P4.2

THE FEEDBACK BETWEEN ENTRAINMENT FLUX AND SEA SURFACE TEMPERATURE

Jeremiah Brown *, Carol Anne Clayson
Florida State University, Tallahassee, Florida

1. INTRODUCTION

Feedback systems have been identified as a primary research focus for climate modeling. The bulk of the literature focuses on the need for improved understanding of cloud feedback, with most cloud feedback studies focusing on the effect of clouds on the top of the atmosphere (TOA) fluxes (Stephens 2005). Cloud feedback models are also used in general circulation models (GCMs) and between simple component processes contained in more complex models (Committee 2005). More generally, Zhang attributes model uncertainties in GCMs to cloud-climate feedback (Zhang 2004). Held notes the need for continued study of cloud feedbacks to enhance modeling and improve study of climate sensitivity (Held 2005). Two recent publications discuss at length the emergent role of feedback in climate modeling and the need for improved understanding of feedback systems (Stephens 2005, Committee 2005). Stephens even calls for alternative, new methods of feedback analysis that incorporate observations (Stephens 2005).

Although the importance of feedback in climate modeling is identified, the literature encompasses only limited attempts at feedback analysis comprised primarily of sensitivity studies. Curry and Webster, and Zhang point to control systems for treatment of feedbacks; yet, both sources abandon the complexities allowed in controller design in favor of simplified single-input/single-output (SISO) feedback systems with a “feedback factor” or gain block in the feedback loop (Curry and Webster 1999). Zhang references similar approaches by other researchers with the net result the same in each instance: the feedback factor is either a positive or negative gain (Zhang 2004).

The reverse controller design methodology presented in this paper relies on classical controller design techniques as used in mechanical engineering (Ogata 1997). This new method deviates from classical controller design in that the authors seek to estimate the feedback system from observational data rather than prescribed performance objectives as is the norm in controller design. It is believed by the authors that no other attempt to incorporate the signals and systems frequency domain design approach exists in the relevant literature. A more concerted use of control system analysis provides tools useful in characterizing the feedback as a dynamic system, rather than a simple gain. In addition to more descriptive feedback relationships, control system analysis also includes provisions to estimate climate systems using complicated system models with loop interactions and multiple feedbacks, or multiple-input/multiple-output (MIMO) systems.

The work presented in this paper will be applied to the problem of estimating the feedback between the entrainment flux and the sea surface temperature through the use of the reverse controller design method. Though the observed system is far more complicated than the SISO system suggested herein, the authors believe that the simplified use of this problem will serve as a point-of-entrance for future analysis of increasingly complex feedback design.

* *Corresponding author address:* Jeremiah Brown, Florida State University, Geophysical Fluid Dynamics Institute, Tallahassee, FL, 32306-4360; email: JLB04ac@fsu.edu

2. REVERSE CONTROLLER DESIGN

Though feedback analysis is recognized as a significant but under-studied area of climate modeling, there is yet to be research attempting to estimate the feedback using classical controller design techniques as presented in mechanical controls engineering (Ogata 1997). Beyond inclusion of a feedback system consisting solely of a feedback factor, no work has been found that attempts to identify the actual feedback dynamics from the frequency response of the observed system. The method proposed herein attempts to estimate the feedback system by comparing the frequency response of the observed system with that of a comparable model in a three step process: 1) Estimate the Open Loop Frequency Response, 2) Estimate the Closed Loop Frequency Response, and 3) Design the Feedback Controller. These three steps compose the reverse controller design methodology and require data from the observed system, as well as that of a representative computer model.

2.1 Open Loop Frequency Response Estimate

The first step in determining the feedback system is to estimate the frequency response of the open loop (OL) system. The data to be used to estimate the OL frequency response is taken from a computer model comparable to the system to be observed. Figure 1 shows a typical OL system, where P is the plant (or computer model in this instance), $u(t)$ is the input to P, and $y(t)$ is the output of P. The OL frequency response of the plant can be estimated from time histories using any of a number of methods, including methods that use only the output $y(t)$.

