11.7 Waves in a Cloudy Vortex

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1. Introduction

There have been many attempts to understand the role of wave dynamics in regulating hurricane intensity. A notable deficiency of past hurricane wave theories is their neglect of moisture. Since clouds pervade hurricanes, such an approximation is not soundly justified. Traditionally, the basic effect of moisture has been viewed as the reduction of static stability (buoyancy frequency) in the primitive equations (e.g., Durran and Klemp 1982). Here, we examine how moisture induced buoyancy reduction influences vortex Rossby wave dynamics.

In particular, we consider vortex Rossby waves in a non-precipitating cloudy vortex whose basic state has no secondary circulation. In general, the vortex has fluctuations of cloudy and unsaturated air, but those fluctuations are assumed to occur on smaller scales than the waves of interest. Section 2 presents the model that is here used to approximate the wave dynamics.

We have shown that by reducing buoyancy, moisture can slow the growth of phase-locked counter-propagating vortex Rossby waves in the eyewall of a hurricane-like vortex (see Section 3). This suggests that increased fractional cloud coverage might inhibit asymmetric eyewall breakdown. The consequence on hurricane intensity is a topic of ongoing study. On the one hand, potential vorticity mixing after eyewall breakdown can directly reduce the maximum tangential wind speed of a hurricane (e.g., Schubert et al. 1999; Kossin and Schubert 2001). On the other hand, the thermo-fluid dynamics connected to eyewall breakdown is subtle, and may actually lead to a stronger hurricane (Emanuel 1997; Montgomery et al. 2002; Persing and Montgomery 2003; Montgomery et al. 2006).

We have also shown that cloud coverage can either cause or accelerate the decay of discrete vortex Rossby waves in a cyclone whose potential vorticity decreases monotonically with increasing radius (see Section 4). If the Rossby number of the cyclone exceeds unity, the discrete vortex Rossby waves can resonantly excite outward propagating spiral inertia-gravity waves in the environment. By damping the vortex Rossby waves, cloud coverage can inhibit the loss of angular momentum by the associated inertia-gravity wave radiation. Such loss might otherwise merit some consideration in the angular momentum budget of a hurricane (e.g., Chow and Chan 2003).

One type of discrete vortex Rossby wave is an azimuthally propagating tilt mode. By causing or accelerating the decay of a tilt mode, cloud coverage improves the resilience of a monotonic cyclone after it is misaligned by ambient vertical shear. Such was anticipated from earlier work that showed or implied that decreasing the (dry) static stability of the atmosphere increases the realignment rate of certain vortices that resemble tropical cyclones (Jones 1995; Reasor and Montgomery 2001; Schecter et al. 2002; Reasor et al. 2004).

2. The Moist Wave Equations

In this section, we present the model that is here used to study waves in a cloudy vortex. Subtleties of its derivation are explained in a forthcoming paper that is currently under review.

The wave equations are for small hydrostatic perturbations of an axisymmetric vortex in gradient balance. As mentioned earlier, the basic state of the vortex has no secondary circulation. The only forms of moisture in the vortex are liquid cloud and vapor. Air parcels are assumed to move moist adiabatically, conserving total water mass while not allowing vapor to exceed its saturation level at any instant.

As usual, let primes and overbars denote perturbation and basic state variables. In addition, let r, φ and p denote the radial, azimuthal and pressure coordinates. The wave model consists of three prognostic equations and two diagnostic equations, given immediately below.

