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VALIDATION OF SODAR REAL-TIME SENSING AND VISUALISATION OF WAKE VORTICES

Stuart G Bradley*
University of Auckland, Auckland, New Zealand

1. INTRODUCTION

Acoustic wind and turbulence profilers (SODARs) have proven to be particularly effective, across many applications, in real-time observation of the lowest few hundred meters in the atmosphere. This is largely because of operational reliability in virtually all weather conditions. SODARs obtain strong reflections of acoustic pulses from atmospheric turbulence, and the Doppler shift allows wind vector components to be measured, generally about every 10-15 m up to a height of 200-500m. In contrast to LIDARs, SODARs measure through fog and in clean air and at least *operate* in precipitation (but with mixed effectiveness).

One particularly challenging wind measurement goal is to adequately characterise the position and strength of the vortices produced behind aircraft. The lift maintaining aircraft in flight comes from a bound circulation around the aircraft wings. At the wing tips this circulation becomes a pair of trailing vortices, with a strong combined downwash between them. The vortices are relatively long-lived because of their angular momentum, and interact with each other and the prevailing atmospheric conditions (particularly wind shear and temperature inversions), making their longevity, strength, and location difficult to predict. Since a sudden unpredicted downwash is a hazard to following aircraft at the critical times of landing or taking off, considerable effort has been expended by many workers in attempting to provide real-time remote sensing measurement tools.

Recently Bradley et al. (2006a) described a method for continuous real-time operational monitoring of wake vortices using an array of four SODARs at a major European airport. Each vertical wind speed profile comprises a matrix of wind speeds at each height range (determined by acoustic pulse characteristics) and each horizontal position (determined by SODAR spacing). These 'snap-shots', every 2 seconds, are individually analysed so as to estimate best values of vortex circulation and position. This is achieved using least-squares fitting, of the data matrix against a simple vortex model, which also returns reliability of the vortex strength and position estimates. The resulting time-sequence of vortex development can then be presented in a number of ways as a real-time visualisation. Important features of this work are: use of a proven technology; validation at vortex generation against known aircraft characteristics (before substantive vortex-meteorological interaction); use of very simple assumptions in the parameter-extraction model so that the estimated parameters are not constrained; and real-time processing and display, with reliability estimates.

This is an unusual and demanding use of SODAR technology since no signal averaging is used to obtain these ‘snap-shots’ of the vertical wind structure. Normally SODARs average 50 or more power spectra at each height range in order to obtain good estimates of the Doppler-shifted frequency at the spectral peak. In order to track development and position of individual vortices it is however necessary to obtain wind profiles every few seconds. For an initial vortex height of 100 m, the return time for a sound pulse is about 0.6 s, so only about 4 profiles can be averaged. The decrease in signal-to-noise ratio (SNR) is however compensated to some degree by doing a non-linear least-squares fit of the instantaneous vertical velocity field to a simple vortex model. This inherently emphasizes signal in comparison to noise, providing that the model adequately describes the underlying physical situation.

The purpose of this paper is to extend the work of Bradley et al. (2006a) through investigating the sensitivity to acoustic noise and through optimizing the SODAR array design in terms of SODAR spacing and numbers. A companion paper, Bradley et al., (2006b) considers factors leading to signal loss in an operational system.

2. VORTEX PARAMETER ESTIMATION

Bradley et al. (2006a) use a four-vortex model based on an inviscid incompressible uniform atmosphere in which each vortex has tangential velocity v_θ given by the potential flow solution

$$v_\theta = \frac{\Gamma}{2\pi r} \qquad (1)$$

where Γ is the circulation and r the radial distance from the vortex core. The Kutta-Joukowski Lift Theorem gives lift per unit length as

$$\frac{M_{ac} g}{2s} = \rho V_{ac} (2\Gamma) \qquad (2)$$

where M_{ac} is the aircraft mass, g is the gravitational acceleration, s is the half-spacing of the trailing vortices, ρ is air density, V_{ac} is aircraft speed, and Γ is the circulation associated with *one* of the trailing vortices (Anderson, J. D., 2001). In order to satisfy the boundary condition of zero flow through the ground surface, two ‘image vortices’ are included in the model, below the ground surface and rotating in the opposite sense to the vortices above ground (see Fig. 1). The total flow is the composite of the four vortex flows. Parameters describing this flow are Γ, s, and the position of the core of the right-hand upper vortex (x_c, z_c). This model is simply extended to allow for a uniform horizontal wind U by writing $x_c = x_0+Ut$ where t is time.