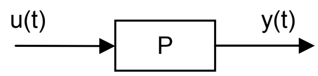

Figure 1. OL System

2.2 Closed Loop Frequency Response Estimate

After estimating the OL frequency response, the next step in determining the feedback system is to estimate the closed loop (CL) frequency response. This estimation is accomplished using observed data. There is an implicit assumption that the feedback system of interest is included in the observed system and absent from the computer model. However, the CL system is assumed to include the OL plant, P, as well as the CL feedback. Figure 3 shows a typical CL feedback system, where C is the controller providing the feedback signal and P is the OL plant as before. Henceforth, the controller C refers to the feedback block in Figure 3 containing the dynamics effecting the output y(t). As was done for the OL system, the time histories are again used to estimate the CL frequency response.

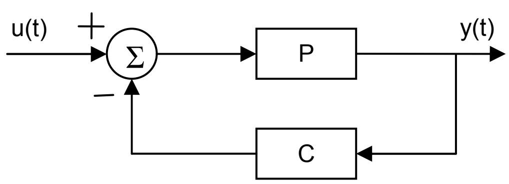

Figure 2. CL Feedback System

2.3 Feedback Controller Design

Once the model and observed data sets have been associated with the open loop and closed loop systems, respectively, the final step is to design the controller, C, responsible for the feedback. Employing a frequency-based controller design strategy, C is designed as the feedback dynamics required to shape the OL frequency response to match the frequency response of the CL system. The design is achieved by assuming, *a priori*, that the CL system has the feedback structure shown in Figure 2. For climate systems, this appears to be the most reasonable configuration.

3. EXAMPLE

As a simple demonstration of the concept, an OL frequency response is estimated in Figure 3. Assuming the data is a result of an appropriate computer model, the frequency response is that of the plant, P. Figure 3 is a bode plot, detailing the estimation of both the magnitude and phase of the frequency response plotted for each frequency.

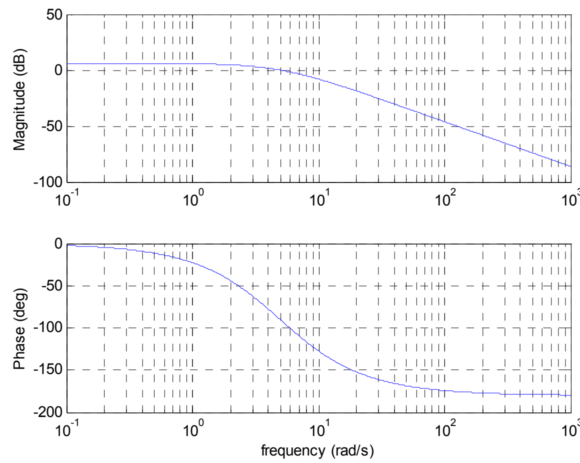

Figure 3. OL Frequency Response

Next, observation data is used to estimate the CL frequency response shown in the bode plot of Figure 4. Comparing the response for the OL and CL systems, it is apparent that the CL system contains dynamics not present in the OL system.

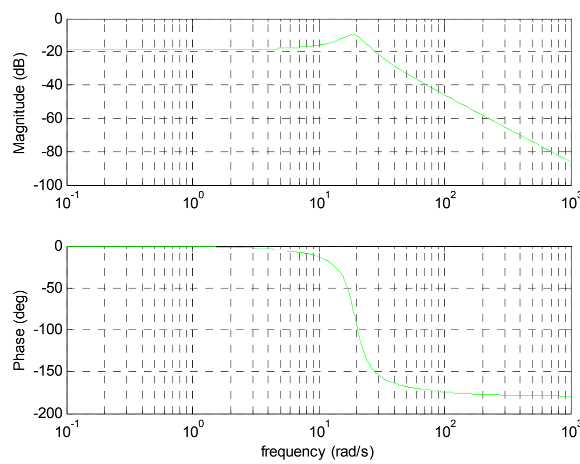

Figure 4. CL Frequency Response

Plotting both the OL and CL frequency response on the same bode plot allows for a direct comparison of magnitude and phase at each frequency. The assertion is that the difference between the two responses (Figure 5) is caused by the dynamics of the controller, C, in the feedback loop.

