JP 1.6 SIMILARITY ON TURBULENT TRANSFERS OF HEAT, WATER VAPOR, AND CARBON DIOXIDE OVER A SUBURBAN SURFACE UNDER WEAKLY UNSTABLE CONDITION

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1. INTRODUCTION

The use of Monin-Obukov similarity theory, hereafter MOS theory, to predict surface fluxes from associated variances, known as the flux-variance method, is a practical method to estimate the turbulent fluxes of momentum, heat, and water vapor. This method has been well developed for smooth, homogeneous surfaces such as grass, crop, and forest (e.g., Tillman, 1972; Weaver, 1990). Ohtaki (1985) found that the method has also been used to determine the turbulent statistics of carbon dioxide (CO_2) concentration over homogeneous crop. The urban field experiment of Roth and Oke (1995) indicated that the empirical flux-variance relationship is not applicable for water vapor transfer due to the heterogeneity of water vapor sources. However, knowledge on the flux-variance relationships in urban surface layers is still much less than that in low-roughness homogeneous surfaces. In addition, the monitoring of CO_2 flux in urban areas has become an important issue in urban surface studies (e.g., Grimmond et al., 2002; Moriwaki and Kanda, 2004). In urban environment there exists the difference in ground sources and sinks of heat, water vapor, and CO_2: e.g., Moriwaki and Kanda (2004) showed that the main source of H_2O is from vegetation and that of CO_2 is from houses, transportation, and human exhalations in a suburban area in Tokyo Japan. Therefore, further inquiries on the features of turbulent transfer of scalars are needed not only for heat and vapor, but also for CO_2.

In this study, we analyzed data on the turbulent transfer of heat, vapor, and CO_2 over the suburban surface. We examined the similarity of the vertical transfer of scalars that had different source distributions and different physical behavior by using the turbulent linear correlation coefficients, a method previously used by Roth and Oke (1995). Also, we used wavelet analysis to determine the time-scale characteristics of the turbulent transfer of scalars for the evaluation of the 'local' similarity, and quadrant analysis to determine the contributions of sweep or ejection motion to the total fluxes.

2. METHODS

2.1 *Site Description*

The experiment was done during July 2001 in a low-storied residential area in Kugahara, Tokyo, JAPAN (35º34'N, 139º41'E). A 29-m tower (without

Figure 1. 29-m height tower (left) and view to the west (right).

guy lines) was installed in the backyard of a home. The residential area consists of houses with a mean height of 7.3 m, paved roads, and small playgrounds. A view from the tower is shown in Figure 1. Flat, uniform terrain extended over 1-km to the south, west, and north from the tower. The terrain within 200-m to the east is gently slanting down with an inclination angle of 5.7º. Within a circle of 500-m radius centered on the site, about 33% of the area was buildings, 22% was vegetation, and 23% was impervious space such as paved roads and concrete. A general description of the climatology and energy exchanges between the surface and the atmosphere in this site are described in Moriwaki and Kanda (2004).

2.2 *Experimental Setup*

A sonic anemometer (USA1, Metek GmbH) and an infrared CO_2/H_2O open-path analyzer (LI-7500, LI-COR) were installed at the tower top (29 m), which is about 4 times the canopy height. The measurement height was found to be above the urban roughness layer according to the analysis for the flux-gradient relationships of momentum and heat (Moriwaki and Kanda, 2005). The instantaneous horizontal and vertical wind velocities *u*, *v*, and *w*, sonic temperature *T*, humidity *q*, and CO_2 concentration *c*, were sampled at 8 Hz. To avoid the use of flux corrections arising from density effects (Webb et al., 1980), the measured CO_2 mole concentrations were converted into mixing ratios using the sonic temperature. These data were logged to a data logger (CR10X, Campbell Sci.) and stored on a computer.

