**4.4** THE MOMENTUM AND TURBULENT KINETIC ENERGY BUDGETS WITHIN THE OKLAHOMA CITY PARK AVENUE STREET CANYON

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## 1. INTRODUCTION

Urban areas dramatically affect the flow characteristics of the atmospheric surface layer (ASL) through a variety of mechanisms such as enhanced shear, wake diffusion, etc. (Roth 2000). The urban roughness produces a flow in the urban canopy sub-layer (UCSL) and urban roughness sub-layer (URSL) which is three-dimensional (3D) and heterogeneous, invalidating the assumptions typically used to simplify budget analyses in the inertial sub-layer (ISL). As such, very few terms in the turbulent kinetic energy (TKE) and momentum budgets can be systematically neglected a priori. Some of the few examples of TKE budget analyses within the UCSL and URSL are found in Louka et al. 2000 and Christen et al. 2004. This is principally due to the complicated nature of the flow in urban areas and the inherent difficulties in sighting instruments in actual urban areas. This lack of data leaves the dominant physical processes and the extent to which these processes interact with each other largely subject to conjecture. The flow characteristics within the UCSL are often treated in the area-averaged sense (Bentham and Britter 2003, Gayev and Savory 1999, Cheng and Castro 2002, and MacDonald 2000) and are therefore only crude approximations to the actual flow phenomena that occur locally.

This work uses the 3D sonic anemometer data taken from multiple towers in Oklahoma City's Park Avenue street canyon during the Joint Urban 2003 (JU2003) field campaign to explore the TKE and momentum budgets within the UCSL of a real-world central business district (CBD). The contribution from all the components of the storage, advection, buoyant production/destruction, mechanical shear production, and turbulent transport terms in the TKE budget are calculated. In addition, all of the terms of the storage, advection, and turbulent transport terms of all three velocity components are calculated. The results from the 3D budget analyses are compared with the commonly used ISL assumptions that have been employed in previous urban TKE budget analyses.

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## 2. EXPERIMENTAL DETAILS

The Joint Urban 2003 field campaign was performed from 29 June through 30 July 2003 in Oklahoma City (OKC), OK (Allwine et al. 2004 and Brown et al. 2004). OKC provided a relatively large urban area located in idealized terrain devoid of major topological features. A section of Park Avenue found in the urban core of OKC was selected to concentrate a large number of measurements in a single urban canyon. The average building height ($H$) for the selected urban canyon was ~50 m with a corresponding canyon width ($W$) of ~25 m and canyon length ($L$) of ~150 m, making $H/W$ ~ 2. An analysis of the building data sets in the urban core of OKC performed by Burian et al. (2003) found the plan area fraction ($\lambda_p$) to be 0.35 and the frontal area index ($\lambda_f$) to range between 0.14 and 0.22 depending on wind direction. These values characterize the flow through the urban core as skimming flow using the thresholds suggested by Oke (1987).

Fig. 1 shows the relative locations of the wind instruments deployed in the Park Avenue urban canyon. Forty-three 3D sonic anemometers were placed in and around the urban canyon, acquiring nearly continuous data throughout the entire month of July. During ten intensive observation periods (IOP), when dispersion experiments were performed, an additional two 3D sonic anemometers and seven two-dimensional (2D) sonic anemometers were deployed on tripods at the ends of the canyon. Further details regarding the actual locations can be found in Nelson et al. (2004 and 2005a).

The computation of the TKE and momentum budgets within the Park Avenue street canyon center around four sonic anemometers. These four anemometers, two on the Defense Science and Technology Laboratory (DSTL) tower and two on the University of Utah (UU) tower both at the western end of the canyon interior, were chosen for this analysis due to the fact that they could be paired with other anemometers aligned in all three principal directions within the canyon: along-canyon ($x$), cross-canyon ($y$) and vertical ($z$). This allows the gradients in all three directions to be approximated as shown in Fig. 2. The principal towers used are shown as green rings while the towers used to calculate the along-canyon gradients are orange rings. The vertical gradients were approximated from a $2^{nd}$-order polynomial curve fit of the data from three sonic anemometers following the algorithm found in Chapra and Canale (1998),

