8D.1 HIGH RESOLUTION NUMERICAL MODELLING OF DEEP MOIST CONVECTION IN STATISTICAL EQUILIBRIUM: BUOYANCY AND VELOCITY SCALES

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1 INTRODUCTION

Buoyancy and velocity scales for dry convection in statistical equilibrium were derived long ago by Prandtl (1910, 1925), but the question of convective velocity and buoyancy scales (Robe et al. 1996, Robe et al. 2001), as well as the topic of fractional area coverage of convective clouds, are still unresolved in moist convection. In this study, high resolution simulations of an atmosphere in radiative convective equilibrium are performed using the Lokal Model, a three-dimensional, nonhydrostatic, convection-resolving, limited area model. Prescribing different constant cooling rates to the system, we characterize the velocity and buoyancy scales for moist convection in statistical equilibrium. The influence of domain size on radiative convective equilibrium statistics is assessed. The dependence of fine-scale spatio-temporal properties of convective structures on numerical and physical details such as horizontal grid spacing and microphysics is investigated. As a further assessment of the reliability of the results and of the proposed scalings, we are currently performing similar simulations using the WRF model.

2 NUMERICAL EXPERIMENTS

The study of buoyancy and velocity scales is addressed in this work by performing high resolution numerical simulations of deep moist convection in statistical equilibrium over a horizontally uniform water surface. In this way, the convection dynamics can be resolved explicitly and we can bring to bear a detailed representation of cloud microphysics processes. We employ version 3.9 of the Lokal Model (LM), which is a fully compressible and nonhydrostatic limited area model. The Lokal model, created in 1998 by the DWD (Germany) and developed continuously since, is used for operational and research purposes in the context of the COSMO consortium. For a comprehensive description of this model, the reader is referred to Steppeler et al. (2003).

The model in this work is run on a doubly periodic domain of different sizes (L=50, 100, 200 and 400 km) and having an height of 20 km. Uniform horizontal spatial resolution of 2 km is adopted here, but we plan to use even higher resolution (1 km and 500 m). The vertical grid spacing stretches gradually from 40 m near the bottom boundary to 500 m near the top one. The microphysics processes are parameterized according to the scheme of Kessler (1969), with some modifications according to the underlying assumptions of this work, discussed in the next paragraph. The lower boundary is a passive ocean with constant temperature $T_S=300$ K. The subgrid-scale turbulence is represented using a level 1.5, one-dimensional, moist turbulence parameterization, while horizontal mixing relies on the use of a 4th order linear horizontal diffusion formulation. The use of a 1D turbulence parameterization scheme at these fine resolutions (<1-2 km) might be questionable, thus we are presently undertaking further work in order to assess the reliability on the present results by means of the WRF model, which provides both three-dimensional (1.5
order) TKE turbulence and Smagorinsky closures.

In order to partially account for the effect of a mean wind, which is absent in our model, and also to accelerate the approach to moist statistical equilibrium, a minimum wind speed of 5 m/s is added to the actual wind speed for the computation of moisture and heat fluxes, using a constant roughness length $z_0=0.001$ m.

A constant cooling rate $Q_{\text{rad}}$ is applied from the surface to 15 km, while above 15 km a sponge layer which relaxes back to the initial temperature profile is adopted. The moist convection scaling has been studied for three different cooling rates, $Q_{\text{rad}}$ (-2 K/day, -4 K/day and -6 K/day), but here we here present the results only for $Q_{\text{rad}} =-4$ K/day.

3 RESULTS

The underlying hypothesis of the work is that convective updraft velocities and rainfall intensity scale with the terminal velocity of raindrops, $V_T$. To test this hypothesis, the original Kessler microphysics scheme has been modified in order to allow for fixing a constant value of $V_T$ independent of the raindrop size distribution. The evaporation and accretion rates have been reformulated according to this hypothesis.

Then, for prescribed cooling rate $Q_{\text{rad}}$, a set of simulations with different raindrop terminal velocities ($2 < V_T < 50$ m/s) has been performed and the results are compared. The expected value of raindrop terminal velocity is around 5 m/s but the explored range of $V_T$ has been extended considerably in order to test the proposed scaling hypothesis and to explore the properties of convective dynamics with respect to different values of this key microphysical variable.

The first part of this study was devoted to an evaluation of the effect of computational domain size on the statistical properties of moist convection in statistical equilibrium. In particular, we show here, for $Q_{\text{rad}} =-4$ K/day and $V_T =5$ m/s, that some simple statistical properties of the flow field (eg: mean value of maximum vertical and horizontal velocities) converge in a statistical sense when the domain size increases from $L=25$ km to $L=400$ km (figure 1). Similar results are valid for other values of $V_T$.

This finding is important in considering the typical domain sizes of around 50 – 100 km used in previous research on radiative-convective equilibrium, since it suggests a strong underestimation of the intensity of velocity field in the core of the convective cells when $L$ is too low. The domain size also seems to affect (not shown here) the number of convective cells generated in the flow field.

In this work, as a good compromise between computational expense and statistical reliability of the results, a reference domain size $L$ equal to 200 km has been assumed.

