4B.1 TROPICAL CYCLONE RESPONSE TO TIME DEPENDANT, AXIALLY SYMMETRIC HEATING

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1. INTRODUCTION

The purpose here is to study transient convective heating in a realistic Tropical Cyclone (TC) -like vortices. The strategy is comparison between solutions obtained using the classical Sawyer-Eliassen Equation (SEE) (Smith 1981, Scubert and Hack 1982, Shapiro and Willoughby 1982) for axially symmetric, temporally steady heating and linear perturbations induced by axially symmetric heating that varies sinusoidally in time. The mean flow is a realistic, baroclinic vortex in hydrostatic and gradient balance (Fig. 1). The governing equation for the second case is the Azimuthal component of the Vorticity Equation (AVE). Both yield the mass-flow streamfunction in the radius-height plane. In contrast with the analogous problem where the forcing is an initially imposed temperature anomaly (Nolan and Montgomery 2002, Nolan et al. 2003), this approach allows clear separation of slow- and fast- manifold solutions.

2. FORMULATION

The two governing equations are closely analogous. Their principal differences lie in the coefficients of the second partial derivatives of the streamfunction with respect to radius and height. In the SEE, these coefficients are, respectively, the squares of the buoyancy frequency (N^2) and of the local inertia frequency (I^2); in the AVE they are $N^2-\omega^2$ and $I^2-\omega^2$, the differences between the corresponding SEE terms and ω^2, the square of the frequency with which the heating varies. The coefficient of the mixed partial with respect to height and radius is the baroclinic term $2B^2$. It is the same in both equations.

For low frequencies, where the discriminant of the AVE (the product of the coefficients of the unmixed partials minus the square of the coefficient of the mixed partial) is positive, the AVE is an elliptic partial differential equation yielding Poisson-like solutions in response to imposed heating. When the discriminant is negative, the AVE is hyperbolic, and the heating excites inertia-buoyancy (IB) wave solutions. Shear widens the IB passband such that the approximate frequency range over which the AVE is hyperbolic is $I^2 - B^4/N^2 \le \omega^2 \le N^2 + B^4/N^2$. Generally, the inertia frequency varies from large values (period < 1/2 h) in the eye and eyewall

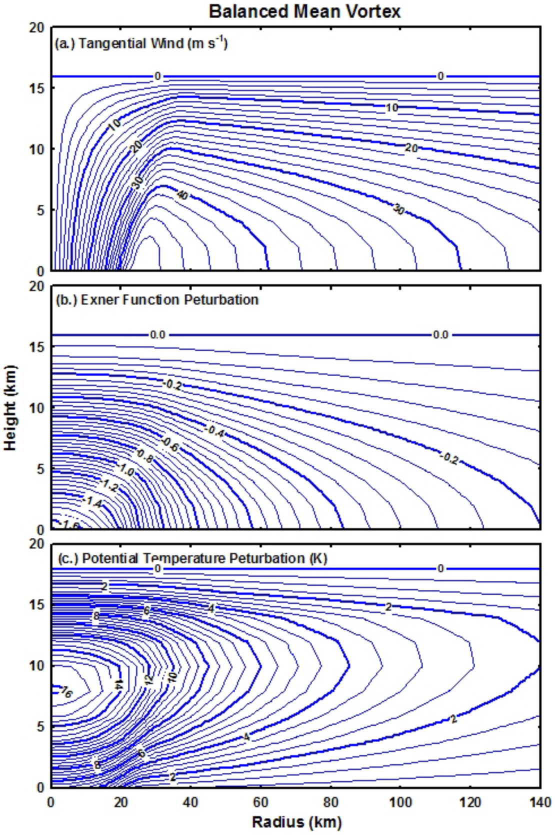

Figure 1. (a.) Tangential (swirling) velocity component (m s^{-1}). (b.) Gradient-balance Exner function minus its environmental vertical profile. (c.) Hydrostatic balance potential temperature (K) minus its environmental profile. The maximum swirling wind is 50 m s^{-1} at the surface and 25 km radius.

**Corresponding author address:* Hugh E. Willoughby, Dept of Earth Sciences, Florida International University, Miami, FL, 33199, e-mail hugh.willoughby@ fiu.edu to the Coriolis frequency far from the vortex center. Thus, the sign of the discriminant is often positive in the vortex core and negative at large radius.

3. RESULTS

As a first approximation, TC intensification can be modeled in terms of the vortex response to an axially symmetric imposed heat source that induces a secondary radial-vertical circulation. Since the primary swirling motion is much stronger than the secondary circulation, the response is quasi-linear or actually linear. The realistic basic vortex used here is a statistical fit of a piecewise continuous profile to aircraft observations of actual TCs obtained between 1977 and 2000 (Willoughby et al 2006).

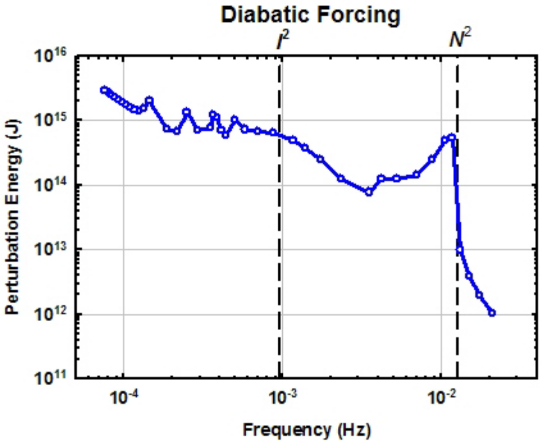

Figure 2. Domain-integrated perturbation energy (J) as a function of frequency forced by a sinusoidally varying heat source centered at 50 km radius and 6 km altitude. The heat amplitude of the source, Q, is scaled such that $Q/N^2 = 1$ m s^{-1}.