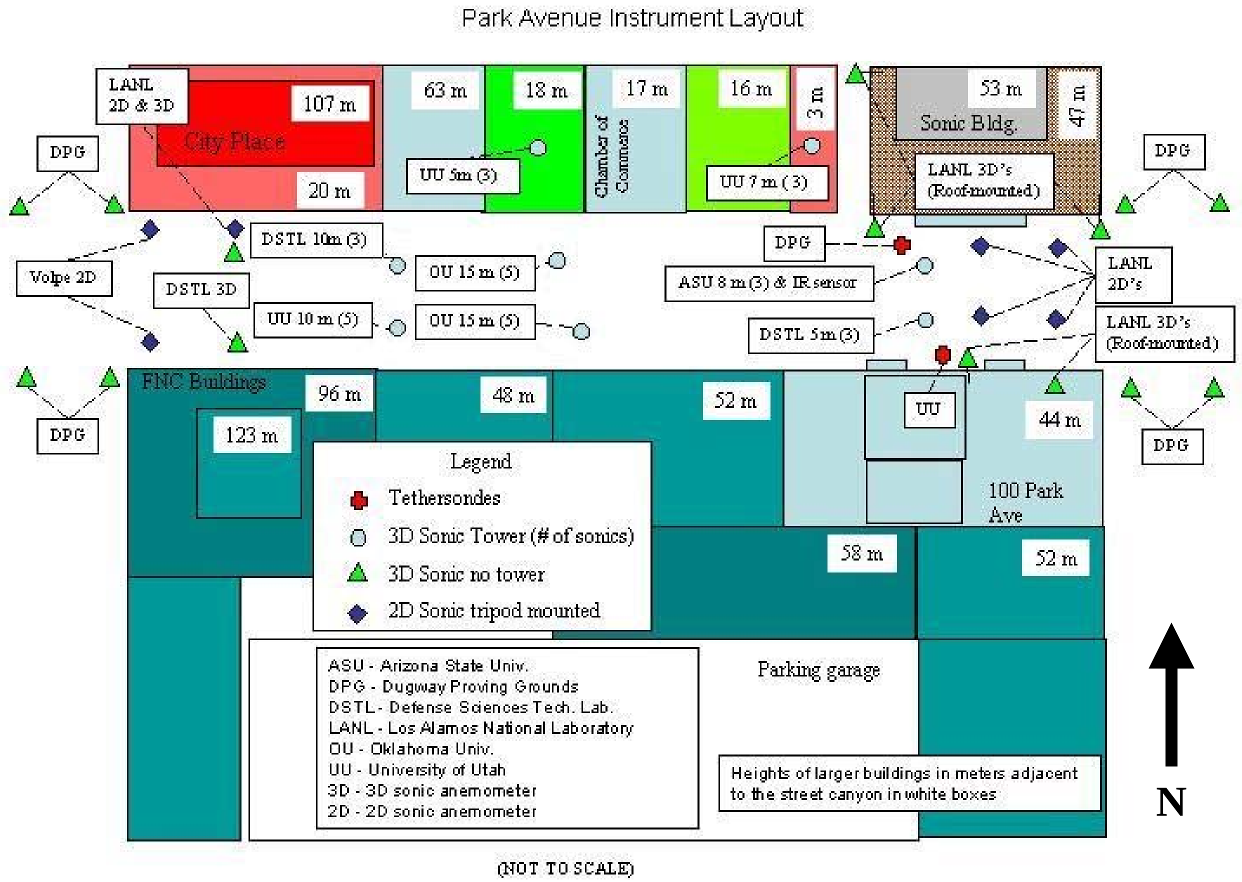


**Figure 1** Schematic of the relative locations of the wind instrumentation on Park Avenue including the type of instrument and the organization responsible for its operation.

while the horizontal gradients are calculated using simple forward or backward differences depending on location. Note that in the Park Avenue street canyon, the along-canyon and cross-canyon directions correspond to the standard meteorological coordinates (i.e. positive *U* for winds blowing west to east and positive *V* for winds blowing from south to north).

