P4.3 ENTRAINMENT PARAMETERIZATION BASED ON ENTRAINMENT ZONE RICHARDSON NUMBER

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1. INTRODUCTION

Entrainment at the top of a convective or transitional boundary layer is a first-order term in the budgets of heat, moisture, momentum, and pollutants. Model treatments of entrainment, however, tend to be somewhat haphazard. Either entrainment is expected to emerge correctly from a 1.5 or higher order boundary layer scheme, or it is crudely parameterized as a fraction of the surface flux. The latter approach has some justification for buoyancy flux only, and only under pure free convection conditions. Because entrainment is difficult to measure, only a few observational studies are available to test model formulations, but those measurements that are available strongly suggest that entrainment as a fraction of the surface flux is quite variable. Large-eddy simulations tend to show less entrainment than the measurements, even under conditions that depart substantially from free convection (strong shear).

Throughout this paper, I am primarily talking about the (negative of the) ratio of buoyancy flux at the top of the boundary layer to buoyancy flux at the surface. Notation for this is notoriously non-standard. For this paper, I will just call it A. Buoyancy and virtual potential temperature are interchangeable in this context. I am assuming that no clouds are present, because even small amounts of cloud invalidate all this analysis.

Should we parameterize entrainment flux in terms of the surface flux at all? Can we assume that we have a single, surface-based, convective boundary layer? Our models must switch between a convective BL scheme and a scheme for other types of BLs, which will likely introduce discontinuities and transients, and requires that we have a well-justified trigger function. Schemes that seamlessly handle all of the types of layers found in the atmosphere are likely to give better performance. However, given that computational efficiency is still an issue for large-scale models, parameterizing entrainment may still be necessary for some time.

It is widely believed (for example, (Stull 1988)) that, in pure free convection conditions, A = 0.2. I am unable to convince myself that the justification for this belief is strong enough to explain why it is so widely held. Worse yet, this value is often used for other quantities than buoyancy. It is easy to show that, for example, the ratio of potential temperature fluxes is not equal to A unless the humidity profile takes a specific form.

Free convection is unusual. It is much more common to have significant wind and wind shear. This is particularly true when we consider boundary layers over moist or transpiring surfaces, or early or late in the day. All such effects result in an increase in the entrainment flux relative to the surface flux. In other words, even if we know the lower limit of A, we also know that it rarely takes that lower limit value.

Let us consider some limiting cases. When there is no gradient of buoyancy above the turbulent BL, the situation is called "free encroachment" (Sorbjan 2004) and the entrainment buoyancy flux is identically zero. This is a common situation during morning BL growth. At the other extreme, when the BL is capped by a very strong inversion, the BL growth rate is small, but what is the entrainment buoyancy flux? Small amounts of entrained air, carrying large relative buoyancy,

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could produce a large flux. On the other hand, most or all of the incident energy could go into producing gravity waves on the inversion (Stull 1976). For the moment, I consider this limit unknown. When wind and wind shear are strong, the BL is considered convective even if the surface buoyancy flux is small, but the entrainment flux may be independent of the surface flux.

Several approaches to finding A have appeared in the literature. Seibert et al. (1997) give a detailed discussion. One common approach is to add terms depending on the surface wind and entrainment zone wind shear. Unfortunately, the coefficients of these terms are very uncertain (see for example (Barr and Betts 1997)).

A different approach has been suggested by Fairall (1984) and by Sorbjan (2004). This involves a Richardson number of the entrainment zone, based on the buoyancy and wind speed jumps across the entrainment zone. This should not be confused with other Richardson numbers based on the convective velocity scale. A conceptual argument goes something like this: A small Richardson number indicates a dynamically unstable interface, which will entrain even without any influence of large eddies from below. This can arise because of strong shear, weak static stability, or some combination. A large Richardson number indicates an interface that will not entrain unless it is strongly influenced from outside (below). In some intermediate range of Richardson number, entrainment will be due to the interaction of local dynamics with the influence from below.

The entrainment zone Richardson number is clearly not the whole story, because the same values can arise in different ways. This is particularly clear in the free encroachment case, where the Richardson number is zero but there may be no shear (and therefore no turbulence) at all.

2. MEASUREMENTS

There are a few sets of measurements of entrainment into convective boundary layers over land. They rely on budget methods, since direct measurements of entrainment flux are rendered impossible by even small amounts of heterogeneity. The prototype of these measurements is the paper by Betts and Barr (1996). They concluded that A was considerably larger than 0.2 over the prairie. I came to the same conclusion (Angevine 1999) based on measurements over flat farmland in Illinois. Barr and Betts (1997), however, found smaller values over the boreal forest.

