

THE INTENSIFICATION OF CYCLONES FROM ASYMMETRIC HEATING REVISITED:  
ENERGETICS AND WEAKLY NONLINEAR EFFECTS

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## 1. INTRODUCTION

The intensification of tropical cyclones is driven in large portion by the latent heating released in cumulus convective clouds around the center of the cyclone. While the structure of mature tropical cyclones can be quite axisymmetric, the arrangement of heating sources around the center of the storm is typically asymmetric.

In previous studies (Nolan and Montgomery 2002, hereafter NM02; and Nolan and Grasso 2003, hereafter NG03), the evolution and dynamics of three-dimensional, nonhydrostatic, asymmetric temperature perturbations and the response of tropical-cyclone-like vortices to these perturbations were examined with a linear model. It was found that heat energy released in asymmetric perturbations was converted into kinetic energy of a tropical-cyclone-like vortex through a series of asymmetric and symmetric adjustment processes. When an unbalanced, asymmetric thermal perturbation is introduced, asymmetric gravity waves and asymmetric vorticity anomalies are generated along with the adjustment to hydrostatic and gradient wind balances. Then these quasi-balanced vorticity perturbations are shared by the basic-state flow, producing heat and momentum fluxes as they are axisymmetrized. The parent vortex is perturbed in response to these fluxes, and after generating additional gravity waves, the axisymmetric vortex becomes adjusted to a new balanced state, resulting in intensity and structure change.

One conclusion of NM02 and NG03 was that the net effect of the instantaneous, purely asymmetric perturbation is to decrease the intensity of the parent vortex. This work is a continuation of the aforementioned studies. Unlike NM02 and NG03, thermal perturbations in this study are allowed to last over time and rotate around the storm. This would seem a more realistic representation of the heating sources because convection in tropical cyclones lasts over

the order of a few hours and is often observed to move around the center. The goal of this work is to examine if the claims by NM02 and NG03 remain valid. Our approach includes a careful examination of the energy budgets of the asymmetric perturbations and the parent tropical storm vortex.

## 2. A LINEARIZED, NONHYDROSTATIC MODEL

The effects of asymmetric thermal perturbations on tropical-cyclone-like vortices are examined by using a linearized, nonhydrostatic model, 3D Vortex Perturbation Analysis and Simulation (3DVPA), that was developed based on the work of NM02 and NG03. The model is based on the linearization of the dry, anelastic equations of motion in cylindrical coordinates with no friction. The validity of these equations has been further established by a multiple scaling technique as shown by Hodyss and Nolan (2006). The tropical-storm-like vortex used in this study (Fig. 1) is constructed by using a Gaussian vorticity profile which results in maximum tangential velocity and radius of maximum wind that are typical to tropical cyclones.

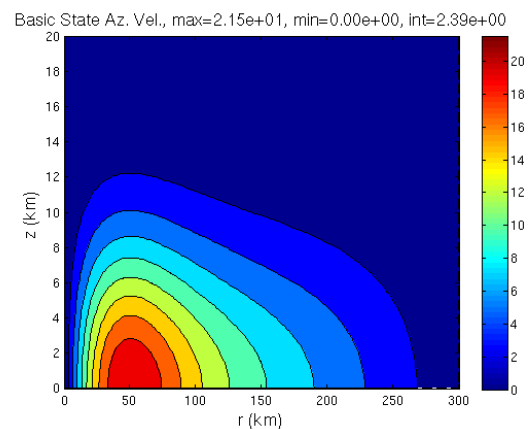


Fig. 1: Velocity profile of a tropical-storm-like vortex. Units are in  $\text{ms}^{-1}$ .

Temperatures and pressure fields are calculated to satisfy hydrostatic and gradient balance, and damping regions are placed along the upper and outer boundaries to suppress the reflection of gravity waves. A secondary circulation is absent in the basic-state vortex. The basic state vortex has a

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maximum wind speed of  $21.5 \text{ ms}^{-1}$  at a radius of 49 km. The asymmetric heat sources are characterized in the form of a Gaussian bubble (Fig. 2) with the radial and vertical half-widths of 20 km and 3 km.

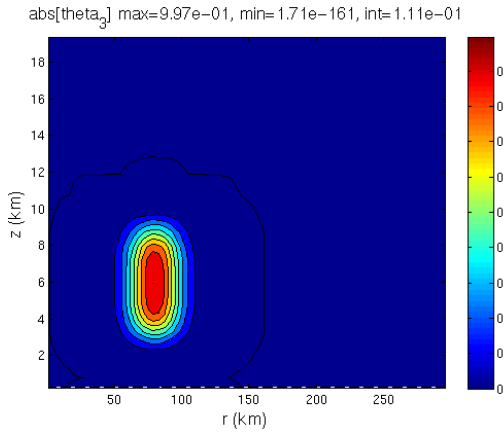


Fig. 2: Temperature perturbation centered at the radius of 80km and the altitude of 6 km. Units are in K.