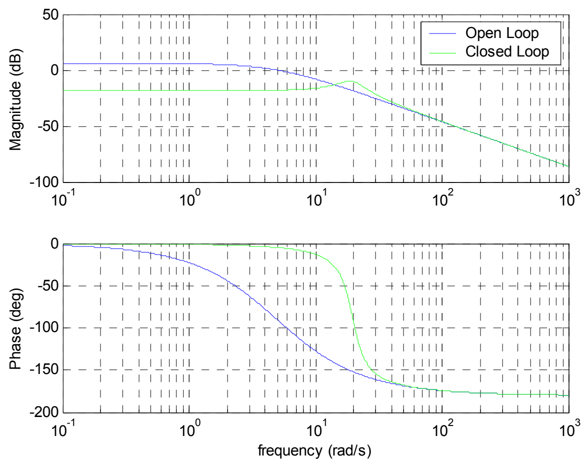

Figure 5. OL and CL Frequency Response

The final step in the reverse controller design process is to design the controller, C, that will yield the estimated CL response (Figure 4), using the OL response (Figure 3) and the structure in Figure 2. Classical controller design typically occurs on a Nichols Chart whereby a series of lead and lag compensators are enlisted to shape the OL response. The Nichols Chart, or gain-phase plane, combines the information from the bode plot into a single space; thus, the controller design process is simplified because of the reduction in the number of design parameters from two with the bode plot, to one with the Nichols Chart. Figure 6 shows the OL and CL frequency responses on the Nichols Chart.

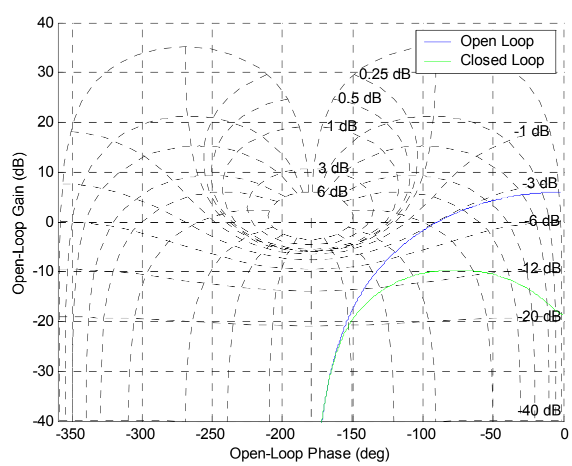

Figure 6. OL and CL Frequency Response in the Gain-Phase Plane

Once the controller design is completed, the dynamics of the controller can be evaluated. Figure 7 shows a bode plot of the controller designed for this example. Examining Figure 7, the controller amplifies the incoming signal 18 dB until 10 rad/s. The controller begins to amplify at a different rate, 16 dB, after 100 rad/s, with a modest phase lag associated with the change in amplification.

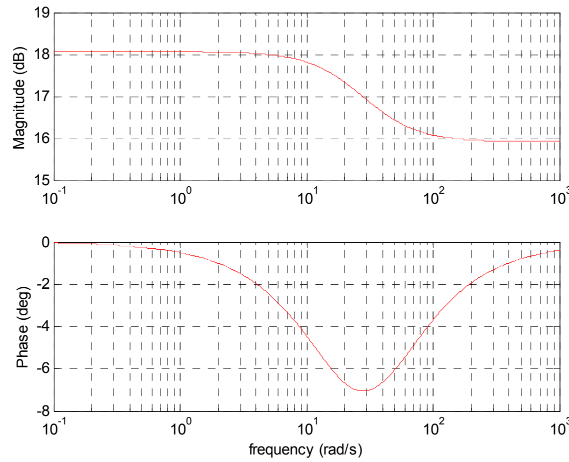

Figure 7. Controller Frequency Response

4. APPLICATION FOR ENTRAINMENT FLUX

The controller design technique discussed in Section 3 works quite well for linear mechanical, SISO systems. The difficulty in applying such a process adapted from another area comes in manipulating the dynamic system in question to achieve a form for which the reverse controller design technique can be applied. As a demonstration of the concept, the newly described process presented in this paper will be applied to estimate the feedback between entrainment flux and the buoyancy flux, which is a function of the sea surface temperature. From Curry and Webster, the entrained heat flux, F_Q^{ent}, is defined by

