# JP4.8 ENERGY-SIMILARITY – A NEW TURBULENCE CLOSURE MODEL FOR THE STABLY STRATIFIED ATMOSPHERIC BOUNDARY LAYER

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### **1 INTRODUCTION**

Increasing interest is being given to the stably stratified atmospheric boundary layer (SBL). It has been identified as one of the main challenges through the GEWEX Atmospheric Boundary Layer Study (GABLS), Holtslag (2006). The SBL is important in both numerical weather prediction (NWP) and climate simulations (IPCC, 2001; ACIA, 2005).

Within the framework of GABLS two intercomparison studies have been performed. The first case, described in Cuxart et al. (2006) and Beare et al. (2006), is an idealized study based upon a profile observed in the Arctic. It was classified as a weakly stable case. State-ofthe-art operational models had difficulties capturing basic features, such as boundary layer height and momentum- and heat-fluxes, while some research models and large-eddy simulations (LES) agreed better. While the first GABLS case was a highly idealized study, the second GABLS case (Svensson and Holtslag, 2006a) aimed at a closer relation to observations, by targeting a few daily cycles of the CASES-99 field campaign. Again, the diversity of the model results is enormous. In particular none of the models are able to replicate the magnitude of the daily cycles of turbulent fluxes, temperature and wind, despite the fact that the surface temperature was prescribed.

Recently, an observational study of stably stratified turbulence at both weak and strong

flow stability has been conducted by Mauritsen and Svensson (2006). They used the gradient Richardson number (Ri), rather than the widely used Monin-Obukhov length (L) as a stability parameter, thus avoiding the problem of self-correlation inherent to Monin-Obukhov based studies. Mauritsen and Svensson (2006) found that even at Ri > 1 the turbulent stress is finite, while the turbulent heat flux is indistinguishable from zero. Thus, not supporting the assumption that there is no turbulence at Ri beyond some critical value.

## 2 MODEL

Prognostic equations for the mean state wind and the mean potential temperature of the atmospheric turbulent boundary layer in their Reynolds averaged form are:

$$\frac{DU}{Dt} = -\frac{\partial \overline{uw}}{\partial z} - f(V_g - V),$$

$$\frac{DV}{Dt} = -\frac{\partial \overline{vw}}{\partial z} + f(U_g - U),$$

$$\frac{D\Theta}{Dt} = -\frac{\partial \overline{w\theta}}{\partial z},$$

assuming horizontal homogeneity and making the Boussinesq approximation. Symbols have their usual meaning. In addition to the mean state variables we study the turbulent part of the field. Of the budget equations for the second order moments of wind and temperature, we keep those for turbulent

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kinetic energy (  $E_K$  ) and the potential

temperature variance (  $\sigma_{\theta}^2$  ). We combine these two equations to obtain the total turbulent energy equation:

$$\frac{DE}{Dt} = \tau \cdot \frac{\partial \mathbf{U}}{\partial z} - \gamma - \frac{\partial F_E}{\partial z} + \begin{cases} 0 & \text{for } N^2 \ge 0\\ 2\beta \overline{w\theta} & \text{for } N^2 < 0 \end{cases}$$

where  $\tau$  is the stress vector,  $\gamma$  is the total turbulent energy dissipation,  $F_E$  is the third order energy flux,  $\beta = g/\theta$  is the buoyancy parameter and N is the Brunt-Väisälä frequency. The definition of the total turbulent energy is:

$$E = E_K + E_P = E_K + \frac{g}{\theta^2} \frac{\sigma_{\theta}^2}{2N^2}$$

We note that, in stably stratified flows, there is roughly a balance between the shear production and small scale dissipation. For unstable stratification the buoyancy production term appears, increasing the turbulence level. Near neutral stratification the relative importance of  $E_P$  is small. However, at strong stratification or convective instability  $E_P$  may be a large fraction of E.