The heat source may either be steady or vary sinusoidally with time. For steady heating, the SEE yields the gradual evolution of the mean vortex. In the case of unsteady heating, when the forcing frequency is substantially below the local inertia frequency, the AVE is elliptic and the solutions approach the solutions of the SEE (Fig. 3). At higher frequencies, the AVE is hyperbolic and solutions are wavelike (Fig.4). These solutions are generally IB waves, with relatively high frequencies (periods reckoned in tens of minutes) relative to the Coriolis frequency, so that their group velocity has a substantial vertical component. Thus, most of the perturbation energy supplied by the heat source is carried away into the stratosphere. Eddy flux convergences due to the waves are small. The streamfunction lags a few degrees behind the diabatic forcing. The perturbation buoyancy is small, equivalent to 0.1-0.3 K, consistent with recently published aircraft measurements (Eastin et al 2005a, b).

The local inertia frequency is dominated by the relative rotation in the vortex core where the Rossby number is >> 1. Since it decreases outward, approaching the Coriolis frequency far from the center, there is often a neighborhood in the vortex core where the AVE is elliptic, surrounded by the rest of the domain where the AVE is hyperbolic. Only when the forcing frequency is below the Coriolis frequency, is the AVE elliptical throughout the domain. In the elliptical neighborhood, the streamfunction is like that simulated with the SEE, but more extended radially because time dependence increases the effective Rossby radius of deformation. The vertical and radial perturbation velocities are in phase with the diabatic forcing. The perturbation tangential velocity and buoyancy lag the heating by 90°, so that they reach their largest values at the end of the heating phase, as one would expect for SEE-like solutions.

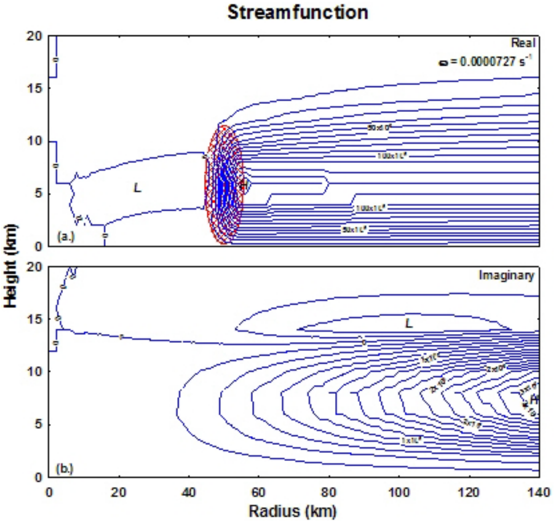

Figure 3. Real (a) and imaginary (b) parts of the mass-flow streamfunction (blue) induced by the same heat source as in Fig. 2 (red). The frequency, 0.0000727 Hz, corresponds to a period of 24 h.

Outside the neighborhood where the AVE is elliptic, the solutions are IB waves that match onto the balanced secondary circulation solutions along the boundary where the AVE is parabolic (ie the locus of zero discriminant). Since these waves have relatively low frequencies, they propagate more nearly horizontally than those described above.

These results are important for assessing the role of transient convection in hurricane intensification (e.g. Heymsfield et al 2001). Net convective heating is generally the sum of a steady (lasting a time longer the local inertia period) component and at least one Fourier component. In a sequence of convective outbursts, only the steady heating changes the vortex intensity permanently. Low-frequency sinusoidal components produce SEE-like additional intensification by the end of their heating phases and mirror image weakening at the end of their cooling phases. High-frequency components excite IB waves that propagate out of the vortex with little effect on intensity. The reported association of asymmetric convective outbursts with the transitions from weakening to intensification or intensification to weakening may stem from the existence of an optimum amount of shear through which the vortex passes on its way either from shear-limited intensity to intensification or vice-versa in increasing or decreasing shear, respectively. Follow-on work with asymmetric, rotating heat sources should cast some light on this process.

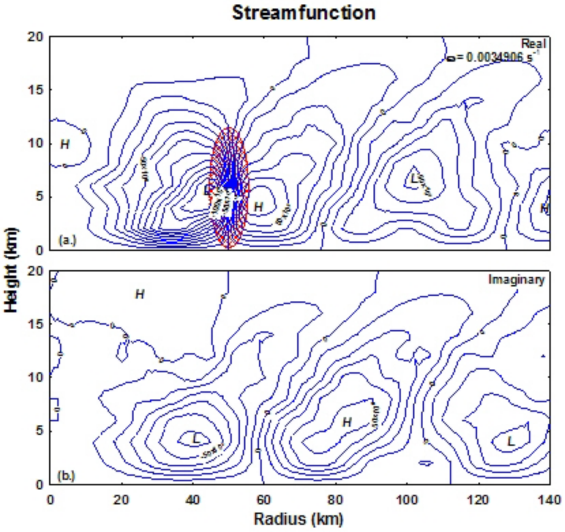

Figure 4. As in Fig. 3, but for frequency 0.0034906 Hz, equivalent to a period of 30 min.

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