2.3 *Data Processing*

In this study, we used data only from July 2001. To ensure high data quality, we only used data from fair weather days in which the percentage of sunshine exceeded 80%. To determine whether or not a given day was fair, we used weather data from a Japan Meteorological Agency weather station 12.5-km from the tower. In addition, we used only data obtained at local times from 9:00 to 16:00 during which the meteorological conditions can be regarded as a quasi steady state; the temporal changes of wind, temperature, humidity, and CO_2 are relatively small at these times. Turbulent statistics were calculated for every 60-min period. There were a total of 43 such periods. We first eliminated the trend of *T*, *q*, and *c* in each 60-min data set by subtracting a linear fit to the data and then calculated the following fluctuations from these fits: *u'*, *v'*, *w'*, *T'*, *q'*, and *c'*. Then we transformed the velocity coordinates using a method proposed by McMillen (1988) so that $\overline{v'} = 0$, $\overline{w'} = 0$, and $\overline{v'w'} = 0$.

a) *Linear correlation coefficient*

Following Roth and Oke (1995), we examined the turbulent transfers of heat, vapor, and CO_2. The analysis was done using the following turbulent linear correlation coefficients:

$$r_{wT} = \overline{w'T'} / \sigma_w \sigma_T = 1/(\frac{\sigma_w}{u_*}\frac{\sigma_T}{T_*}) \quad , \qquad (1)$$

$$r_{wq} = \overline{w'q'} / \sigma_w \sigma_q = 1/(\frac{\sigma_w}{u_*}\frac{\sigma_q}{q_*}) \quad , \qquad (2)$$

$$r_{wc} = \overline{w'c'} / \sigma_w \sigma_c = 1/(\frac{\sigma_w}{u_*}\frac{\sigma_c}{c_*}) \quad , \qquad (3)$$

where the overbar means time-mean value, prime denotes the instantaneous deviation from the time-mean value, σ is the standard deviation, and u_*, T_*, q_*, and c_* are the surface layer scales defined as $(-\overline{u'w'})^{0.5}$, $\overline{w'T'}/u_*$, $\overline{w'q'}/u_*$, and $\overline{w'c'}/u_*$, respectively. The correlation coefficients can be viewed as a measure of the overall efficiency of the transfer process and varies between 0 (no correlation) and 1 (optimally efficient transfer) (Roth and Oke, 1995).

b) *Wavelet analysis*

Wavelet analysis provides both scale and time information of analyzed signals and allows an objective separation of patterns in the data that have different time scales at different time. The wavelet transform of a time series can be written as

$$W_x(a,b) = \frac{1}{a}\int_0^T f\left(\frac{t-b}{a}\right)x(t)dt \quad , \qquad (4)$$

where f represents a wavelet function, a is the dilation parameter, hereafter the time-scale, that determines the duration and amplitude of the wavelet, b is the center of the wavelet, and T is the length of the data record. The wavelet variance is defined as

$$V_x(a) = \int_0^T W_x(a,b)^2 db \quad . \qquad (5)$$

We applied wavelet analysis to the time series of w'/σ_w and the turbulent transfers $w'T'/\sigma_w\sigma_T$, $w'q'/\sigma_w\sigma_q$, and $w'c'/\sigma_w\sigma_c$. The time series that was transformed were normalized by the standard deviations. Therefore, the wavelet coefficients measure the relative intensity of fluctuation to the total turbulent deviation. This enables us to directly compare the efficiency of turbulent transfer of scalars. The analyzing wavelet in this paper is the "Mexican hat" function, a second derivative of a Gaussian. Because of its functional form, the wavelet transform has large positive values when upward transfers are observed in the original time trace.

c) *Quadrant analysis*

The turbulent transfers were also inferred using quadrant analysis, a method that is often used for the Reynolds stress fraction in laboratory studies (e.g., Nakagawa and Nezu, 1977), large eddy simulations (e.g., Kanda, 2005), and field experiments (e.g., Shaw et al., 1983; Oikawa and Meng, 1995).

Quadrant analysis can also be used for the turbulent transfer of scalars. For example, for heat transfer, $w'T'$ is divided into four different quadrants. Quadrants one and three, representing ejection and sweep, respectively, make positive contributions to the vertical heat flux, whereas quadrants two and four make negative contributions to the upward flux. A fraction S_{iwT}, which is the conditionally averaged stress $< w'T' >_i$ in quadrant i normalized by the total mean averaged heat flux, is defined as

$$S_{iwT} = < w'T' >_i / \overline{w'T'} \quad . \qquad (6)$$

A similar procedure can be used for vapor flux $w'q'$ and CO_2 flux $w'c'$ as

$$S_{iwq} = < w'q' >_i / \overline{w'q'} \quad , \qquad (7)$$

$$S_{iwc} = < w'c' >_i / \overline{w'c'} \quad . \qquad (8)$$

3. RESULTS AND DISCUSSION

3.1 *Correlation Coefficients*

First we investigated the vertical transfer of scalars using the correlation coefficients between w' and T', w' and q', and w' and c', which are defined in Equations (1) - (3) as r_{wT}, r_{wq}, and r_{wc}, respectively. In Figures 2a, 2b, and 2c, they are plotted versus atmospheric stability z'/L, where L is the Obukov length:

$$L = -\left(-\overline{u'w'}\right)^{2/3} T / gk\overline{w'T'} \quad , \qquad (9)$$

where *g* is the acceleration of gravity, and *k* is the von Karman constant. Here, *k* = 0.4. Momentum transfer efficiency $r_{uw} = -\overline{u'w'}/\sigma_u\sigma_w$ is also plotted by black triangle as reference. The linear correlation coefficient for heat r_{wT} has a positive bias for unstable conditions, compared with the MOS function over flat surfaces (estimated from the combination of Panofsky et al. (1977) and Wyngaard et al. (1971)) indicated by curves in Figure 2. The increased heat transport over that for the flat surfaces is due to the relatively low σ_w/u_* (not shown here). A low σ_w/u_* is commonly

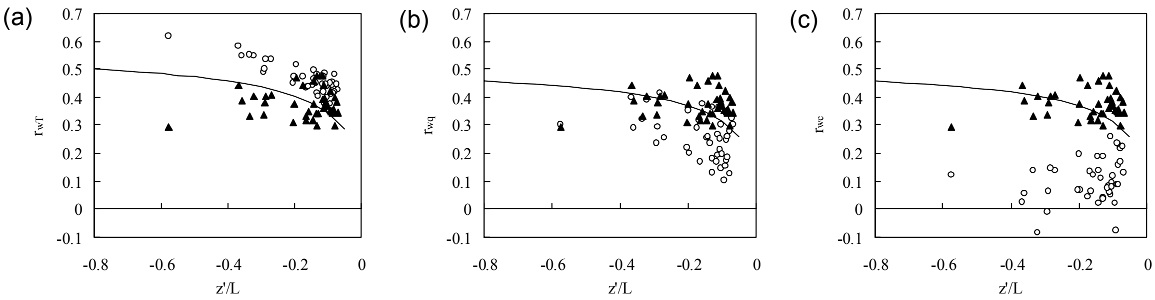

Figure 2. Efficiencies of turbulent transfer for various measured values of atmospheric stability z'/L. Each circle represents one hour of data. (a) Heat transfer efficiency r_{wT}. (b) Water vapor transfer efficiency r_{wq}. (c) CO_2 transfer efficiency r_{wc}. The solid curves are prediction from MOS theory. Momentum transfer efficiency $r_{uw} = -\overline{u'w'}/\sigma_u\sigma_w$ is plotted by black triangle as reference.

observed in urban surface layers (e.g., Roth and Oke, 1995; Clarke et al., 1982). The reason for the low σ_w/u_* here is that the relatively high urban roughness increases the momentum transport over that for typical rural areas (Roth and Oke, 1995). The stability dependency of the linear coefficient for vapor r_{wq} is similar to that of r_{wT}, but it has a smaller magnitude. Also, the linear coefficient for CO_2 mixing ratios r_{wc} is much less than that of r_{wq}. Ohtaki (1985) concluded that temperature and humidity statistics have similar characteristics over fairly homogeneous surfaces. But for more complex surfaces, Beljaars et al. (1983) showed that the nondimensional standard deviations of temperature and humidity have different values at the same stability, as we have found. Katul et al. (1995) suggested that, even over homogeneous surfaces, temperature and humidity statistics could differ because temperature is an active scalar quantity whereas humidity is generally not. Such an argument logically implies then that r_{wT} and others would be relatively alike in near neutral stability but would diverge as $|z'/L|$ increases and temperature assumes a more active role in the dynamics (Andreas et al., 1998). Therefore the stability dependency of r_{wq}/r_{wT} was expected, but it was not found in Figure 3a. On the other hand, the ratio of CO_2 mixing ratio to heat r_{wc}/r_{wT} decreases as $|z'/L|$ increases. Such a stability dependency is accordance with the suggestion by Katul et al. (1995). By a similar argument to vapor, CO_2 might be called 'passive' because the effect of CO_2 on buoyancy is negligible due to the extremely low mixing ratios of CO_2.

