JP5.7 EVALUATION OF DYNAMIC SUBGRID-SCALE MODELS IN LARGE-EDDY SIMULATIONS OF TURBULENT FLOW OVER A TWO-DIMENSIONAL SINUSOIDAL HILL

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1. INTRODUCTION

Large-eddy simulation (LES) can provide valuable high-resolution spatial and temporal information necessary to understand the effects of topography on turbulent transport in the atmospheric boundary layer (ABL). It consists of explicitly resolving all scales of turbulent transport larger than the grid scale Δ (on the order of tens of meters in the ABL), while the smallest (less energetic) scales are parameterized using a subgrid-scale (SGS) model. Despite the potential of LES, the strong spatial heterogeneity and flow anisotropy associated with topography hinder the performance of commonly used subgrid-scale models.

In this study, large-eddy simulation (LES) is used to study turbulent boundary-layer flow over rough two-dimensional sinusoidal hills. Three different subgrid-scale (SGS) models are tested: *(a)* the standard Smagorinsky model with a wall-matching function, *(b)* the Lagrangian dynamic model, and *(c)* the recently developed Lagrangian dynamic scale-dependent model (Stoll and Porté-Agel, 2006). The simulation results obtained with the different models are compared with turbulence statistics obtained from experiments conducted in the meteorological wind tunnel of the AES (Atmospheric Environment Service, Canada) (Gong et al., 1996). Next, a brief description of the three models is given.

1.1 *The Smagorinsky model*

The eddy-viscosity (Smagorinsky) model is commonly used in LES to parameterize the SGS stresses τ_{ij} as

$$\tau_{ij} = -2[C_S\Delta]^2 \left|\widetilde{S}\right| \widetilde{S}_{ij}, \tag{1}$$

where $\widetilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$ is the resolved strain rate tensor, $\left|\widetilde{S}\right| = 2\left(\widetilde{S}_{ij}\widetilde{S}_{ij}\right)^{\frac{1}{2}}$ is the magnitude of the resolved strain-rate tensor, Δ is the filter width, and C_S is a non-dimensional parameter called the Smagorinsky coefficient.

The value of the model parameter C_S is well established for isotropic, homogeneous turbulence ($C_S \sim 0.17$). However, anisotropy of the flow due to strong mean shear near the surface makes the optimum value of C_S depart from its isotropic counterpart. In order to account for these effects, application of eddy-diffusion models in LES of the ABL has involved the use of various types of *ad hoc* wall damping corrections. For example, Mason and Thomson (1992) proposed to use the equation

$$\frac{1}{\lambda^n} = \frac{1}{\lambda_o^n} + \frac{1}{[\kappa(z+z_o)]^n}, \tag{2}$$

where κ is the von Karman constant, $\lambda = C_S\Delta$ is the length scale in the model, $\lambda_o = C_o\Delta$ is the length scale far from the wall, z_o is the roughness length, and C_o and n are adjustable parameters. This matching function has been used with different values of C_o (from 0.1 to 0.3) and n (1, 2, and 3).

1.2 *The Lagrangian dynamic model*

The dynamic procedure (Germano et al., 1991) provides a systematic way to calculate the value of the model coefficient (C_S^2) at every time and position in the flow based on the dynamics of the smallest resolved scales. By applying the Smagorinsky model at two different resolved scales (typically the filter scale Δ and twice the filter scale, 2Δ) and assuming scale invariance of the model coefficient (i.e. $C_S^2(2\Delta) = C_S^2(\Delta)$), one can optimize the value of C_S^2 (Germano et al., 1991). In order to implement the dynamic model, some sort of averaging needs to be used to guarantee numerical stability of the procedure. Typically averaging is done over directions of flow homogeneity (e.g., horizontal planes over flat homogeneous terrain), or along flow pathlines using the Lagrangian averaging procedure developed by Meneveau et al. (1996). Lagrangian dynamic models are therefore suitable for simulations of the ABL over complex terrain, where there is no direction of homogeneity in the flow.