$$F_Q^{ent} = \rho c_p u_e \Delta T \tag{1}$$

where ρ is the density, c_p is the specific heat, ΔT is the change in temperature across the base of the mixed layer, and u_e is the entrainment velocity. The entrainment velocity is then given by

$$u_e = \frac{c_1 u_*^3 - c_2 F_B h_m}{h_m\left(\alpha g \Delta T - \beta g \Delta S\right)} \tag{2}$$

where c_1 and c_2 are constants, u_* is the friction velocity, F_B is the buoyancy flux, h_m is the depth of the ocean mixed layer, α is the thermal expansion coefficient, and β is the salinity expansion coefficient.

The relationship between the entrainment flux and the buoyancy flux is then realized by combining (1) and (2). Simplification of the resulting equation gives the following relationship

$$F_Q^{ent} = D - PF_B \tag{3}$$

where D is the signal

$$D = \frac{c_1 u_*^3}{h_m\left(\alpha g \Delta T - \beta g \Delta S\right)} \quad (4)$$

and P is the signal

$$P = \frac{c_2 h_m}{h_m\left(\alpha g \Delta T - \beta g \Delta S\right)} \quad (5)$$

The system in (3) can then be written as a block diagram for the OL system as was done in Figure 1. For the OL system in Figure 8, P is the system plant and D appears as a disturbance to the system. In accordance with the procedure outlined in Section 3, (3) and subsequently Figure 8 are the OL system, or the computer model.

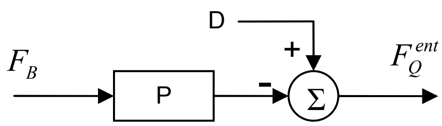

Figure 8. OL System for Entrainment Flux

Following the procedure outlined in Section 3, the observed system is presumed to include a feedback, yielding the structure in Figure 9. The reverse controller design technique described herein is then used to estimate the controller, C, in Figure 9 that will shape the OL system response to be that of the CL system response.

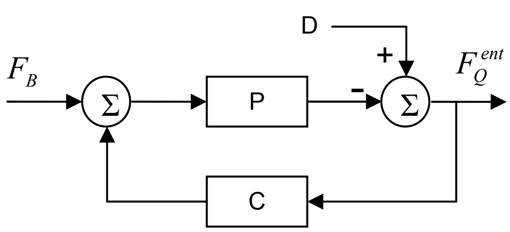

Figure 9. CL System for Entrainment Flux

5. REFERENCES

Committee on Challenges in Representing Physical Processes in Coupled Atmosphere-Land-Ocean Models, National Research Council, 2005: Improving the scientific foundations for atmosphere-land-ocean simulations: report of a workshop, 84 pp. [Available online at http://www.nap.edu/catalog/11266.html].

Curry, J.A., and P.J. Webster, 1999: *Thermodynamics of Atmospheres and Oceans*. Academic Press, 471 pp.

Held, I.M., 2005: The gap between simulation and understanding in climate modeling. *Bull. Amer. Meteor. Soc.*, **86**, in press.

Ogata, K., 1997: *Modern Control Engineering*. 3rd ed. Prentice-Hall, Inc., 997 pp.

Stephens, G. L., 2005: Cloud feedbacks in the climate system: a critical review. *J. Climate*, **18**, 237-273.

Zhang, M., 2004: Cloud-climate feedback: how much do we know? *Observation, Theory and Modeling of Atmospheric Variability*, River Edge, NJ, World Scientific Co. Pte. Ltd., 161-183.