1.10 MEDIUM-RANGE PLUME DISPERSION CALCULATED DIRECTLY FROM SURFACE-DERIVED SHEAR AND TURBULENCE ESTIMATES

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1. INTRODUCTION

Classical plume and puff dispersion models are derived from differential equations that assume that homogeneous turbulence alone is responsible for dispersing pollutants away from the plume centerline downwind of point sources. However, observed dispersive behavior of plumes cannot be explained in terms of turbulent diffusion alone, requiring empirical adjustment of horizontal and vertical dispersion parameters (σ) used in regulatory plume models.

In this paper, a more general derivation of the dispersion behavior of a plume is provided by considering the effects of shearing motions on plume dispersion. In section 2, the mathematical formulation of a steady-state plume emitted into an environment containing wind shear is presented. Section 3 specifically addresses the effects of shear on horizontal "size" or dispersion of pollution plumes. Section 4 provides explicit derivations of the diffusion coefficients and shear parameters required to assess plume dispersion based on less empirical first principles of diffusion and shear.

2. DERIVATION: PLUME WITH SHEAR

The initial transport and dispersion of pollutants in plumes downwind of point sources in the atmosphere can be mathematically quantified using a steady-state three-dimensional advection-diffusion equation:

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = K_h\frac{\partial^2 c}{\partial y^2} + K_z\frac{\partial^2 c}{\partial z^2}. \quad (1)$$

where c is the pollutant concentration, K_h and K_z are the horizontal and vertical turbulent diffusion coefficients, and u and v are the wind speeds parallel to and perpendicular to the mean wind. Classical Gaussian plume formulations represent one solution to this equation under conditions when there are no motions perpendicular to the mean motion (v=0). Analytical solutions to Eq. (1) exist for some simple configurations of v. For example, If v varies linearly with height (z)

$$v = z\frac{\partial v}{\partial z}, \quad (2)$$

and the shear perpendicular to the mean ($\partial v/\partial z$) is constant, the following analytical solution exists for a point source emitted at x=y=z=0.

$$c = \frac{Q}{2\pi u\sigma_z\sigma_y\sqrt{1+s^2/12}} \bullet \exp\left[\frac{-y^2}{2\sigma_y^2\left(1+s^2/12\right)} + \frac{-z^2\left(1+s^2/3\right)}{2\sigma_z^2\left(1+s^2/12\right)} + \frac{yz}{2\sigma_y\sigma_z}\left(\frac{s}{1+s^2/12}\right)\right] \quad (3)$$

where Q is the emission rate (mass s^{-1}), and lateral and vertical dispersions are given as

$$\sigma_y = \sqrt{\frac{2K_h x}{u}}, \quad \sigma_z = \sqrt{\frac{2K_z x}{u}}, \quad (4)$$

and s is a nondimensional shear factor

$$s = \frac{\partial v}{\partial z}\frac{x}{u}\frac{\sigma_z}{\sigma_y} = \frac{\partial v}{\partial z}\frac{x}{u}\sqrt{\frac{K_z}{K_h}}. \quad (5)$$

Cursory analysis of (3) shows that it reverts to the classical Gaussian plume formulation when shear is neglected. When $\partial v/\partial z$=0, s=0 and the third "yz" term in the exponential factor of Eq. (3) drops out. Fig. 1 shows schematically the coordinate system and configuration of winds used for this derivation.

The solution presented here was originally derived by Konopka (1995) and Dürbeck & Gerz (1996) to describe the cross-section of a stationary plume segment in a sheared environment. Here the "time" variable used in their formulation is substituted with the distance downwind of the point source divided by the mean wind speed transporting the plume downwind (t=x/u).

3. HORIZONTAL DISPERSION WITH SHEAR

Visual inspection of the terms in Eq. (3) shows that the horizontal dispersion is enhanced relative to purely turbulent diffusion, and this "effective" horizontal dispersion (σ_y') can be represented by:

$$\sigma_y' = \sigma_y\sqrt{1+s^2/12} = \sqrt{\frac{2Kx}{u}}\sqrt{1+\frac{1}{12}\left(\frac{\partial v}{\partial z}\frac{x}{u}\frac{\sigma_z}{\sigma_y}\right)^2} \quad (6)$$

where σ_y is dispersion resulting from a purely diffusive process.

Fig. 1 shows the effective horizontal dispersion quantified in Eq. (6) as a function of distance downwind of a point source. Here a wind speed of 10 m s^{-1} is used, shear in ranges from 0 – 10 m $s^{-1}km^{-1}$, and diffusion coefficients range from nominal stable night values (1 m^2s^{-1}) to typical unstable daytime (100 m^2s^{-1}) conditions. Gray area denotes range of observed horizontal plume dispersion from standard Gaussian plume empirical formulations for daytime "class A" (greatest) to night "class F" (lowest) dispersion.