The data analysed by Bradley et al. (2006a) was recorded from four SODARs spaced by 25 m on one side of, and at right angles to, the flight path, and about 80 m below landing aircraft. Simultaneous profiles for the SODAR array were recorded every 2s, with vertical wind speeds w recorded every 10 m up to 80 m, giving a total of 8x4 = 32 data points every 2 s, from which to estimate the 4 parameters (Γ, s, x_c, and z_c). A key feature of the parameter estimation was allowance for the volume-averaging inherent in the SODAR data which yields $\overline{w}$ rather than w. Between 35 and 50 profiles were obtained during the lifetime of each vortex pair, giving a total of 1100-1600 observations per event.

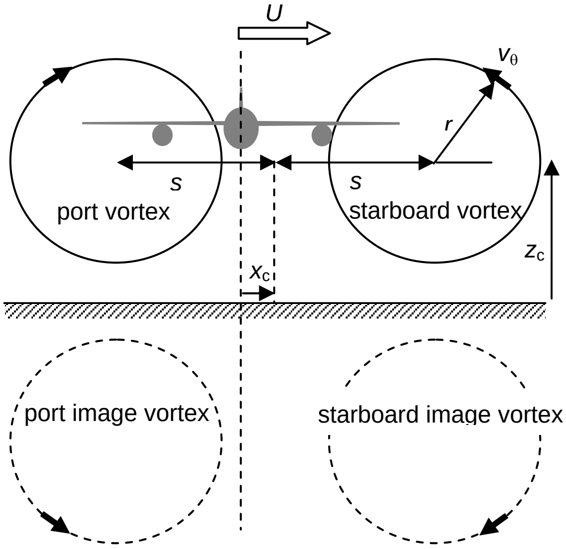

Figure 1. The 4-vortex configuration used to model wing vortex pairs near a ground surface.

A corrected version of the MathWorks non-linear least-squares routine was used to minimise

$$\chi^2 = \frac{1}{32}\sum_{h=1}^{8}\sum_{m=1}^{4}\left[\overline{w}_{h,m} - \overline{w}(x_m, z_h; \Gamma, s, x_c, z_c)\right]^2 \qquad (3)$$

where $\overline{w}_{h,m}$ are the observations at height z_h (h = 1,2,…,8) and SODAR horizontal position x_m (m=1,2,3,4), and $\overline{w}(x_m, z_h; \Gamma, s, x_c, z_c)$ are the corresponding model estimates based on an assumed parameter set. Fig. 2 shows examples of estimated parameters and their temporal evolution.

3. SOURCES OF PARAMETER ERROR

The non-linear least-squares fitting process produces estimates of Γ, s, x_c, and z_c and associated error estimates σ_Γ, σ_s, σ_x, and σ_z. These errors arise from two sources:

- noise in the SODAR $\overline{w}_{h,i}$ estimates
- fit of model to reality.

Since the 'model noise' is not easy to evaluate, Bradley et al. (2006a) solved the dynamics of their 4-vortex group in a simple atmosphere. Comparison with other observations of vortex behaviour was good, lending credence to their model.

We therefore concentrate here on the uncertainties in SODAR vertical velocity estimates. Bradley et al. (2004) have given an exhaustive account of errors and uncertainties arising in obtaining high-accuracy SODAR wind measurements (to 1% accuracy) for wind energy applications. Here we are not concerned so much with high absolute accuracy, and the main source of measurement noise arises from noise in the Doppler spectrum causing uncertainties in estimating the position of the signal peak in the Doppler spectrum.

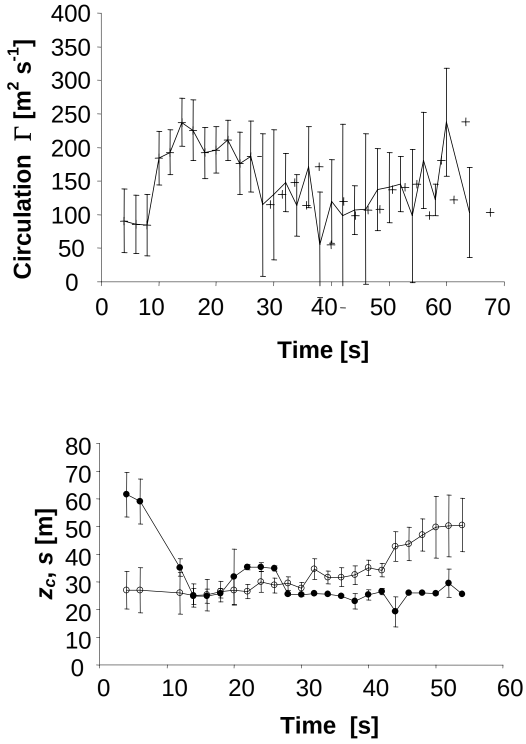

Figure 2. Example of estimated vortex parameters from a landing aircraft.

