J4.6 A FORMULA FOR THE DEPTH OF THE STABLE BOUNDARY LAYER: EVALUATION AND DIMENSIONAL ANALYSIS

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1. INTRODUCTION

The stable boundary-layer (SBL) height (h) is important to understand SBL development and vertical structure. Clearly, h influences the SBL mixing properties. Model formulations with an explicit prescription of the vertical profile of the eddy diffusion coëfficient, require an explicit expression for h (e.g. Troen and Mahrt, 1986, Holtslag and Boville, 1993). For stable conditions, many models of this type overestimate the vertical mixing and thus h. So there is a clear need for an alternative h formulation for Numerical Weather Prediction models.

Furthermore, the dispersion of pollutants is strongly affected by h. Release of pollutants below h during periods of weak winds results in very high concentrations of primary and secondary pollutants, which can cause serious consequences for the environment. This means that for meteorological preprocessors in air quality models, h is the most critical quantity to estimate (Lena and Desiato, 1999).

Measuring h is not straightforward because the turbulence is suppressed at night and can be of intermittent nature or ill-defined (e.g. Holtslag and Nieuwstadt, 1986). In addition to turbulence, radiation divergence, gravity waves, wave breaking and baroclinicity influence the SBL structure for the very stable case. In that case, problems occur in measuring h, since no universal relationship exists between the profiles of temperature, wind speed and turbulence variables.

We evaluate the performance of two multi-limit equations (Zilitinkevich and Mironov, 1996; henceforth ZM96) against four observational datasets and Large Eddy Simulation (LES). Secondly, we present an alternative, robust and practical formulation for h. Finally, we will show that the Coriolis parameter is not *a priori* a necessary quantity for h estimation.

2. BACKGROUND

ZM96 identified rotation, surface buoyancy flux and free flow stability to be the key physical processes that govern h. Consequently, ZM96 derived a formula for h by inverse quadratic interpolation of the relevant boundary-layer height scales that represent these three processes. The formula uses the friction velocity (u_*), buoyancy flux ($B_s = g/\theta\,\overline{w\theta_s}$), Coriolis parameter ($f$) and free flow stability ($N^2 = g/\theta\,\partial\theta/\partial z$) ($g$ is the gravity acceleration, θ the potential temperature and z the height above ground) and reads:

$$\left(\frac{fh}{C_n u_*}\right)^2 + \frac{h}{C_s L^*} + \frac{Nh}{C_i u_*} = 1 \qquad (1)$$

Herein $L^* = -u_*^3/B_s$ is the Obukhov length (without Von Kármán constant). The main advantage of Eq. (1) is its multi-limit behaviour, i.e. both for $f \to 0$, or $N \to 0$ or $L^* \to \infty$, Eq. (1) remains defined.

Based on Zilitinkevich (1972) and Pollard et al. (1973), ZM96 add two additional terms to “include the cross interactions” between f, B_s and N:

$$\left(\frac{fh}{C_n u_*}\right)^2 + \frac{h}{C_s L^*} + \frac{Nh}{C_i u_*} + \frac{\sqrt{|fB_s|}\,h}{C_{sr} u_*^2} + \frac{\sqrt{|fN|}\,h}{C_{ir} u_*} = 1 \qquad (2)$$

with C_n = 0.5, C_s = 10, C_i = 20, C_{sr} = 1, C_{ir} = 1.7.

Apart from the benefits discussed above, Eqs. (1) and (2) have several drawbacks. Firstly, a large amount of parameters is required in both equations. Several of these coefficients are hard to determine (ZM96, Joffre et al., 2002, Vickers and Mahrt, 2004). For example, C_n ranges from 0.045 to 0.6 and C_s ranges from 1.2-100 (ZM96) in the literature.

Secondly, it is not a priori clear that the method of inverse quadratic interpolation gives the proper weight to the relevant

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length scales. Alternative interpolation methods will give different results.

Thirdly, the rules of dimension analysis are violated in Eq. (2) because five groups are used while only three are allowed since we have 5 variables and 2 elementary units.

3. DATASETS

The validation material consists of four observational datasets of turbulent surface fluxes and radiosonde profiles for a broad range of latitude and surface roughness:

- Sodankylä (NOPEX/WINTEX, Finland (67.4° N, 26.7° E, 180 m ASL), forest.
- CASES-99, Kansas USA, (37.6º N, -96.7º E, 430 m ASL), prairie grass.
- Cabauw, The Netherlands,(51.9 ºN, 4.9 ºE; -0.7 m ASL), grass.
- SHEBA, 75° N, 144° W to 80° N, 166° W, sea ice.
- GABLS Large Eddy Simulation (Beare et al, 2005)

The θ profile and the LLJ height were used to obtain *h*, except for Cabauw where *h* was obtained by sodar.

4. RESULTS

a) Evaluation

Fig. 1 shows the performance for, Eq. (1). *h* is satisfactorily estimated for thick SBLs, although for h > 400 m, an overestimation is seen. Contrary, for small *h*, we find a clear offset: the model predicts *h* of only several meters where the observations still show heights of 50-80 m. The mean RMSE and median of the absolute error (MEAE) over all locations amounts to 133.0 m and 86.3 m respect-tively. For the LES model results, Eq. (1) gives *h* = 218 m, while 180 m was given by the LES. Note that in the current analysis, the impact of subsidence was neglected.