3.2 *Wavelet Analysis*

The wavelet transform is useful for detecting patterns in the data that have different time scales at different times. We investigated the 'local' similarity of

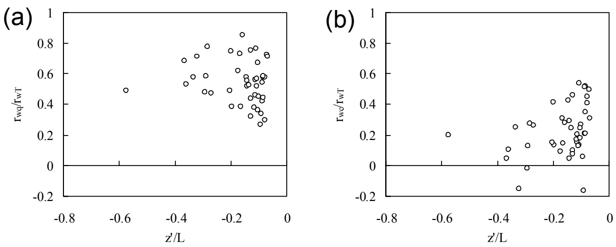

Figure 3. Efficiencies of water vapor and CO_2 turbulent transfer relative to heat transfer versus atmospheric stability. Each circle represents one hour of data. (a) Water vapor. (b) CO_2.

the turbulent transfer of scalars using the wavelet transform. A typical wavelet transform for the turbulent transfer of scalars is shown in Figure 4. The color-coding scale at bottom right indicates the value of the wavelet transform. Wavelet analysis was done for the time series data normalized by the standard deviation of the quantity that was transformed; w'/σ_w, $w'T'/\sigma_w\sigma_T$, $w'q'/\sigma_w\sigma_q$, $w'c'/\sigma_w\sigma_c$, T'/σ_T, q'/σ_q, and c'/σ_c. Hence, the value shows the intensity of turbulent transfer relative to the total flux of scalars (see Section 2.3b). We now compare the turbulent transfers of scalars.

First, we point out the common features between the scalar transfers. Small time-scale turbulence (i.e., a < 1 second) has no specific organized structures for the vertical velocity w'/σ_w as shown in the upper most region of Figure 4a (between 1 and 0.25 seconds). The same applies to the turbulent transfer of $w'T'/\sigma_w\sigma_T$, $w'q'/\sigma_w\sigma_q$, and $w'c'/\sigma_w\sigma_c$ (Figures. 4b to 4d). This disorganized small-scale turbulence field is contributed primarily by random background turbulence at small scales. As the time scale increases, the distribution tends to be organized into discrete structures. These organized structures can be divided into two time scales. Figure 5 shows the resulting scalogram for the wavelet variance of

