes to the functional are not yet complete as this remains a work in progress. The error that applies to the partial derivative terms has no accepted value at this time. It might be possible to better estimate the error terms by analytic methods similar to the tests conducted in this paper. One could impose random error in a synthetic field and then from this, compute the partial derivative error estimates.

5. ACKNOWLEDGEMENTS

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