Fig. 2 depicts the modeled Eq. (2) and observed *h*. In this case, *h* is systematically underestimated over the whole range for all datasets. For the LES case the modeled *h* = 90 m. The off-set for small *h* as with Eq. (1) is present here as well. It generally seems that Eq. (2) underestimates the data by a factor 2.

b) Calibration

With the large available dataset, it is tempting to recalibrate the coefficients in Eqs (1) and (2). The proportionality constant for a truly neutral boundary layer (B_s = 0 and N = 0), C_n is hard to obtain from atmospheric observations truly neutral boundary layers are generally absent in the atmosphere.

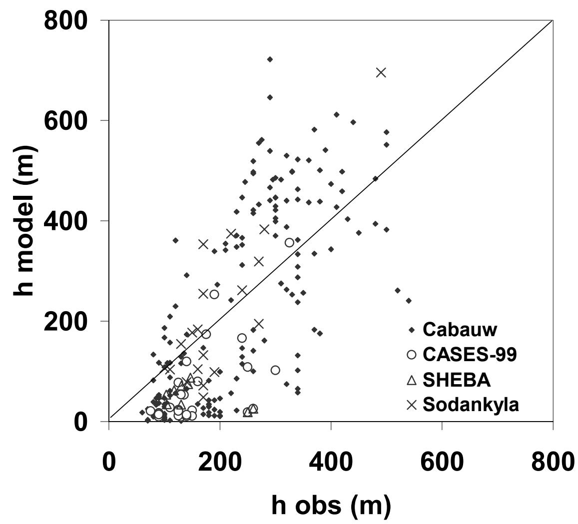

Figure 1: Model performance for Eq. (1).

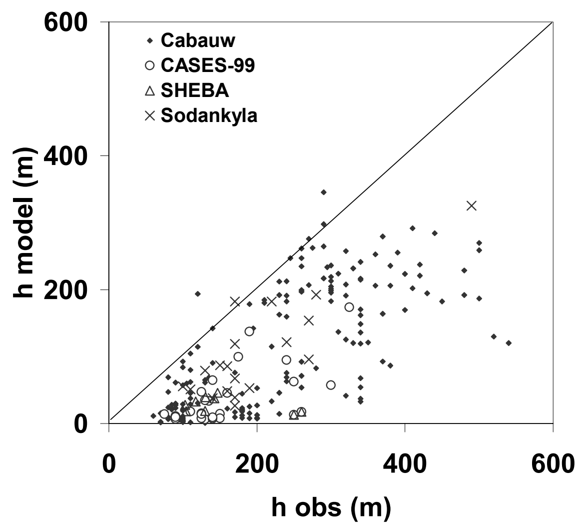

Figure 2: Model performance for Eq. (2).

Therefore, we obtain C_n from LES studies by e.g. Mason and Thom-son (1987). They find C_n = 0.6 and we will use this value further on. The remaining coefficients (C_s and C_i) are calibrated on the Sodankylä dataset with a Monte Carlo approach. We prefer the MEAE to prevent outliers to affect the quality too much. Fig. 3 shows a surface plot of the MEAE for parameters ranges $C_i \in [5,20]$ and $C_s \in [0,100]$. The MEAE has a minimum for C_i = 11, which agrees with findings by Van Pul et al., Kitaigorodskii and Joffre (1988), and VH96 who found C_i = 7-13, and Joffre et al. (2002) who found C_i = 10. Vickers and Mahrt propose C_i = 15.

Since no clear minimum is found in the contour plot, the model performance is insensitive for coefficient C_s for C_s > 40. This also explains the large range of proposed values for C_s. Also, no unique parameter set for C_i and C_s was found with 0.1 < C_n < 0.7 (not shown).

Thus, Eqs. (1) and (2) show a clear bias against observations and the parameters cannot be calibrated robustly.

c) Alternative formulation

We apply Buckingham Π theory on the relevant quantities in Eq. (1) to find three (instead of five) dimensionless groups:

$$\Pi_1 = B_s / hfu_* N ,$$
$$\Pi_2 = hB_s / u_*^3 = h / kL^* = h/L ,$$
$$\Pi_3 = N/f .$$

Consequently we determine a relationship between Π_1, Π_2 and Π_3 from observations. Fig. 3 shows Π_1 versus Π_2 for different classes of Π_3 on a linear scale. Despite the small number of data per class, Π_2 increases obviously with Π_1, but levels off at different values for different classes of *N/f*. This relevance of *N/f* was already mentioned by Kitaigorodskii and Joffre (1988). After rearrangement of the relevant groups, we find for *h*:

$$h = L\left(\frac{g}{\theta}\overline{w\theta_s} \Big/ \alpha u_* fNL\right)^{\lambda} \qquad (3)$$

with α = 3, $\lambda = \left(C_1 - \frac{N}{1000f}\right)^{-1}$ and C_1 = 1.8.