In order to obtain closure of the four prognostic equations, we need to diagnose the fluxes and the dissipation. The fluxes are obtained from empirical functions based on observations for the normalized stress and heat flux, found by Mauritsen and Svensson (2006):

$$\frac{\frac{\gamma}{E_k}}{\frac{\overline{w\theta}}{\sqrt{E_k\sigma_{\theta}^2}}} \approx -f_{\theta}(0) \cdot (0.75/(1+2Ri)^2 + 0.25),$$

Here Ri is again the gradient Richardson number and only positive values are considered. Mauritsen and Svensson (2006) found with good precision that  $f_{\tau}(0)=0.17$ , while the near-neutral value for the normalized heat-flux was more uncertain. We here use the water vapour flux value, as a proxy for a passive tracer at small Ri, giving  $f_{\theta}(0)=0.145$ . The partitioning of E between  $E_{K}$  and  $E_{P}$  is diagnosed partly from the budget equations at near-neutral, and partly from very stratified observed values, when Ri > 1. Under these conditions two thirds of E is in  $E_K$  and one third is in  $E_p$  based on observations (Mauritsen and Svensson, 2006). When Ri < 0we use the neutral values in the above functions.

The dissipation is parameterized following the ideas of Kolmogorov (1941):

$$\gamma = C_{\gamma} \frac{E\sqrt{E}}{l},$$

where  $C_{\gamma} = f_{\tau}(0)^{3/2}$  is a dependent constant and l is the dissipation length scale:

$$\frac{1}{l} = \frac{1}{kz} + \frac{f}{C_f \sqrt{\tau}} + \frac{N}{C_N \sqrt{\tau}},$$

where k = 0.4 is the von Karman constant, and  $C_f$  and  $C_N$  are constants to be determined empirically. Another option is to use squared reciprocals.

This formulation takes into account the distance to the ground, the Earth rotation and the local stratification, following Zilitinkevich and Esau (2002). Whereas they considered the bulk of the boundary layer, using the surface stress and a bulk stratification, the present approach is strictly local. This is an advantage in more complex situations, e.g. low-level jets, breaking gravity waves and baroclinic flows, that occur in reality.

The third-order flux of turbulent energy is here parameterized using the mixing-length concept,  $F_E = l^2 S dE/dz$ . This is a local formulation. The closure allows for more advanced expressions, even the non-local scheme suggested by Zilitinkevich (2002).

The equations are solved in a single-column model (SCM) setup with a staggered grid, having the first mean level at the roughness length,  $z_0$ , and the second level at  $z_0+dz$ , usually about one meter, utilizing intermediate turbulence levels. The fluxes and gradients at the first level of the model are calculated using logarithmic finite differences, while in the rest of the model linear finite differences are applied. The flux-profiles below the first level

are assumed to be linear, rather than constant. This approach was found to make the model results less sensitive to the grid resolution and yielded good conservation of heat.

The prognostic equations are solved using standard numerical methods. Each time-step the distribution of E between  $E_K$  and  $E_P$  is diagnosed. With a given Ri from the mean profile, the turbulent fluxes can be calculated. Finally, the dissipation length is calculated, and a new prognostic time-step can be made.



## **3 TUNING AND VALIDATION**

The two free constants,  $C_f$  and  $C_N$ , in the dissipation length-scale formulation could be attempted to be determined from observations. However, we suggest to 'tune' (Randall and Wielicki, 1997) the model to give reasonable results. While 'tuning' is not very popular and may be rightfully critiziced, we believe that in the present situation outlined above, this is a valid way to improve the performance of our NWP and climate models.

As a target for the tuning we have chosen to use the boundary layer height, H. This single entity has been found, to a large extent, to control for instance the surface fluxes and the ageostrophic mass-transport, which are key parameters in NWP, Svensson and Holtslag (2006b). We define H as the lowest height were the turbulent stress is below 10 % of the surface stress, divided by 0.9, and identify idealized classes of neutral and stratified boundary layers:

$$\begin{aligned} N_{+}^{2} &= 0 \qquad N_{+}^{2} > 0 \\ \hline \overline{w\theta}_{0} &= 0 \qquad \text{Neutral Long-lived} \\ \hline \overline{w\theta}_{0} &< 0 \qquad \text{Nocturnal Combined} \end{aligned}$$

Here the N refers to the initial conditions. A database of 91 LES results for these classes with varying parameters such as initial stratification, geostrophic wind, rotation and surface heat flux, was prepared. The simulations were made with  $64^3$  gridpoints due to computational limitations. This resolution is only modest with present day standards for SBL (Beare et al. 2006). They found the need to have at least  $200^3$  gridpoints for the first GABLS case. However, the particular LES employed here, was found to be rather insensitive to resolution giving results at  $64^3$ similar to the others at  $200^3$ . Further, the need for a large number of simulations and limited computational resources dictated this modest resolution. Each run was 15 hours long, comparing only the quasi-stationary last hour. The SCM was run with a fine resolution, to avoid the numerical methods to influence the

Figure 1 shows H from the LES compared to the SCM, for  $C_f = 0.18$  and  $C_N = 2.5$ . The values were chosen using the database to give zero average bias on H. The former was first chosen using the neutral runs only, while the latter was found using all runs. It is seen that the SCM is able to replicate the LES heights within 20 %, which is reasonable considering the uncertainty in LES for stably stratified conditions (Beare et al. 2006). They also point to the fact that increasing resolution in LES tends to make them less diffusive, and therefore give lower H.

results.