### 3. ENERGY BUDGET ANALYSIS

#### a. Injecting forcing over time

To examine the effects of injecting the thermal perturbation over time, 1.0 K thermal forcing is introduced over a period of either 1 hour or 2 hours. Perturbation kinetic energy (KE) and available potential energy (APE) were computed as the simulations evolved. Our expression for KE and APE are based on the work of Chagnon and Bannon (2001) and are also validated by Hodyss and Nolan (2006). In this work, all asymmetric perturbations are located at an altitude of 6 km with a wavenumber 3 structure.

Fig. 3 shows the time evolution of asymmetric thermal perturbations and the response of the axisymmetric vortex to these perturbations in terms of their kinetic energy. Calculations (Table 1) show that the total amount of APE injected from the forcing and the maximum KE of the perturbation decreases as the time over which thermal forcing is introduced increases (Fig. 3a). The response of the tropical storm vortex (Fig. 3b) shows that there is a decrease in the net kinetic energy of the tropical storm vortex in the early period, followed by an increase in the later period. However, the magnitudes of extracted KE and returned KE are not the same (Table 2). Introducing the forcing over a longer time period results in a smaller KE decrease. The magnitude of returned KE decreases as the forcing is injected over a longer time.

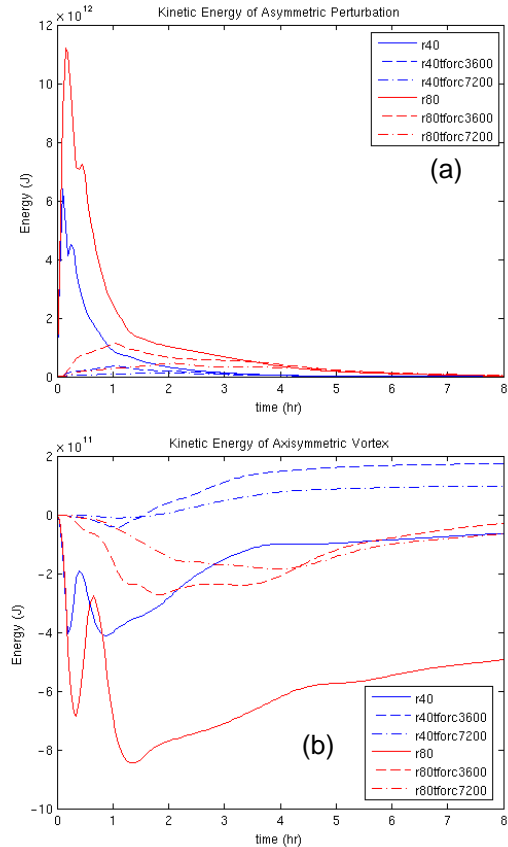


Fig. 3: (a) Kinetic energy of asymmetric perturbations injected at  $r=40\text{km}$  (blue) and  $r=80\text{km}$  (red). 1.0 K thermal forcing is introduced instantaneously (solid), over 1 hour (dash), and over 2 hours (dash-dot). (b) Kinetic energy change of the axisymmetric tropical storm vortex in response to asymmetric thermal forcing. Only the first 8 hours of the simulations are shown.

Since a perturbation grows at the expense of the parent vortex during a period of transient growth, the KE decrease in the early period may be inferred as the amount of KE extracted from the parent vortex and then transferred to the perturbation. As can be seen in Fig. 3a, the transient growth of the perturbation at the radius of 80 km is larger, and this is due to the larger basic-state shear. The axisymmetrization of the perturbation by the basic-state shear may be inferred to be responsible for the KE increase in the later period, as energy is transferred to the axisymmetric vortex through eddy momentum fluxes. Then the "total" energy made available for a perturbation is the total APE injected (Table 1) plus the KE extracted from the parent vortex (Table 2). The ratio of KE returned to this "total" perturbation is calculated in Table 2 and this value decreases as the duration of forcing increases. The difference between these two values may be the amount of

energy lost due to diffusion and gravity wave radiation. Therefore, the overall effect of injecting the forcing over time is to strengthen the storm by returning more energy to the vortex than energy extracted from the vortex in comparison to when the forcing is injected instantaneously. However, it is unclear at this point whether increasing the duration of forcing always has a negative effect on the strength of the parent vortex.

Table 1: Simulations of asymmetric perturbation injected over different time periods.