## 3. DATA PROCESSING

Nelson et al. (2005a) found that southerly and southeasterly winds produced very complicated flow patterns in the canyon interior and hypothesized that the complicated flow patterns were due to downdrafts of high momentum fluid into the canyon. It was also found that southwesterly winds produced westerly channeling throughout the canyon interior. Since the method used to approximate the horizontal gradients is necessarily crude due to the instrument spacing, the approximations of the gradients are insufficient to resolve complicated flow phenomena. IOP 10, which ran continuously from 2300 July 28, 2003 to 0700 July 29,2003 Central Daylight Time (CDT), was chosen for analysis in this work, since westerly channeling was found to dominate the flow throughout the canyon during this time period (see Fig. 2). The approximations to the gradients are much more likely to be representative of the actual flow in the absence of highly complicated phenomena. In some sense the method of estimating the gradients within the canyon can be thought of as the field data analog to a rough large-eddy simulation of the canyon. It is hypothesized that this approximation to the gradients within the canyon may be sufficient in the absence of the complicated downdraft phenomena since Nelson et al. (2005b) found that most of the dominant flow scales were generally of the order of the canyon width or larger.

The velocity data were sampled at 10 Hz, however, the only data available to the authors for the DSTL tower were one-second averages of the original 10 Hz data. As such, the processed 1 Hz data from the DSTL tower was used to compute the flow statistics used in the budget analyses in this work. While this is unlikely to have a large effect on the mean velocities, it does potentially affect the turbulent fluxes. In addition, the flow within the UCSL tends to be highly heterogeneous and 3D. Recent studies

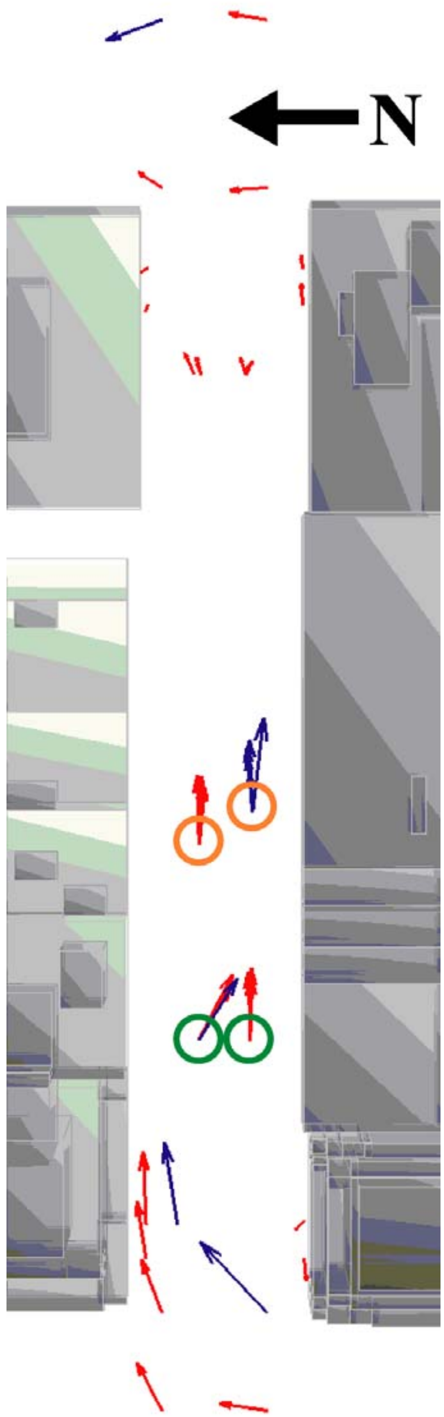


**Figure 2** Mean wind vectors within the Park Avenue street canyon during the 8 hours of IOP 10. Green rings indicate the location of the principal towers used for budget analyses and orange rings indicate the location of towers used to compute along canyon gradients. Vectors are colored by mean vertical velocity, red and blue are respectively positive and negative mean vertical velocities. The figure is to scale.

have shown that these conditions can cause errors in sonic anemometry measurements due to the fact that sonic anemometers are typically calibrated under conditions with little or no mean vertical velocity (see van der Mollen et al. 2004). Correcting for these effects requires the calibration of each of the various sonic anemometer geometries for a large range of angles of attack. The data presented here have not been corrected for these effects since the authors do not have the proper calibration algorithms for each of the various makes and models of sonic anemometers used.