The dynamic model avoids the need for a-priori specification or tuning of the coefficient because it is evaluated directly from the resolved scales in the LES. However, recent studies have shown that the dynamic models have problems to reproduce the correct flow statistics over both flat surfaces (Porté-Agel et al., 2000) as well as complex terrain (Iizuka and Kondo, 2004).

1.3 *The Lagrangian scale-dependent dynamic model*

Recently, Porté-Agel et al. (2000) proposed a scale-dependent dynamic model, a modification of the dynamic procedure that allows the model coefficient to change with scale (i.e. not assuming that $C_S^2(\Delta) = C_S^2(2\Delta)$). By using information on the dynamics of the flow corresponding to an additional test-filter scale (e.g. 4Δ) the scale-dependent model has the ability to detect and account for

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scale dependence in a dynamic manner (based on the information of the resolved field and, thus, not requiring any tuning of parameters). In particular, the scale-dependent dynamic model is used to dynamically calculate not only $C_S^2(\Delta)$, but also the value of the scale-dependence coefficient $\beta = C_S^2(2\Delta)/C_S^2(\Delta)$.

Lagrangian scale-dependent dynamic models have successfuly been implemented in simulations of ABLs over flat heterogeneous terrain (Bou-Zeid et al., 2005; Stoll and Porté-Agel, 2006). In this paper, we study the performance of the Lagrangian scale-dependent dynamic model in simulations of a boundary layer over rough two-dimensional sinusoidal hills.

2. NUMERICAL EXPERIMENTS

The large-eddy simulation code is a modified version of the code described by Albertson and Parlange (1999), Porté-Agel et al. (2000), and Stoll and Porté-Agel (2006). The code uses a mixed pseudospectral finite-difference method. Consequently, periodic boundary conditions are assumed in the horizontal directions. The upper boundary condition is a fixed stress-free lid. The lower boundary condition consists of using similarity theory (the logarithmic law) to calculate the instantaneous (filtered) surface shear stress as a function of the velocity field at the lowest computational level.

The simulated physical domain corresponds to two sinusoidal waves with elevation

$$z_s = a\ \cos 2x, \tag{3}$$

where $a = 0.249$ is the normalized wave amplitude, and x is the normalized streamwise position (see Figure 1). The flow direction is perpendicular to the wave crests. The coordinate transformation developed by Clark (1977) has been used to transform the sinusoidal wave bounded physical domain into a rectangular computational domain. In order to match the wind-tunnel experimental conditions of Gong et al. (1996), the computational domain, after normalization with the boundary layer height $L_z = 194$ mm, is of size (2π, 2π, π). The non-dimensional aerodynamic surface roughness height is $z_o/L_z = 2.06 \times 10^{-3}$. The computational domain is divided into $80 \times 80 \times 80$ uniformly spaced grid points. Wind velocities are normalized using the free stream wind tunnel velocity, $U_o = 10$ m/s.

A horizontal pressure gradient is exerted on the flow in the streamwise direction. The magnitude of this pressure gradient is set to balance the drag forces (surface stress and form drag) measured during the experiment (Gong et al.,1996). The value of the non-dimensional pressure gradient is 0.654.

3. RESULTS

Figure 2 shows the simulated non-dimensional velocity profiles over the wave crests obtained with the different SGS models under consideration: the Smagorinsky model with two different matching functions (SMAG-1: $Co = 0.1$ and $n = 2$; and SMAG-2: $Co = 0.17$ and $n = 1$

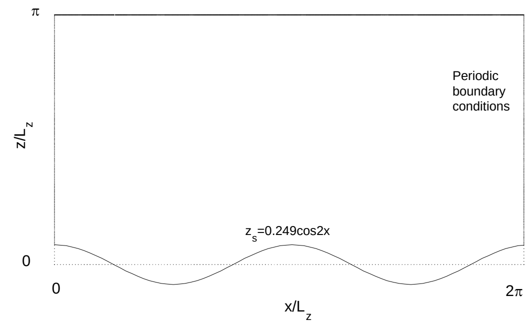

FIG. 1: Schematic of computational domain over the two-dimensional sinusoidal hills.

in Eq. 2), the Lagrangian Dynamic model, and the Lagrangian Scale-Dependent Dynamic model. Results are compared with wind tunnel data (symbols) of Gong et al. (1996). The Lagrangian dynamic model clearly overestimates the average velocity near the surface by as much as 20%. A similar behavior of the velocity over the crest was reported by Iizuka and Kondo (2004) in a numerical study of flow over a single two-dimensional hill. The scale-dependent dynamic procedure substantially improves the results.