L is the Obukhov length (with Von Kármán constant).

A similar Monte-Carlo strategy as in the previous section was followed to estimate the coefficients α and C_1. A clear minimum in the MEAE is found in the contour plot with C_1 = 1.8 and α = 3 as optimal parameter values. So in contrast to Eqs. (1) and (2), it appears that Eq. (3) can be calibrated in a robust way.

Fig. 4 shows the performance of Eq. (3). The model agrees well with the CASES-

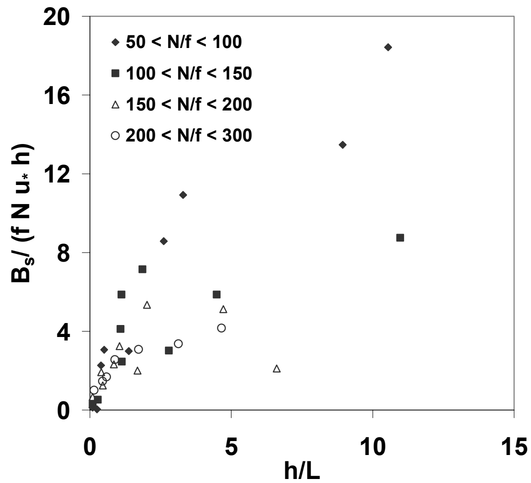

Figure 3: Dependence of $B_s / hfu_* N$ vs h/L and N/f.

99 and Cabauw observations, although the scatter is larger for Cabauw than for the other data sets. This relatively large scatter is probably inherent to the sodar based observations for Cabauw instead of radio sounding profiles for the other datasets. For SHEBA the model performance is good, although the model seems to slightly overestimate the observations.

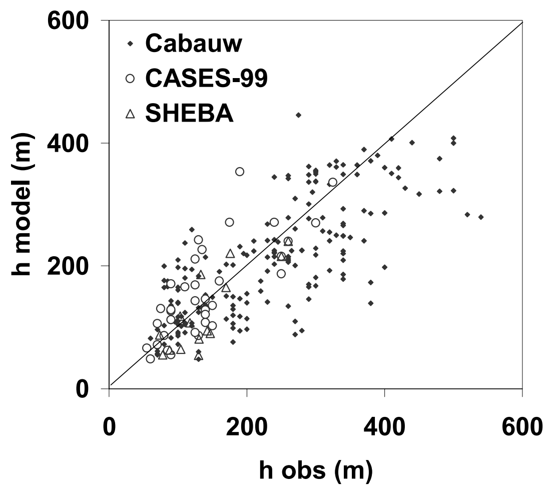

Figure 4: Model performance of Eq. (3)

b) Two dimensionless groups.

The recent literature discusses the relevance of *f* for the boundary-layer height (e.g VH96, Mahrt, 1998). Since *N* is typically O(10^{-2}) while *f* is O(10^{-4}) in the mid-latitude atmosphere, VH96 suggest that the impact of *f* can be neglected in practice. Accordingly, we assume that *f* can be excluded from the list of relevant variables for *h* estimation in practical applications. Consequently, two dimensionless groups: hN/u_* and h/L^* remain, as shown in Fig.

5. Two regimes can be clearly distinguished. For $h/L^* < 1$ (towards the near neutral limit) $h \propto u_* / N$, which is in agreement with earlier findings by Kitaigorodskii and Joffre (1988), Van Pul et al., (1994), and VH96. For $h/L^* > 1$ the two groups are linearly related on the log-log scale. This implies $h \propto \sqrt{B_s / N^3}$. Consequently a diagnostic equation for h reads as:

$$h = \begin{cases} 10 u_* / N & \text{for} \quad u_*^2 N / B_s > 10 \\ 32 \sqrt{B_s / N^3} & \text{for} \quad u_*^2 N / B_s < 10 \end{cases} \qquad (4)$$

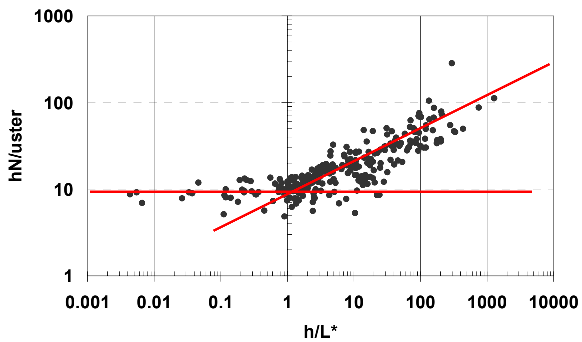

Figure 5: Scaling without f with hN/u vs h/L. A clear regime change occurs for h/L =1.*

5. CONCLUSIONS

This study shows that the multi-limit equation for the stable boundary-layer height does not work satisfactorily against four observational datasets that originates from a broad range of latitude and surface roughness. An alternative formulation is proposed on formal dimension analysis. The formulation is robust and gives unbiased estimates. Furthermore it is shown that if the Coriolis parameter is disregarded as a relevant quantity, the SBL height scaling shows two different regimes according to *h/L*.

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