### 3.1 *Turbulent Kinetic Energy Budget*

For this analysis, the full turbulent kinetic energy (TKE) equation,

$$\frac{\partial\langle e\rangle}{\partial t}+\langle U_j\rangle\frac{\partial\langle e\rangle}{\partial x_j}=+\delta_{i3}\frac{g}{\langle\theta_v\rangle}\langle u_i'\theta_v'\rangle$$
$$-\langle u_i'u_j'\rangle\frac{\partial\langle U_i\rangle}{\partial x_j}-\frac{\partial\langle u_j'e\rangle}{\partial x_j}-\frac{1}{\rho}\frac{\partial\langle u_i'p'\rangle}{\partial x_i}-\varepsilon \qquad (1)$$

has been rearranged, the partial derivatives replaced with finite differences, and the terms which cannot be calculated from the existing data (i.e. the pressure correlation term and the dissipation rate) have been lumped together to form a residual term ($R_{TKE}$) which takes on the value required to satisfy the TKE budget equation. (Eq. 2).

$$0=S+A+B+P+T+R_{TKE} \qquad (2)$$

Specifically, each of the terms has been approximated as follows:

$$S=-\frac{\Delta\langle e\rangle}{\Delta t}$$ Storage

$$A=-\langle U_j\rangle\frac{\Delta\langle e\rangle}{\Delta x_j}$$ Advection

$$B=\delta_{i3}\frac{g}{\langle\theta_v\rangle}\langle u_i'\theta_v'\rangle$$ Buoyant Production

$$P=-\langle u_i'u_j'\rangle\frac{\Delta\langle U_i\rangle}{\Delta x_j}$$ Mechanical Production

$$T=-\frac{\Delta\langle u_j'e\rangle}{\Delta x_j}$$ Turbulent Transport

$$R_{TKE}=-\frac{1}{\rho}\frac{\partial\langle u_i'p'\rangle}{\partial x_i}-\varepsilon$$ Residual

The angled brackets represent the one-hour time averages and the values that are presented in this work are ensemble averages of the individual one-hour time periods.

While the dissipation rate can be calculated from the velocity power spectral energy density in the inertial subrange as was done in Christen et al. (2004), this method relies heavily on the validity of Taylor's frozen turbulence hypothesis. The validity of the use of the hypothesis is highly questionable deep within the UCSL. Hence, the authors feel that it is best to leave the dissipation rate as part of the residual term. Unfortunately this leaves a large amount of ambiguity in the interpretation of the residual term due to the fact that both terms are likely to have significant contributions to the budget within the street canyon. Fluctuating quantities were computed using a 30-minute running-block average to remove large-scale meteorological effects. The data were then separated into 1-hour bins for use in the calculation of the various terms with the exception of the storage term, which was further subdivided into two half-hour periods, which were then used to compute the time derivative.

### 3.2 *Momentum Budget*

Similarly the full 3D momentum equation,

$$\frac{\partial\langle U_i\rangle}{\partial t}+\langle U_j\rangle\frac{\partial\langle U_i\rangle}{\partial x_j}=-\delta_{i3}g+f_c\varepsilon_{ij3}\langle U_j\rangle -\frac{1}{\rho}\frac{\partial\langle P\rangle}{\partial x_i}+\nu\frac{\partial^2\langle U_i\rangle}{\partial x_j^2}-\frac{\partial\langle u_i'u_j'\rangle}{\partial x_j} \qquad (3)$$

has been rearranged, the Coriolis term neglected due to the size of the flow scales of interest, the partial derivatives have been replaced with finite differences, and the remaining terms which cannot be calculated from the existing data (i.e. the pressure gradient and the viscous terms) lumped together to form a residual term ($R_m$) which takes on the value required to satisfy the momentum budget equation.

$$0=S+A+G+T+R_m \qquad (4)$$

Specifically the various terms of the momentum budget have been approximated as follows:

$$S=-\frac{\Delta\langle U_i\rangle}{\Delta t} \qquad \text{Storage}$$

$$A=-\langle Uj\rangle\frac{\Delta\langle U_i\rangle}{\Delta x_j} \qquad \text{Advection}$$

$$G=-\delta_{i3}g \qquad \text{Gravitational Body Force}$$

$$T=-\frac{\Delta\langle u_i'u_j'\rangle}{\Delta x_j} \qquad \text{Turbulent Transport}$$

$$R_m=-\frac{1}{\rho}\frac{\partial\langle P\rangle}{\partial x_i}+\nu\frac{\partial^2\langle U_i\rangle}{\partial x_j^2} \qquad \text{Residual}$$

Although the residual term is once again the sum of two other terms, the ambiguity of the residual term in the momentum budget is likely to be much smaller due to the fact that viscous effects are generally negligible over the flow scales in question leaving the residual term to be principally due to the pressure gradient. The same procedures used to compute the values in the TKE budget were also employed in calculating the momentum budget.