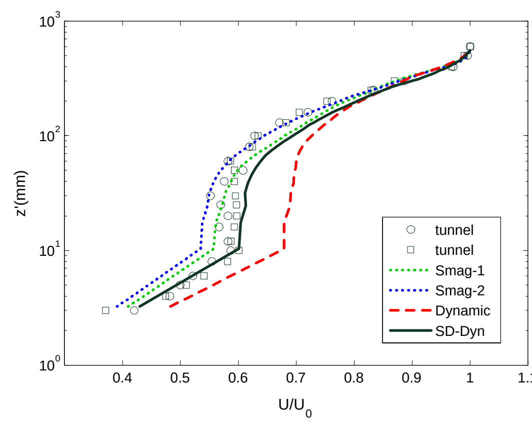

FIG. 2: Non-dimensional velocity profiles over the wave crests from wind tunnel data (symbols) and from LES with different SGS models: Smagorinsky model with two different matching functions (dotted lines), dynamic model (dashed line), and scale-dependent dynamic model (solid line).

The non-dimensional standard deviation of the vertical velocity over the wave crests is presented in Figure 3. The dynamic model overpredicts the level of fluctuations of the vertical velocity and also the horizontal velocity components (not shown here). The overestimation of the velocity variance is consistent with the idea that the dynamic model is not dissipative enough, and it is in good agreement with previous studies over flat terrain (Porté-Agel et al., 2000).

The dynamically calculated values of the model coefficient C_S^2 obtained using the Lagrangian dynamic and

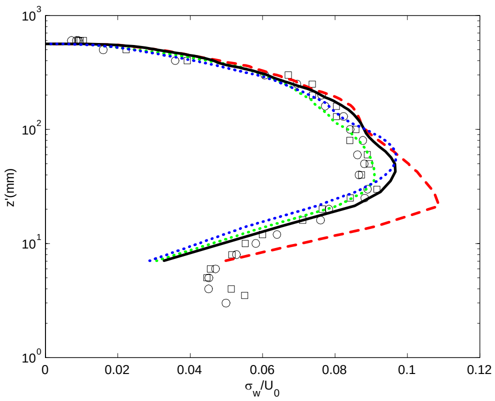

FIG. 3: Non-dimensional standard deviation of the vertical velocity over the wave crests from wind tunnel data (symbols) and from LES with different SGS models: Smagorinsky model with two different matching functions (dotted lines), dynamic model (dashed line), and scale-dependent dynamic model (solid line).

Lagrangian scale-dependent dynamic models are presented in Figures 4 and 5, respectively. It is important to note that the value of the coefficient is substantially larger for the scale-dependent model. As expected, both coefficients decrease as the distance to the surface decreases. In addition, there is a clear dependence of the coefficient on horizontal position. For the same distance to the ground, the coefficient is smaller near the crest, where the flow undergoes strong straining. Alternatively, the coefficient is larger downwind of the crest, where the flow detaches from the surface (recirculation region) and is subject to smaller strain rates.

Figure 6 shows the value of the scale dependence parameter β obtained dynamically with the Lagrangian scale-dependent dynamic model. The value of β is close to 1 away from the surface, where the flow is more isotropic at the smallest resolved and subgrid scales and,

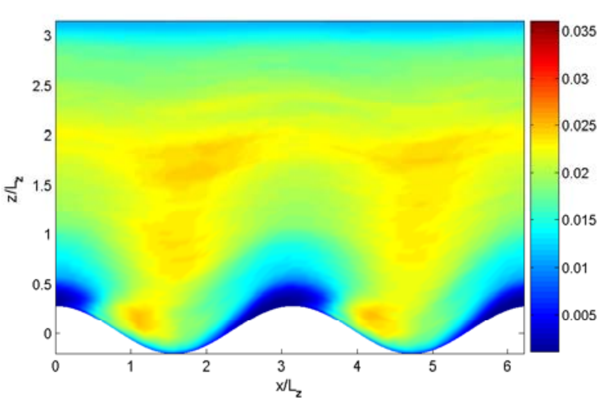

FIG. 4: Smagorinsky coefficient (C_S^2) obtained with the dynamic model. Results are averaged over time and spanwise direction.