Fig. 1 shows that in the limit of shear-dominated dispersion, the shear term in Eq. (6) involving $\partial v/\partial z$ is >>1, and plumes grow in proportion to the 1.5 power of the downwind distance. The range of observed plume sizes encompasses both the magnitudes and power-law of distance expressions derived here.

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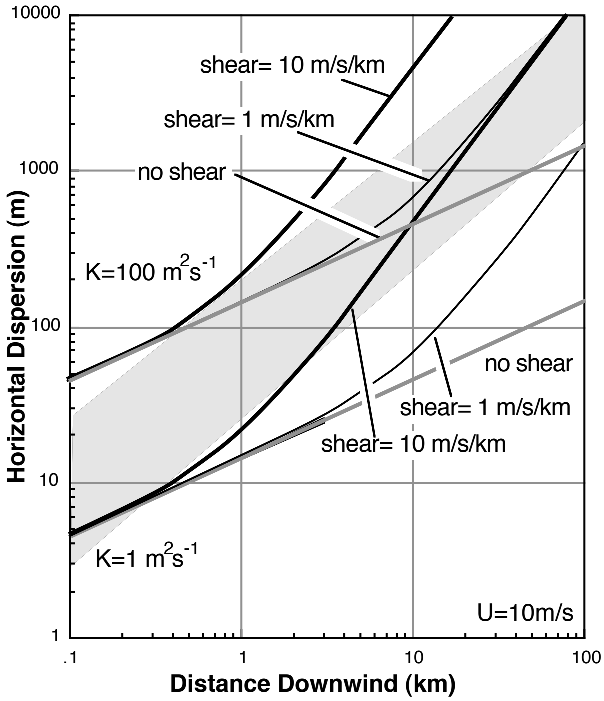

Fig. 1. Effective horizontal dispersion as a function of distance downwind of a point source. Wind speed =10 m s^{-1}, shear in range of 0 – 10 m $s^{-1}km^{-1}$, diffusion coefficients range from nominal stable night values (1 m^2s^{-1}) to typical unstable daytime (100 m^2s^{-1}). Gray area denotes range of observed horizontal plume dispersion from standard Gaussian plume empirical formulations for daytime "class A" (greatest) to night "class F" (lowest) dispersion.

Near the source, shear is not important, and plume dispersion is governed by purely turbulent processes, thus yielding $\sigma \sim x^{0.5}$. At larger distances downwind, growth rates that spread pollutants with distance of $x^{1.5}$ power result from the fact that the plume is growing vertically by a turbulent process ($\sim x^{0.5}$), but this vertical growth exposes the plume to shearing motions that grow linearly with distance ($\sigma \sim \partial v/\partial z x/u \sim x^1$). These two effects are essentially multiplicative, yielding plumes that grow in proportion do the 3/2 power of distance from release or emission point.

Fig. 1 clearly shows that for distances beyond about 1 km downwind of a point source, shearing motions will under many conditions dominate the horizontal dispersion process relative to the dispersion caused by turbulence alone. For distances beyond 1-2 km downwind, even small amounts of shear are shown to enhance plume horizontal size by factors of 5-10 relative to plumes growing without shear.

Another note of interest is that under stable, night conditions, the only way the theory derived here can match observed dispersion 1-10 km downwind of the release point is for there to be considerable shear present under night conditions, which is consistent with many observations. At night, with diffusion coefficients of ~1 m^2s^{-1}, in the downwind range 1-10 km, plumes only grow to 15-40 m in width in environments without shear. Observations under "class-F" stability conditions are considerably greater than this (30-300 m), suggesting that relatively high shear (~10 m $s^{-1}km^{-1}$) must occur in order to obtain such large horizontal spread.

4. ESTIMATING K AND SHEAR (∂v/∂z)

In order to explicitly calculate horizontal plume dispersion based on first principles of diffusion and shear, it is necessary to estimate the turbulent diffusion coefficients (K_y, K_z) in the PBL, and the shear perpendicular to the mean wind (∂v/∂z). Here similarity theory is used to estimate representative PBL diffusion coefficients, and typical diurnally-varying measurements of shear are presented. Ideally, one would directly measure shear perpendicular to the mean wind to assess shear.

Similarity theory suggests that diffusion coefficients can be specified as

$$K = \frac{ku_*z}{0.74}\sqrt{1-9\frac{z}{L}} \quad , \tag{7a}$$

under daytime (unstable, L<0) conditions, where u* is the friction velocity, k is the Von Karman constant, and L is the Monin-Obukhov length scale given below. Under stable conditions (L>0, night)

$$K = \frac{ku_*z}{0.74+5\dfrac{z}{L}} \quad . \tag{7b}$$

In Eq. 7, L is given by

$$L = \frac{-u_*^3}{k(g/T_o)w'T'} \quad . \tag{8}$$

where To is a reference temperature (~288K), g is gravity acceleration (9.81 m s-2), and w'T' is the sensible potential temperature (heat) flux. The largest turbulent diffusion coefficients experienced by a diffusing plume will govern lateral dispersion, so diffusion coefficients evaluated at height z= 10-20% of the PBL depth should be used in calculating K.