## 4. RESULTS AND DISCUSSION

### 4.1 *Turbulent Kinetic Energy Budget*

The TKE budget on the DSTL and UU towers during IOP 10 is shown in Fig. 3. Note that the lines between the lower- and upper-level data points are not intended to describe the behavior of the flow in between the data points. Instead they are used on all of the data plots in this work to emphasize differences between the two levels. It is immediately apparent that the storage and buoyancy terms are negligible during this time period when compared to the production, advection, turbulent transport, and residual terms. Even though IOP 10 took place during the night with stable upstream conditions, buoyancy contributes very little to the TKE budget. The advection and turbulent transport terms also have a very small mean contribution to the budget over all 8 hours. However, unlike the storage and buoyancy terms this is due to significant positive and negative contributions in the individual one-hour periods

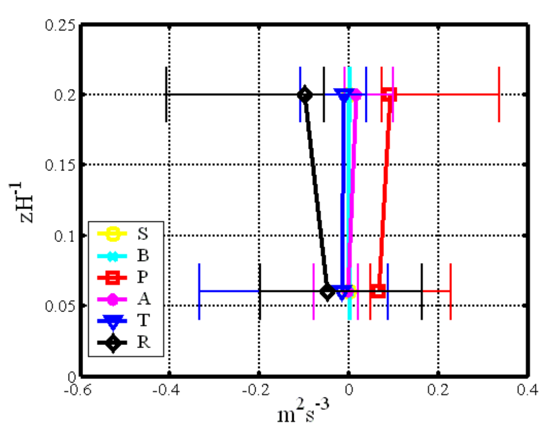


**Figure 3** Average turbulent kinetic energy budget on the DSTL and UU towers for IOP10. Error bars denote the extreme one-hour average values for each term.

yielding a small net contribution rather than simply having small one-hour averages as was the case with the storage and buoyancy terms. Thus, on average, the TKE budget within the Park Avenue street canyon appears to principally be a balance of mechanical shear production, pressure correlation, and dissipation over the 8 hours of IOP 10, while the advection and turbulent transport terms play important roles over smaller periods of time.

In order to gain further insight into the mechanisms dominating the production and transport of TKE within the street canyon, the advection, mechanical shear production, and turbulent transport terms have been further separated into the individual components of the terms in Figs. 4-6 respectively. The along canyon component dominates the advection of TKE (Fig. 4). This is not unexpected given the westerly channeling flow within the canyon during IOP 10 (see Fig. 2).

There are nine terms that are summed together to produce the bulk mechanical shear term (Fig. 5). In standard ISL TKE budget analyses the coordinate system is rotated into the mean wind and horizontal homogeneity and negligible subsidence are assumed reducing the bulk term to a single significant term $\langle u'w' \rangle \frac{\partial \langle U \rangle}{\partial z}$. In the non-rotated coordinate system of the street canyon, this result might lead one to believe that the dominant terms are likely to be $\langle u'w' \rangle \frac{\partial \langle U \rangle}{\partial z}$ and $\langle v'w' \rangle \frac{\partial \langle V \rangle}{\partial z}$. Interestingly, Fig. 5 shows that only one of these quantities, $\langle u'w' \rangle \frac{\partial \langle U \rangle}{\partial z}$, is really significant and it contributes about as much to the net production of TKE as the term related to subsidence, $\langle w'w' \rangle \frac{\partial \langle W \rangle}{\partial z}$. $\langle v'v' \rangle \frac{\partial \langle V \rangle}{\partial y}$ obviously dominates the production of TKE at this location in the street canyon. It is also interesting to note that some of the quantities are actually negative contributions to the production of TKE, e.g. $\langle u'v' \rangle \frac{\partial \langle U \rangle}{\partial y}$. This being the case requires the Reynolds stress to work against the gradient of the mean wind and may suggest that the Reynolds stresses are not due entirely to local mean wind gradients. Fig. 5 also suggests that the methods and assumptions typically used in the ASL to determine the mechanical shear production of TKE as outlined above, which were employed in Louka et al. (2000) and Christen et al. (2004), are unlikely to capture all of the contributions and may in fact be omitting the dominant contributions to the production term in the UCSL.