Figure 2 shows composite profiles for all the cases by class. It is seen that the SCM is able to capture the varying structure between the classes. Minor differences between the SCM and the LES are seen for the 'long-lived' and 'nocturnal' classes, which is a consequence of an average 4 % overprediction in former case and 8 % underprediction of H in the latter (Figure 1). These errors for the 'combined' class average 3 % overprediction for SCM. The results are consistent in the sense that a stronger downward heat-flux results in higher surface temperatures. This makes the boundary layer less stable, and thus deeper.

Figure 3 shows a comparison with the first GABLS case (Cuxart et al. 2006). It is seen that the SCM performs well. As noted above, tuning the model with the moderate resolution LES in the present study yields a somewhat diffusive behaviour, particularly in the inversion zone. At the same time the results are well within the spread between the participating LES codes. These LES used 128<sup>3</sup> gridpoints.

Also shown, and probably most important for applications, is a coarse resolution version of the SCM, with the first level at 30 m and only three levels within the boundary layer. The results are hardly discernible from the highresolution version of the SCM and the LES. Placing the first level at 30 m in applications is however questionable, since shallower boundary layers do occur (Mahrt, 1999). We recommend having as high resolution close to the ground as can be computationally afforded.

#### **4 CONCLUSIONS**

We have developed a new parameterization for stably stratified turbulence. The model differs from previous work by applying the total turbulent energy equation. Further, it is based upon observations, which are free from the selfcorrelation problem, inherent to Monin-Obukhov based models.



The closure contains two unknown constants. We propose to use LES to find these two constants. The approach allows us to 'tune' the model to give boundary layer heights close to those of the LES for a wide stability range. Future sensitivity studies should provide uncertainty ranges on these two constants.

The model performs well in comparison with other LES than the one used for 'tuning'. Preliminary results, compared to observations, look promising. The model is only weakly sensitive to resolution provided only a few computational levels are within the boundary layer. This property makes the parameterization attractive in applications such as numerical weather prediction and climate modelling, where computational power is a limitation.

## REFERENCES

- ACIA, 2005. Arctic climate impact assessment: Scientific report.
- Beare, R. J. et al. 2006. An intercomparison of large-eddy simulations for the stable boundary layer. *Bound-Layer Meteorol.* in press.
- Cuxart, J. et al. 2006. Single-column model intercomparison for a stably stratied boundary layer. *Bound-Layer Meteorol.* in press.
- Holtslag, A. A. M. 2006. GEWEX atmospheric boundary layer study (GABLS) on stable boundary layers. *Bound-Layer Meteorol.* in press.
- IPCC, 2001. Climate change 2001: The scientific basis.
- Mahrt, L. 1999. Stratified atmospheric boundary layers. *Bound-Layer Meteorol.* 90, 375-396.
- Mauritsen and Svensson, 2006. Observations of stably stratified shear-driven atmospheric turbulence at low and high Richardson numbers. 17th AMS Conference on Boundary Layers and Turbulence.
- Stull, R. B. 1988. An introduction to boundary layer meteorology. *Kluwer Academic Publishers*, 649 pp.
- Svensson, G. and A. A. M. Holtslag, 2006a. Single column modeling of the diurnal cycle based on CASES99 data – GABLS second intercomparison project. *17th AMS Conference on Boundary Layers and Turbulence.*
- Svensson, G. and A. A. M. Holtslag, 2006b. The turning of the wind with height in the stable boundary layer. In preparation.
- Zilitinkevich, S. S. 2002. Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer. *Q. J. R. Meteorol. Soc.* **128**, 913-925.
- Zilitinkevich, S. S. and I. Esau, 2002. On integral measures of the neutral barotropic planetary boundary layer. *Bound-Layer Meteorol.* **104**, 371-379.