![Figure 1. Dependence of the mean value of the maximum vertical velocity on the computational domain size.](image)

Since the main goal of this study is to address some basic scaling issues in moist radiative-convective equilibrium, a simple algorithm for the extraction of the rainfall intensity of convective cells from each spatial frame field has been developed on the basis of the following procedure (von Hardenberg et al. 2003): a local maximum, in the rainfall intensity field $R$, is defined as a pixel that is larger than any of its 20 nearest neighbors. Each maximum represents a center of a cell. Around each cell center, progressively lower contour levels are traced and the horizontal extent of each rain cell is determined by identifying the connected region around each maximum which has intensity larger than a chosen level.

In this work, we retain only those maxima in $R$ that exceed the threshold of 30 mm/h. The horizontal extent of each rain cell is then determined by identifying the connected region around the maximum that has $R$ larger than 0.5 mm/h. For each cell the spatial average of rainfall intensity over this connected region is
computed. The same connected region (determined in the R field) is used as a mask to determine the spatial average of the vertical velocity field, at z=5000, in the convective core of each cell.

The ensemble mean and standard deviation of the rainfall intensity and vertical velocity are computed during a period of several days in radiative convective equilibrium. Then the scaling of these simple statistical properties against the raindrop terminal velocity $V_T$ is analyzed.

Both rainfall intensity and vertical velocity (mean and standard deviation) exhibit a strong and clear dependence on the value of the raindrop terminal velocity. When $V_T$ grows from a value of 2 m/s to about 20 m/s these two cell statistics increases considerably, while for higher values of $V_T$ an asymptoting of the scaling begins (figure 2).

The intensification of the rain intensity, when $V_T$ increases, is confirmed, from a qualitative point of view, comparing two instantaneous frames of the rainfall intensity and vertical velocity fields for $V_T = 2$ m/s and 20 m/s respectively (figures 3 and 4). When the raindrop terminal velocity is higher, the convective cells turn out to be stronger and smaller in size, in good agreement with previous work (Grabowski 2003, Wu 2002).

In the next section, a physical argument that might support the observed scaling behaviour is presented.

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**SCALING THEORY**

The buoyancy of an air parcel lifted to some level can be broken into two parts: a part owing to temperature excess and a part owing to liquid water loading:

\[
B = g \frac{\alpha'}{\overline{\alpha}} - gl'
\]  

(1)

where $\alpha$ is specific volume, $l$ is the liquid water concentration, $g$ is the acceleration of gravity, and the primes denote deviations (at constant pressure) from the base state, represented by an overbar.

Expressing $\alpha$ as a function of pressure and saturation moist entropy, $s'$, and using one of Maxwell’s relations, the above expression can be written:

\[
B = \Gamma s'' - gl'
\]  

(2)
where $\Gamma$ is the moist adiabatic lapse rate. We suppose that fluctuations of saturation entropy in the cloud are directly related to fluctuations of actual entropy in the boundary layer, allowing us to re-write (2) as:

$$B = \Gamma s' - gl'$$  \hspace{1cm} (3)

where $s'$ is a typical value of the boundary layer entropy fluctuation. If we integrate this altitude, assuming from energy balance, that

$$w^2 \sim \int Bdz$$  \hspace{1cm} (4)

where $w$ is characteristic vertical velocity in the cloud, then we have:

$$w^2 = \Delta Ts' - ghl'$$  \hspace{1cm} (5)

scale height for liquid water concentration. Now to estimate $l'$, we postulate a local balance between the fall of rain water and the cloud and the upward transport of water vapor, so that water substance does not accumulate in any layer:

$$\left( V_T - w \right) l' = wq'$$  \hspace{1cm} (6)

where $V_T$ is the raindrop terminal velocity and $q'$ is a characteristic difference between specific humidity in the cloud and outside the cloud. Eliminating $l'$ between (5) and (6) gives:

$$w^2 \sim \Delta Ts' - ghq' \frac{w}{V_T - w}$$  \hspace{1cm} (7)

If we scale $w$ using:

$$w^2 \to \Delta Ts'w^2$$  \hspace{1cm} (8)

Using this expression and re-arranging (7) gives a cubic equation for $w$:

$$w^3 - Vw^2 - \left( 1 + \chi \right) w + V = 0$$  \hspace{1cm} (9)

where the nondimensional coefficients are given by:

$$V \equiv \frac{V_T}{\sqrt{\Delta Ts'}}$$

$$\chi \equiv \frac{ghq'}{\Delta Ts'}$$

Then (9) has three roots: one is negative and one has $w > V_T$, which is unphysical as it gives negative liquid water. The third root is physical. The graph below show $w$ as a function of $V$ for this root when $\chi$=0.1.

Figure 5. $w$ as a function of $V$ for the physical solution of equation (9) for $\chi$=0.1

We are currently testing this theory for buoyancy and velocity scales, which explains also the asymptoting of the vertical velocity shown before.

The results are promising and support the underlying premise of this work which identifies the raindrop terminal velocity as the physical parameter which governs the intensity of deep moist convection in statistical equilibrium.

5 CONCLUSIONS

In this work buoyancy and velocity scales for moist convection in statistical equilibrium are studied.

We find that convective updraft velocity and rainfall intensity scale with the raindrop terminal velocity. This work is in progress and the results deserve further investigation. We are currently repeating the same analysis with the WRF model.
in order to further assess the reliability of the results and of the proposed scalings. Moreover, we are also testing the theory proposed for this scaling and the results look very promising.

6 REFERENCES