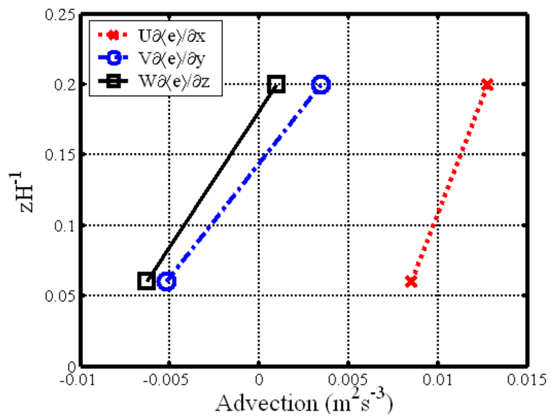


**Figure 4** Average of each of the TKE advection terms on the DSTL and UU towers during IOP10.

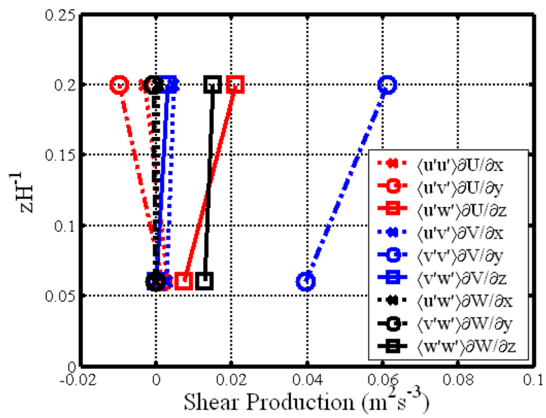


**Figure 5** Average of each of the mechanical shear TKE production terms on the DSTL and UU towers during IOP10.

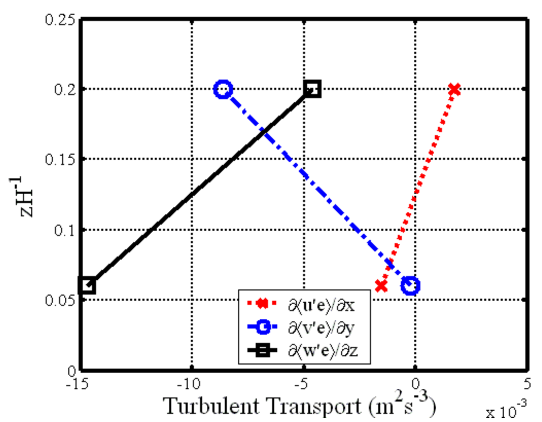


**Figure 6** Average of each of the turbulent transport of TKE terms on the DSTL and UU towers during IOP10.

In the ISL, the bulk turbulent transport of TKE is simplified through similar assumptions to those used to simplify the bulk mechanical shear production term. Employing the typical ISL assumptions and rotating into the mean wind leaves the vertical turbulent transport of TKE as the only significant term. The individual turbulent transport terms shown in Fig. 6 indicate that while the vertical term is significant at both levels in the Park Avenue street canyon, it only dominates the bulk term at the lower level. The along-canyon component does not make much of a contribution at either level but cross-canyon dominates the transport at the upper level. Thus Fig. 6 also suggests that employing the typical ISL simplifying assumptions to the UCSL will not capture all of the significant turbulent transport terms in the TKE budget.

### 4.2 *Momentum Budget*

The mean momentum along-canyon, cross-canyon, and vertical momentum budgets within the Park Avenue street canyon are presented in Figs. 7-9, respectively. Similar to what was found for the TKE budget, the storage terms do not make significant contributions to any of the momentum budgets. The along-canyon (Fig. 7) and cross-canyon (Fig. 8) momentum budgets show that, with the exception of the lower-level cross-canyon budget that simplifies to a balance between advection and pressure forces, the horizontal momentum budgets cannot be reduced to simple balances of two physical processes within the UCSL.

The gravitational body force term has been omitted from the vertical momentum balance in Fig. 9. This was done to facilitate the analysis of the other terms, since the gravitational body force term is a constant 9.8 $ms^{-2}$, making it at least two orders of magnitude larger than the other terms. Thus, the inclusion of the gravitational term would serve only to show that the balance of the pressure gradient and gravitational body force, i.e. the hydrostatic condition, dominates the vertical momentum budget. As presented here, Fig. 9 represents the deviations from the hydrostatic condition. While these deviations are small, within ±5% of the hydrostatic condition, they act to enhance the vertical motions in the UCSL and are thus still significant. It can be seen that the deviations from the hydrostatic condition in the UCSL appear to be due to the turbulent transport of vertical momentum.