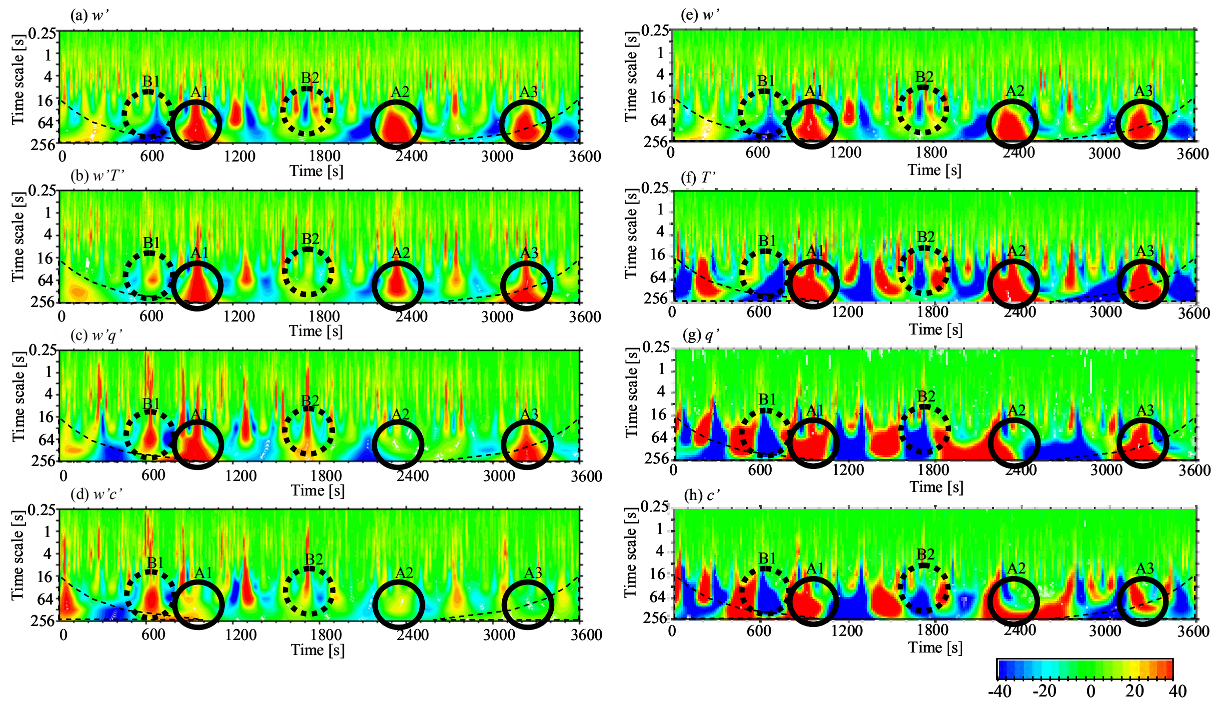

Figure 4. Wavelet transform using various time scales a for variables measured from 13:00 to 14:00 on July 11th 2001. (a) w'/σ_w, (b) $w'T'/\sigma_w\sigma_T$, (c) $w'q'/\sigma_w\sigma_q$, (d) $w'c'/\sigma_w\sigma_c$, (e) w'/σ_w, (f) T'/σ_T, (g) q'/σ_q, and (h) c'/σ_c. Circles mark thermal structures.

$w'T'/\sigma_w\sigma_T$, $w'q'/\sigma_w\sigma_q$, and $w'c'/\sigma_w\sigma_c$ obtained from Figure 4. The scalogram has a plateau near a = 8 seconds and a peak near a = 64 seconds. For the "Mexican hat" wavelet function, as we use here, a can be roughly converted to the spectral frequency $f = (4a)^{-1}$; thus these time scales correspond to 32 and 256 seconds of the period of the sine wave. The structures with a time scale of 32 seconds are probably surface layer eddies, because the nondimensional frequency n_m (= 0.09) of these structures is consistent with the surface layer relationship developed from field experiments (Kaimal and Finnigan, 1994). Another structure group can be found in the time scales from 256 seconds. In these structures, heat is positively transferred from the surface to the atmosphere. These structures correspond to the updraft motions that occurred at the same time and with the same time scale in Figure 4a. The long time scale of these structures (256 seconds) suggests that these structures are thermals or rolls (e.g., Stull, 1988).

Second, we discuss the dissimilarity of turbulent transfers of scalars. In the structures marked by circles (A1-A3) in Figure 4, heat is always transferred upward by structures with positive vertical velocity. However, for some of these structures, vapor and CO_2 are either not transferred or are only weakly transferred (see Figures 4c and 4d). Possible reasons for the dissimilarity for the transfer efficiency among scalars are the physical characteristics and the

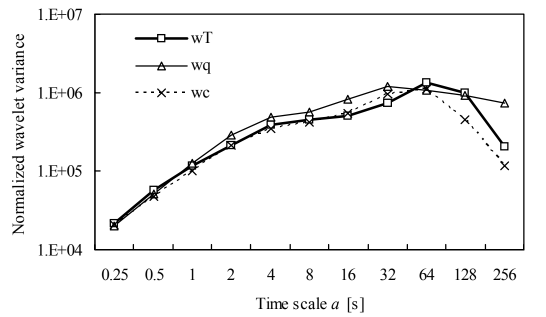

Figure 5. Wavelet variance for $w'T'/\sigma_w\sigma_T$, $w'q'/\sigma_w\sigma_q$, and $w'c'/\sigma_w\sigma_c$ from data in Figure 4.

heterogeneity in the source distribution of scalars. Heat is an active scalar and produces the thermal structures by itself. Hence, the heat is most efficiently transferred. On the other hand, vapor and CO_2 have less contribution to the buoyant convection. If their horizontal distributions were homogeneous near the surface (as they are for crops and forests), they would be transferred as well as heat. However, in urban surface there are complex source and sink patterns of scalars (e.g., Roth and Oke, 1995). Moriwaki and Kanda (2004) showed that the main source of H_2O is from vegetation and that of CO_2 is from houses, transportation, and human exhalations. The heterogeneity of sinks and sources would produce a similar heterogeneity in the concentration in the urban boundary layer. As a result, the heterogeneity increases the variance of that variable but not necessarily its flux (Weaver, 1990). Under such a condition, some thermal structures may also transfer the scalars but other thermal structures would not because the position of a thermal might have little correlation with the region of high humidity of CO_2 mixing ratio. We also analyzed the downward motions. The structures marked by dotted circles (B1-B2) in Figure 4 are downward motions. In these structures, vapor and CO_2 were efficiently transferred upward, whereas heat was not transferred. We analyze these structures further in the next section.