The spectral noise arises from three main sources:

- background acoustic noise (uniformly random, or 'white', over the measurement band)
- echoes from rain (a broad peak within the measurement band)
- echoes from masts, buildings, etc. (spectral shape similar to the transmission pulse, but not generally Doppler-shifted)

The third of these, 'ground clutter' or 'fixed echoes', will not generally be a problem in the airport context because the end of a runway is flat and devoid of major obstacles having any significant height. Echoes from rain are episodic, but are important since rain causes reduced visibility during landing or take-off, and also LIDAR methods of vortex detection fail during fog or precipitation events. However, in the current work we concentrate on the influence of the acoustic background noise, which is usually the dominant limitation to SODAR wind retrievals.

4. DETECTION OF THE SPECTRAL PEAK

Bradley (2006) writes the Fourier transform (the spectral density times frequency bin width) of the received SODAR signal as

$$V_i = \overline{V_i} + E_i \tag{4}$$

where the noise component E_i, centred on frequency f_i, (i =1,2,…,N_f), is Gaussian-distributed. The probability of recording a spectral amplitude magnitude between V_i and $V_i + dV_i$ is

$$p(V_i)dV_i = \frac{1}{\sqrt{2\pi}\sigma_E} e^{-\frac{1}{2}\left(\frac{V_i - \overline{V_i}}{\sigma_E}\right)^2} dV_i \tag{5}$$

where σ_E^2 is the variance of E_i. The power spectral estimate of the noise at f_i is $P_i = V_i^* V_i = |V_i|^2$ [(Volts)^2], giving

$$p(P_i) = p(|V_i|)\frac{|dV_i|}{dP_i} = \frac{1}{2\sqrt{2\pi P_i}\,\sigma_E} e^{-\frac{1}{2}\left(\frac{\sqrt{P_i} - |\overline{V_i}|}{\sigma_E}\right)^2} . \tag{6}$$

The power spectrum of the Doppler-shifted signal can be modelled as

$$\overline{P_i} = P_0 e^{-\frac{1}{2\sigma_f^2}(f_i - f_D)^2} \tag{7}$$

giving (see Bradley, 2006 for details)

$$\mathrm{SNR} = \frac{P_0}{\sigma_E^2} . \tag{8}$$

From this model Bradley (2006) develops methods to estimate the position of the peak in the power spectrum, together with error estimates.

For the current investigation we can use (6) to generate spectra having realistic random fluctuations, superimposed upon the signals expected from vortex circulations at the positions (x_m, z_h). The Doppler shift at each position is then estimated using the method of Bradley (2006), to provide an estimate of vertical wind which simulates the estimates from a SODAR under various noise conditions. Finally, the matrix of wind estimates is used to derive the vortex parameters.

As an example, the mean errors in vertical velocity are shown in Fig. 3 as a function of height for three values of SNR.

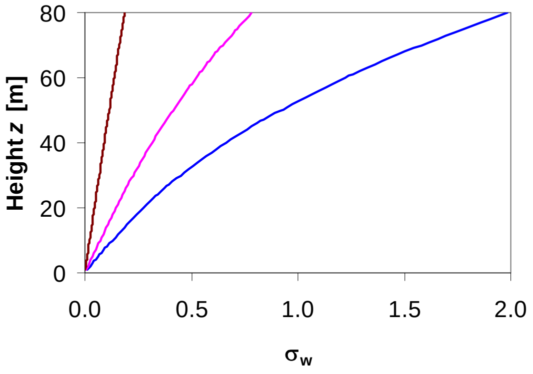

Figure 3. The average uncertainty in vertical velocity estimation for three values of SNR at 100m: -5dB (blue); 0dB (pink); and 5dB (brown).

5. OPTIMISATION OF THE SODAR ARRAY

The full paper at the conference will give results of the simulations. These will show how uncertainties in vortex parameters depend on the background acoustic noise levels. Also, sensitivity to drop-outs of data at some locations will be discussed.

The data presented by Bradley et al. (2006a) were limited to only four SODARs. The results form the present study will be used to determine dependence on the number and the spacing of the SODARs in the SODAR array.

6. SUMMARY

We have described the main sources of noise leading to uncertainties in SODAR wind speed estimates. The most common and persistent of these is background acoustic noise. Based on a robust method for estimation of the position of the spectral peak in the SODAR Doppler spectrum, we simulate noisy spectra typical of SODAR data which might arise from observations of aircraft vortices.

This field of noisy vertical velocity estimates is used as the input to the inversion method described by Bradley et al. (2006a). The resulting error estimates for the vortex position and strength are shown to depend systematically on SODAR SNR, and on the SODAR array configuration (spacing and number of SODARs as well as range-gate size). The outcome provides recommendations as to optimized design of a SODAR array for characterization of aircraft vortices.

7. REFERENCES

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