The shear term in (6) involving ∂v/∂z should be directly measured if possible, but appropriate time-averaged measurements that are typical of the entire turbulent PBL are not widely available. Therefore, Fig. 5 shows the diurnal trend of two components of wind shear in the lowest 1000m above the surface during the entire month of Oct 2003 in Albany NY. Wind shear parallel to the mean wind direction is not important in influencing lateral plume dispersion.

The most relevant information shown in Fig. 5 is typical values of shear PERPENDICULAR to the mean wind direction. Shear perpendicular to the flow maximizes in the early morning hours 7-9 AM with a peak value of about -6 m s^{-1} km^{-1}. During the remainder of the day, the magnitude of the shear perpendicular to the mean wind decreases under the influence of turbulent mixing, reaching a minimum around local sunset time of ~2 m s^{-1} km^{-1}.

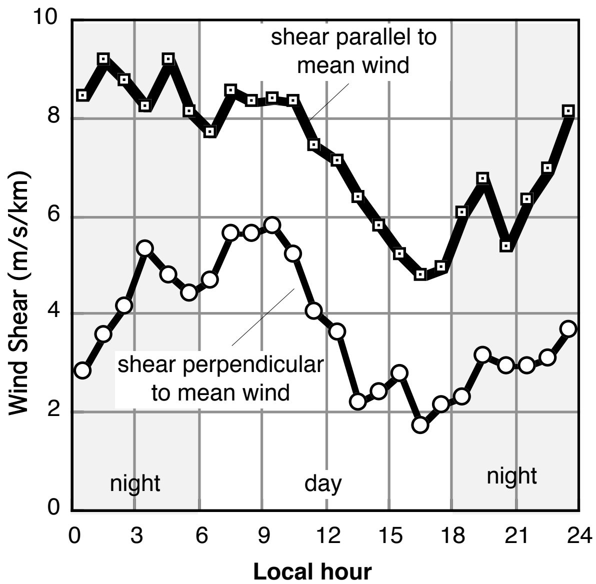

Fig. 5. Diurnal variation of shear in the lowest 1000m over Schenectady NY measured during Oct. 2003. Shear decomposed into vector components parallel and perpendicular to the mean wind over the same layer. For plotting purposes, shear perpendicular to mean wind shown here is the NEGATIVE of this component of the shear, and winds perpendicular to the mean flow nearly always "veer" (rotate clockwise) with altitude.

5. CONCLUSIONS

Classical plume dispersion models estimate lateral and vertical dispersion of individual pollution plumes primarily in terms of empirically-derived Pasquill-Gifford stability categories. Here we describe a more scientifically fundamental method for inferring the lateral plume dispersion based on the assumption that plume dispersion is governed by a combination of random, small-scale turbulent dispersion effects enhanced by lateral wind shear. Unfortunately, wind shear is difficult to accurately measure in the turbulent Planetary Boundary Layer (PBL), however, the vertical shear of the horizontal winds can be estimated from surface-based measurements of turbulence and larger-scale temperature gradients. It is proposed that shearing motions of horizontal winds in the planetary boundary layer result from a superposition of two effects: 1) Ekman-layer induced shear (proportional to boundary layer depth and turbulent intensity); and 2) Baroclinic (i. e. thermal wind) effects (proportional to horizontal temperature gradients). Both of these effects can be inferred from surface measurements. Here estimates of turbulence, wind shear and lateral plume dispersion derived only from surface-based measurements are compared with shear and lateral dispersion calculated using winds directly sampled using radar profilers in the Hudson Valley in upstate New York. It is found that lateral dispersion of plumes is dominated by shearing motions at distances greater than several kilometers from the release point. In contrast, plume dispersion close to release is dominated by turbulent diffusion coefficients that can be inferred from wind speed measurements and similarity-theory descriptions of turbulence in the lower PBL. The magnitude of shear in the PBL can be estimated using thermal wind theory coupled with Ekman-layer physics if several surface temperature measurements are available for assessing horizontal temperature gradients. Therefore it is proposed that the minimum surface observations required in order to accurately infer plume dispersion must include wind velocities at one site, and surface temperature measurements at three or more sites surrounding the wind measurement site in order to estimate lateral temperature gradients. Current guideline plume models require only wind measurements at one site to estimate dispersion, and thus the very important shearing effects on plume dispersion cannot be accurately accounted for.

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