The radial and tangential velocity perturbations u' and v' are governed by

$$\left(\frac{\partial}{\partial t} + \bar{\Omega}\frac{\partial}{\partial \varphi}\right) u' \;=\; \bar{\xi} v' \;-\; \frac{\partial \phi'}{\partial r}, \qquad (1)$$

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and

$$\left(\frac{\partial}{\partial t} + \bar{\Omega}\frac{\partial}{\partial \varphi}\right) v' = -\bar{\eta} u' - \omega' \frac{\partial \bar{v}}{\partial p} - \frac{1}{r}\frac{\partial \phi'}{\partial \varphi}, \quad (2)$$

in which ϕ' is the geopotential perturbation and ω' is the pressure (vertical) velocity perturbation. Equations (1) and (2) introduce three auxiliary functions that are connected to the basic state tangential velocity field $\bar{v}(r, p)$. They are (i) the angular rotation frequency of the vortex $\bar{\Omega} \equiv \bar{v}/r$, (ii) the modified Coriolis parameter $\bar{\xi} \equiv f + 2\bar{\Omega}$, and (iii) the absolute vertical vorticity $\bar{\eta} \equiv f + r^{-1}\partial(r\bar{v})/\partial r$.

Our entropy-related variable of choice is the density potential temperature θ_ρ. Its perturbation is governed by

$$\left(\frac{\partial}{\partial t} + \bar{\Omega}\frac{\partial}{\partial \varphi}\right) \theta_\rho' = -u' \frac{\partial \bar{\theta}_\rho}{\partial r} - \omega' \Upsilon \frac{\partial \bar{\theta}_\rho}{\partial p}. \quad (3)$$

Equation (3) introduces the "buoyancy reduction factor,"

$$\Upsilon(r, p) \equiv 1 - \frac{\overline{(\partial \theta_\rho / \partial p)_{s, q_t}}^{\varphi, t}}{(\partial \bar{\theta}_\rho / \partial p)_r}, \quad (4)$$

which, as explained below, characterizes the fractional cloud coverage in an azimuthal circuit. The second term in the definition of Υ is the ratio of two pressure derivatives of density potential temperature. The numerator involves the pressure derivative at constant moist entropy s and total water mixing ratio q_t. We have derived a formula for this derivative in terms of temperature T, pressure p and and total water mixing ratio q_t. The derived expression varies discontinuously between unsaturated and cloudy air. The overline here represents an average over azimuth φ and time t. The denominator is the pressure derivative of basic state θ_ρ at constant radius. For a dry vortex, the numerator is zero, and $\Upsilon = 1$. For a very cloudy vortex Υ can be much less than unity.

In addition to the above prognostic equations, we have from hydrostatic balance,

$$\frac{\partial \phi'}{\partial p} = -\frac{R_d}{p}\left(\frac{p}{p_o}\right)^{R_d/c_{pd}} \theta_\rho', \quad (5)$$

and from mass continuity,

$$\frac{\partial \omega'}{\partial p} = -\frac{1}{r}\frac{\partial(r u')}{\partial r} - \frac{1}{r}\frac{\partial v'}{\partial \varphi}. \quad (6)$$

Here, R_d is the gas constant of dry air, c_{pd} is the specific heat of dry air at constant pressure, and $p_o = 10^5$ PA is a reference pressure. We need not consider the linearized equation for total water mass, since the above system (1-6) does not depend explicitly on q_t', and is therefore closed.

Note that the above wave equations are the same as those for a dry vortex, with $\partial\bar{\theta}_\rho/\partial p \to \Upsilon \partial\bar{\theta}_\rho/\partial p$. With moisture, the Eulerian rate of cooling/warming by upward/downward motions is reduced by a factor Υ.

Figure 1 shows the Υ-distribution[†] of a hurricane that was simulated with RAMS, the Regional Atmospheric Modeling System developed at Colorado State University (CSU). This particular simulation involved no ice. The data was provided to the authors by Dr. M. Nicholls and Dr. J. Persing at CSU. It suggests that within the context of the present model, the best cloudy vortex to represent a hurricane might have relatively small values of Υ in the eyewall (near the radius of maximum wind).