3.3 *Quadrant Analysis*

A dissimilarity in turbulent transfers was also found using quadrant analyses. Figure 6 shows the relative contributions to flux by ejection and sweep for heat, vapor, CO_2, and momentum transfer for various values of atmospheric stability. The ejection/sweep ratio for heat is larger than unity when z'/L is most negative. This is similar to the results for the atmospheric surface layer over natural ecosystems such as rice paddies, bare soil, water surfaces, and deciduous forests (Maitani and Ohtaki, 1987; Maitani and Shaw, 1990). The ejection dominance in unstable conditions is associated with the thermally generated turbulence. In contrast, the ejection/sweep ratio of water vapor transfer is less than that of heat transfer. Mahrt (1991) and De Bruin et al. (1993) pointed out that when the surface evaporation is lower, the statistics of vapor is affected by downward motions related to the entrainment of warm dry air aloft. Wavelet analysis in the previous section also shows that upward vapor transfer occurs in downward motions (dotted circles in Figure 4). The ejection/sweep ratio of CO_2 transfer is even less than that of vapor. This indicates that downward motion (sweep) transfers CO_2 more efficiently than ejections. CO_2 near the ground has a strong inhomogeneity in its spatial distribution. Therefore, CO_2 is not always transferred by thermals as well as heat or vapor. On the other hand, the heterogeneity of scalars in the upper air is generally less than that near the surface according to concepts of blending height (e.g., Mahrt, 2000). Therefore, in downward motions, the lower CO_2 is efficiently transferred.

4. CONCLUSIONS

The similarity of vertical turbulent transfer of heat, vapor, and CO_2 were examined in an urban surface layer over a residential area in Tokyo, Japan. We investigated the correlation coefficients, and examined the quadrant analysis and the wavelet analysis. The ratios of linear correlation coefficients for vapor/heat and CO_2/heat showed that the transfer efficiencies of vapor and CO_2 were smaller than that of heat in unstable conditions, a result that conflicts with conventional surface layer theory. The wavelet analysis showed that heat was always transferred

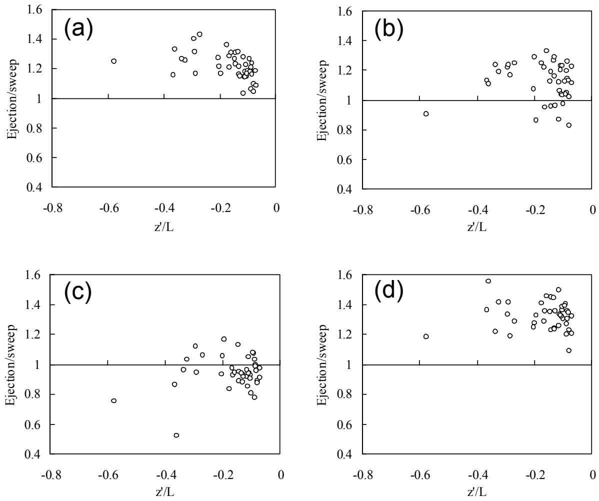

Figure 6. Fraction ratio of ejection to sweep for turbulent transfer of (a) heat, (b) vapor, (c) CO_2, and (d) momentum. Each circle represents one hour of data.

efficiently by the thermal structures and organized structures. Vapor and CO_2 was transferred as well as heat in some of the structures, but not in others. The quadrant analyses also indicated that the vapor and CO_2 were transferred less by upward turbulent motions compared with the heat transport. These differences between heat and both CO_2 and water vapor were probably caused both by the active role of temperature and the heterogeneity in the source distribution of scalars. Heat is an active scalar and produces the thermal structures. Hence, the heat is transferred most efficiently. Vapor and CO_2 are transferred passively. If their horizontal distributions were homogeneous, they would be transferred as well as heat. However, the sink/source of vapor and CO_2 is inhomogeneous. As a result, the heterogeneities of vapor and CO_2 concentration decrease their transfer efficiencies.

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