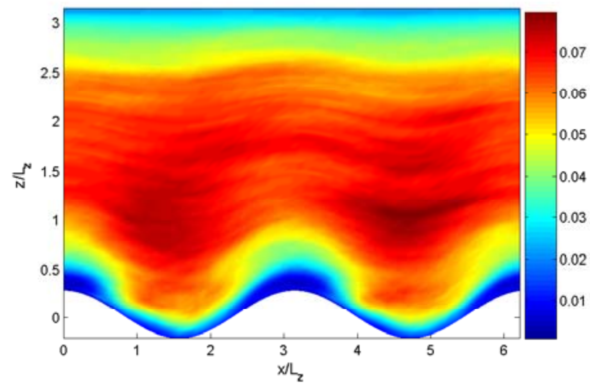

FIG. 5: Smagorinsky coefficient (C_S^2) obtained with the scale-dependent dynamic model. Results are averaged over time and spanwise direction.

consequently, C_S^2 is scale invariant. β becomes smaller as the surface is approached due to increased shear and anisotropy of the flow. The smallest values are found near the crest, particulary in the upwind side, where the shear and anisotropy of the flow are stronger.

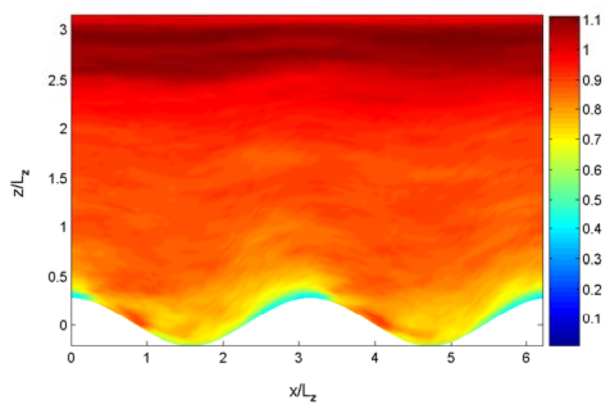

FIG. 6: Scale dependence parameter $\beta = C_S^2(2\Delta)/C_S^2(\Delta)$, obtained with the scale-dependent dynamic model. Results are averaged over time and spanwise direction.

4. SUMMARY

Large-eddy simulation (LES) has been used to simulate turbulent boundary-layer flow over rough two-dimensional sinusoidal hills. Three different subgrid-scale (SGS) models are tested: *(a)* the standard Smagorinsky model with a wall-matching function, *(b)* the Lagrangian dynamic model, and *(c)* the recently developed Lagrangian scale-dependent dynamic model (Stoll and Porté-Agel, 2006). The simulation results obtained with the different models are compared with turbulence statistics obtained from experiments conducted in the meteorological wind tunnel of the Atmospheric Environment Service of Canada (Gong et al., 1996).

Dynamic models have the important advantage of providing tuning-free simulations since the model coefficient is calculated based on the dynamics of the resolved flow scales. However, the flow simulated using the Lagrangian dynamic model shows important differences compared with the wind tunnel experimental data. In particular, the Lagrangian dynamic model is not dissipative enough, leaving too much kinetic energy in the resolved flow. The model overestimates the magnitude of the velocity over the wave crests by about 20%, which is in agreement with the simulation results of Iizuka and Kondo (2004) for flow over a single two-dimensional hill.

By relaxing the assumption of scale invariance in the dynamic model, the scale-dependent dynamic model is able to dynamically (without any parameter tuning) capture the scale dependence of the model coefficient using information from the smallest resolved scales. Our results show that this procedure substantially improves the simulation results with respect to the scale-invariant dynamic model.

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