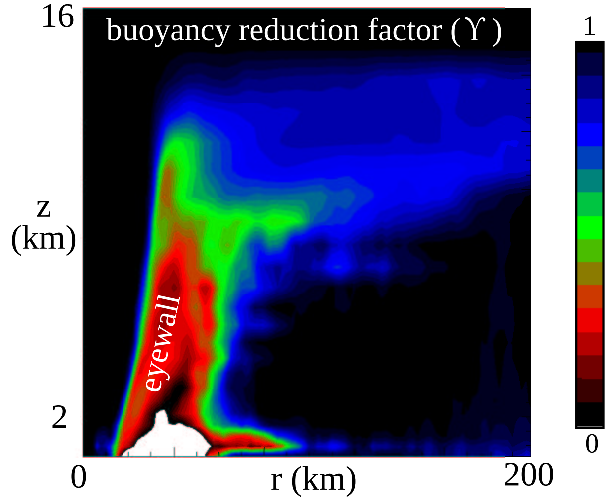

Figure 1: This preliminary computation shows that the "buoyancy reduction factor" of a simulated category 5 hurricane is smallest in the very cloudy eyewall region. The raw simulation data used here was provided to the authors by Dr. M. Nicholls and Dr. J. Persing of Colorado State University.

3. Eyewall Instability

We have used the moist wave equations (1-6) to reexamine the eyewall instability of a baroclinic category-3 hurricane-like vortex (Nolan and Montgomery 2002). In the absence of clouds, the fastest growing eyewall mode of this particular vortex consists of two phase-locked counter-propagating

[†]For this computation, we used an instantaneous azimuthal mean as opposed to the tempero-azimuthal mean that appears in the proper definition of Υ [Eq. (4)].

vortex Rossby waves. The azimuthal wavenumber of the perturbation is 3.

In order to study the effect of moisture on the gravest mode, we developed a computationally stable numerical algorithm for integrating the moist wave equations forward in time. In a sequence of numerical experiments, the buoyancy reduction factor Υ was gradually decreased from unity to bring the vortex toward slantwise convective neutrality. Within the context of our model, a necessary condition for symmetric stability is

$$\Upsilon \geq \Upsilon_{nt}(r,p) \equiv -\frac{[\bar{\xi}(\partial\bar{v}/\partial p)]^2}{R_d\bar{\eta}\bar{\xi}(\partial\bar{\theta}_\rho/\partial p)} p \left(\frac{p_o}{p}\right)^{R_d/c_{pd}}. \tag{7}$$

Slantwise convective neutrality is here defined by $\Upsilon = \Upsilon_{nt}$.

The numerical simulations showed that increasing the fractional cloud coverage in the vortex (decreasing Υ) decreases the growth rate of the gravest mode. This result qualitatively agrees with the stability analysis of potential vorticity rings in a quasigeostrophic shallow-water model. There too, the growing modes are neutralized by reducing buoyancy (the ambient gravity wave speed).

4. Spontaneous Inertia-Gravity Wave Radiation and Vortex Resilience

We have also used the moist wave equations (1-6) to generalize a recent theory of spontaneous inertia-gravity wave radiation from dry cyclones to cloudy cyclones (Schecter and Montgomery 2004,2006). In all cases considered, the cyclones are assumed to have barotropic basic states, Rossby numbers greater than unity, and potential vorticity distributions that decrease monotonically with increasing radius. In dry theory, a discrete vortex Rossby wave will resonantly excite an outward propagating spiral inertia-gravity wave of proportional strength in the environment. The inertia-gravity wave radiation has positive feedback on the Rossby wave, compelling it to grow. However, the radiative instability is often quenched by the negative feedback of potential vorticity stirring in the Rossby wave critical layer. Our specific goal was to understand how cloud coverage affects the quenching.