$r_b$ (km)	Time of forcing (sec)	Total APE injected (J)	Maximum KE of asymmetric perturbation (J)
40	0	$7.91 \times 10^{12}$	$6.39 \times 10^{12}$
40	3600	$4.90 \times 10^{11}$	$3.64 \times 10^{11}$
40	7200	$2.23 \times 10^{11}$	$1.39 \times 10^{11}$
80	0	$1.59 \times 10^{13}$	$1.12 \times 10^{13}$
80	3600	$1.52 \times 10^{12}$	$1.13 \times 10^{12}$
80	7200	$6.41 \times 10^{11}$	$4.61 \times 10^{11}$

Table 2: Simulations of the axisymmetric tropical storm vortex in response to asymmetric perturbations injected over different time periods.

$r_b$ (km)	Time of forcing (sec)	KE extracted (J)	KE returned (J)	$\frac{KE_{returned}}{E_{total}}$
40	0	$4.11 \times 10^{11}$	$3.87 \times 10^{11}$	0.768
40	3600	$4.23 \times 10^{10}$	$2.22 \times 10^{11}$	0.683
40	7200	$1.07 \times 10^{10}$	$1.13 \times 10^{11}$	0.593
80	0	$8.45 \times 10^{11}$	$4.07 \times 10^{11}$	0.670
80	3600	$2.72 \times 10^{11}$	$2.84 \times 10^{11}$	0.630
80	7200	$1.85 \times 10^{11}$	$1.56 \times 10^{11}$	0.558

### b. Rotating heating sources

To examine the effects of rotating the source of heating around the center of the storm, 1.0 K thermal forcing is introduced over a period of either 1 hour or 2 hours, but is allowed to move at different speeds, which are chosen to be 25%, 50%, 75%, and 100% of the surface wind speed at the location of the source. Perturbation KE and APE were computed as the simulations evolved. Again, all perturbations are located at an altitude of 6 km with the wavenumber 3 structure.

Fig. 4 shows the time evolution of asymmetric thermal perturbations (at the radius of 80 km, introduced over 1 hour) and the response of the tropical storm vortex. The surface wind at the radius of 80 km is about  $18.3 \text{ ms}^{-1}$ . Although the total APE injected from the forcing decreases (Table 3) as the rotating speed of the perturbation increases, the maximum magnitude of the asymmetric perturbation increases. The response of the tropical storm vortex (Fig. 4b) shows a similar overall pattern of a decrease in KE in the early period followed by an increase in KE in the

later period. Again, the KE decrease in the early period followed the KE increase in the later period may be understood as the KE extracted from the parent vortex and transferred to the perturbation and the KE returned from the perturbation to the parent vortex during the axisymmetrization process. The ratio of the extracted KE to the returned KE decreases as the rotating speed of the perturbation increases (Table 4). Therefore, the effect of increasing the rotation speed of the thermal forcing is to weaken the parent vortex by extracting more energy than returned later.

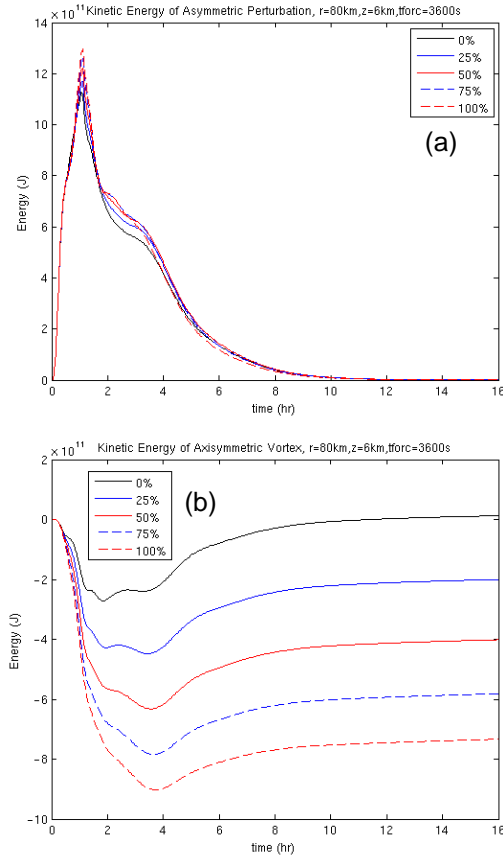


Fig. 4: (a) Kinetic energy of asymmetric perturbations introduced at the radius of 80km. 1.0 K thermal forcing is introduced over 1 hour with different speeds (25%, 50%, 75%, 100% of the surface wind speed at the radius of 80 km. (b) Kinetic energy change of the tropical storm vortex in response to asymmetric thermal forcing.