## 5. CONCLUSIONS

TKE and momentum budget analyses were performed at the base of a street canyon located in the CBD of a city devoid of major surrounding topographical features using only the instruments that could be paired with other instruments to allow for the approximation of gradients in all three principal directions.

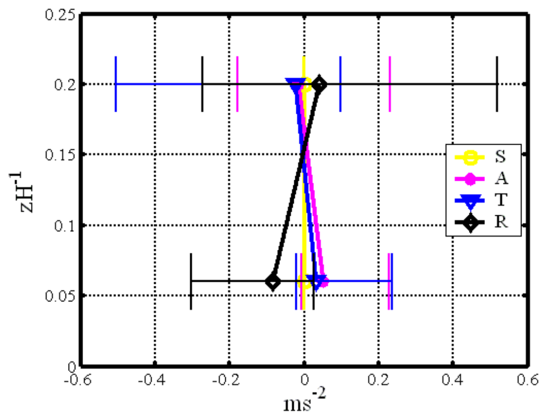


**Figure 7** Along-canyon (*U*) momentum budget on the DSTL and UU towers during IOP 10. Error bars denote the extreme one-hour average values for each term.

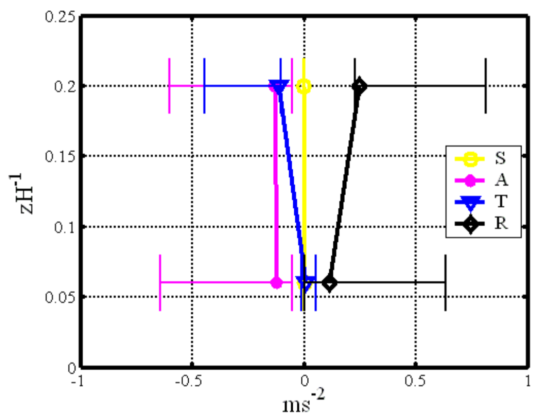


**Figure 8** Cross-canyon (*V*) momentum budget on the DSTL and UU towers during IOP 10. Error bars denote the extreme one-hour average values for each term.

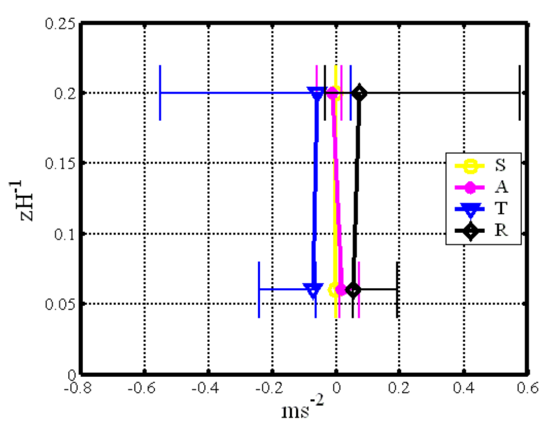


**Figure 9** Vertical (*W*) momentum budget on the DSTL and UU towers during IOP 10. Error bars denote the extreme one-hour average values for each term. Note that the gravitational body force has not been included in the budget.

The buoyancy term was found to have negligible contributions to the TKE budget over the 8 hours that were examined in this work. In addition, on the one-hour time scales considered here, both the momentum and TKE budgets could be considered in a quasi-steady state.

While the method for obtaining the gradients between instruments was admittedly very crude due to the horizontal spacing of instrument towers in the canyon, it showed that the application of typical ISL simplifying assumptions and procedures, which were employed in previous UCSL and URSL TKE budget analyses (Louka et al. 2000 and Christen et al. 2004), will not capture all of the significant contributions to the budget. In some cases the horizontal gradients were found to be more significant than the vertical gradients.

Pressure gradients, turbulent transport, and advection were all found to make significant contributions to the horizontal momentum budgets. The vertical momentum budget showed that the deviations from the hydrostatic approximation within the UCSL are within ±5%. These deviations from the hydrostatic condition were found to be due to the turbulent transport of vertical momentum.

These analyses show that capturing all of significant contributions to the TKE and momentum budgets within the UCSL requires a high density of measurements configured to facilitate the calculation of gradients in all three principal directions.

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