To this end, we derived a growth rate formula for the radiative vortex Rossby waves of a cloudy cyclone. The derivation was based in part on a new flux-conservative equation for the moist angular pseudomomentum of a perturbation. As in dry theory, we converted this equation into one of the following form for the amplitude $a(t)$ of a nearly-neutral vortex Rossby wave:[‡]

$$\frac{da}{dt} = (\nu_{rad} + \nu_{cl})a. \tag{8}$$

The positive radiative pumping rate ν_{rad} is roughly proportional to the outward angular momentum flux of the emitted inertia-gravity wave. The negative critical layer damping rate ν_{cl} is proportional to the radial gradient of potential vorticity at the critical radius r_*, where the angular rotation frequency of the cyclone equals the angular phase velocity of the wave. The critical radius is generally outside the vortex core. The primary effect of cloud coverage is to move r_* inward. The same effect would occur by decreasing the dry static stability of the atmosphere. For monotonic cyclones, moving r_* inward typically increases the absolute value of ν_{cl} by orders of magnitude. As a result, the growth rate becomes negative.

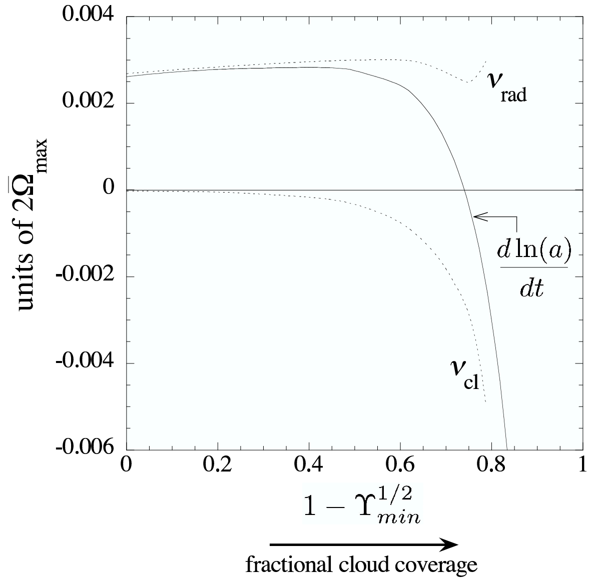

Figure 2: Growth rate (solid curve) of a baroclinic vortex Rossby wave that emits a spiral inertia-gravity wave of proportional amplitude into the environment. See text for discussion.

Figure 2 provides a concrete example of how increased cloud coverage negates the growth rate (solid curve) of a baroclinic vortex Rossby wave in a monotonic cyclone. Details of the Rossby wave and of the cyclone are unimportant for our discussion. As the

[‡]To evaluate the explicit formulas for ν_{rad} and ν_{cl} requires knowledge of the Rossby wave oscillation frequency and wavefunction out to the radiation zone. These are obtained computationally from an eigenmode/quasimode solver that we have written for waves in a barotropic cloudy vortex.

fractional cloud coverage increases in the vortex core (as Υ decreases), the radiative pumping rate (top dotted curve) remains roughly constant. In contrast, for reasons given above, the critical layer damping rate (bottom dotted curve) explosively grows. Ultimately, the growth rate becomes negative and the vortex Rossby wave can no longer produce sustained radiation.

The vortex Rossby wave under consideration could well be a tilt mode. Evidently, adding clouds to the cyclone will either cause or accelerate damping of that mode. Thus, moisture not only inhibits inertia-gravity wave radiation, but it also improves vortex resilience. A cloudy monotonic cyclone will realign more rapidly than its dry counterpart after it is tilted by ambient vertical shear.

Before concluding this section, there is an important caveat worth mentioning: the above discussion pertains to linear theory. If the initial vortex Rossby wave activity (amplitude) exceeds the finite absorption capacity of the critical layer, radiative pumping will ultimately prevail, even if linear theory predicts otherwise (Schecter and Montgomery 2006).

5. Future Plans

We have derived a relatively simple linear model for waves in a cloudy vortex. We have begun using this model to gain further insight into fundamental wave processes that in principle affect hurricane intensity and resilience (see Section 1). We plan more studies along these lines to understand how moisture influences wave transport of energy and angular momentum. In addition, we plan to test at least the qualitative accuracy of our results against more realistic moist vortex simulations. These simulations will most likely be carried out with RAMS. The simple model will be amended (or reformulated) as necessary to explain increasingly complex behavior.

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