Table 3: Simulations of asymmetric perturbation injected at the radius of 80 km over 1 hour with different rotating speeds. The surface wind speed at  $r=80\text{km}$  is  $18.3 \text{ ms}^{-1}$ .

Rotating speed as the percent of surface wind	Total APE injected (J)	Maximum KE of asymmetric perturbation (J)
0%	$1.52 \times 10^{12}$	$1.13 \times 10^{12}$
25%	$1.48 \times 10^{12}$	$1.17 \times 10^{12}$
50%	$1.19 \times 10^{12}$	$1.22 \times 10^{12}$
75%	$7.39 \times 10^{11}$	$1.27 \times 10^{12}$
100%	$2.58 \times 10^{11}$	$1.30 \times 10^{12}$

Table 4: Simulations of the tropical storm vortex in response to asymmetric perturbations injected at the radius of 80 km over 1 hour with different rotating speeds. The surface wind speed at  $r=80\text{km}$  is  $18.3\text{ ms}^{-1}$ .

Rotating speed as the percent of surface wind	KE extracted (J)	KE returned (J)	$\frac{KE_{returned}}{KE_{extracted}}$
0%	$2.72 \times 10^{11}$	$2.84 \times 10^{11}$	1.046
25%	$4.48 \times 10^{11}$	$2.48 \times 10^{11}$	0.553
50%	$6.33 \times 10^{11}$	$2.31 \times 10^{11}$	0.366
75%	$7.85 \times 10^{11}$	$2.04 \times 10^{11}$	0.260
100%	$9.03 \times 10^{11}$	$1.70 \times 10^{11}$	0.187

#### 4. WEAKLY NONLINEAR EFFECTS

In section 3b, the net effect of increasing the rotation speed of the forcing was found to decrease the strength of the tropical storm vortex. When the speed increases from 25% to 100%, the storm ends up with more negative net KE (Fig. 4). However, the changes in the wind field (Fig. 5) show that the tangential wind change inside the RMW at an altitude of 6km becomes less negative. It is unclear at this point whether these changes may have positive or negative feedbacks on the intensification rate of the tropical storm vortex. Our work on these matters is ongoing.

#### 5. CONCLUSION AND FUTURE WORK

This work is a continuation of NM02 and NG03 which concluded the net effect of an instantaneous asymmetric perturbation is to decrease the intensity of the parent vortex. In this work, asymmetric perturbations are introduced over time (rather than instantaneously) and allowed to move with different speeds. It was found that the overall effects of injecting the forcing over time are to decrease the weakening of the parent tropical storm vortex. In contrast, the effect of allowing the thermal forcing to rotate around the center of the storm is to enhance the weakening of the parent vortex. Further analysis is required on the weakly nonlinear impact of increasing the rotation speed of forcing on the symmetric intensification rates of developing cyclones.

#### ACKNOWLEDGEMENTS

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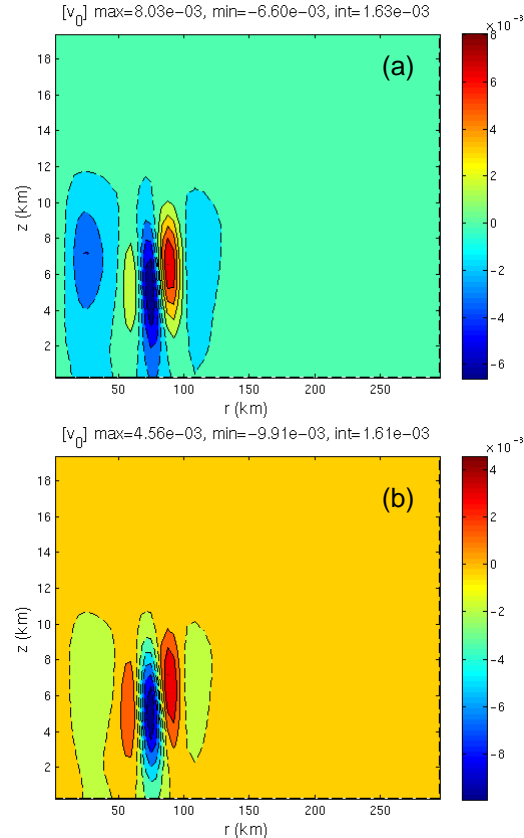


Fig. 5: The wind field change of the tropical storm at the end of simulation (16 hours) with the forcing at  $r=80\text{km}$  allowed to rotate at the speed of (a) 25% and (b) 100% of the surface wind. The surface wind speed at  $r=80\text{ km}$  is  $18.3\text{ ms}^{-1}$  and the forcing is injected over 1 hour.

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