These measurements, taken as a group, suggest that A is not constant and not as small as 0.2 under most realistic conditions. Apparently, shear effects must be taken into account even when the shear is not large. For example, the cases discussed in (Angevine 1999) have surface wind speeds less than 10 m/s and shears across the entrainment zone of less than 6 m/s. Shear influence is more important when the surface sensible heat flux is smaller, for example over prairie or farmland where the Bowen ratio is small. This is less of an issue over the boreal forest where the Bowen ratio is large.

Because the measurements are uncertain, it has been difficult to determine how entrainment depends on other parameters. However, (Angevine 1999) suggests that A is larger when the entrainment zone Richardson number is smaller and when the inversion is weaker. The Richardson number effect was highly non-linear.

Attempts to use the literature formulae involving additive terms to diagnose the shear influence were not successful (Angevine 1999; Barr and Betts 1997).

3. LES RESULTS

One could hope that large eddy simulations could act as controlled experiments on entrainment. Unfortunately, a number of issues have arisen in the course of numerous attempts to do this.

Pino et al. (2003) found that A increased from 0.2 to 0.25 when the geostrophic wind increased from 0 to 10 m/s with modest surface fluxes. Their LES vertical grid spacing was 50 m. Kim et al. (2003) found A varying from 0.13 to 0.30 with small surface fluxes and geostrophic winds varying from 5 to 15 m/s. They used a vertical grid of 15 m. Fedorovich et al. (2004a) tested weak and strong inversions without shear and found A values much less than 0.2, using a vertical grid of 20 m. Fedorovich et al. (2004b) studied the entrainment performance of several LES codes. With no shear, most codes predicted A < 0.2. When a geostrophic wind of 20 m/s was specified, A increased to 0.2-0.3. Sorbjan (2004) found A = 0.2 or slightly larger for four different cases involving a geostrophic wind of 15 m/s and using a vertical grid of 15 m.

4. DISCUSSION

Compared to measurements, it is clear that LES underestimate entrainment. In most cases this is true even without shear. With shear, the underestimate is even larger. The most likely cause is insufficient vertical resolution (Fedorovich et al. 2004). The scale of entrainment varies with inversion strength, becoming smaller with stronger inversions. Shear probably also makes the important scale smaller. In other words, entrainment is not a large-eddy process.

To return to parameterization, it seems appealing to find a formula that takes into account inversion strength and entrainment zone Richardson number. Such a formula was proposed by Sorbjan (2004) in his equation 23b:

$$H_i = c_H w_*^2 \left(N_i \big/ \beta \right) \left[\left(1 + c_2 R_i\right) \big/ \left(1 + 1/R_i\right)^{1/2} \right]$$

where H_i is the entrainment buoyancy flux and N_i is the Brunt-Vaisala frequency of the inversion. Because it is cast in terms of complex scaling parameters, there are some subtleties to this formula. The dependence on surface heat flux, carried by the convective velocity scale, is to the 2/3 power. In other words, one cannot extract a constant ratio A from this formula. It also suggests that the entrainment flux increases as the ½ power of the buoyancy gradient in the entrainment zone (inside the Brunt-Vaisala frequency) and that it increases as the 2/3 power of the BL depth (inside the velocity scale). Of course, these parameters are not independent and only some parts of the parameter space can plausibly be occupied (Fairall 1984).

The Richardson number dependence in this formula gives the desired results, that is, entrainment flux increases without limit when R_i is small, and there is no shear influence when R_i is large.

In models using higher-order turbulence schemes involving TKE, alternative approaches for entrainment parameterization are available, for example (Kim et al. 2006).

The problem of improving entrainment parameterization is challenging. Measurements are few and uncertain; LES results differ from theoretical expectations sufficiently that it is unclear whether we can rely on them. Perhaps the situation can be improved by careful analysis of existing data sets and by more refined LES or even DNS experiments.

Ultimately, the notion of parameterizing entrainment should be abandoned in favor of schemes that handle all types of layering reasonably. This will allow us to get away from issues of definition ("what is a boundary layer", "is the BL stable or unstable") that hamper the understanding and modeling of the full range of atmospheric conditions, including transitional BLs, coastal and complex terrain, "upside-down" BLs, elevated turbulent layers, etc. etc. etc.

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