1. INTRODUCTION

Collisions between small non-spherical ice crystals determine the formation of larger crystals and snowflakes in clouds. Collisions of non-spherical ice crystals and water drops give rise to the formation of graupel in clouds. As a whole, particle collisions are the key processes determining particles size spectrum evolution. As such they are of crucial importance for understanding and appropriate description of ice microphysics and ice precipitation.

At the same time investigation of these processes is far from being completed due to: a) difficulties in estimation of hydrodynamic forces and torques acting on particles; b) realistic representation of turbulent field, characteristic for cloudy conditions. Due to these difficulties there are only a few theoretical investigations on droplets collisions in a turbulent medium at present (e.g. De Almeida, 1976 and 1979; Grover and Pruppacher, 1985; Pinsky et al. 1999). All of them indicate that turbulence increases collision rate of cloud droplets several times. It is natural therefore to expect analogous influence of atmospheric turbulence on ice crystals. However no theoretical investigations on ice crystals collisions were reported in literature up to now (Pruppaher and Klett, 1997).

The case of small Stokes number, or Stokesian, particles simplifies significantly the collision problem. In this study we represent a novel approach for evaluation of Stokesian non-spherical particles collision statistics together with preliminary results. Prolate and oblate spheroids were chosen as an example of non-spherical particles. Strongly elongated spheroids can model needle-like ice crystals, while oblate spheroids – plate-like hexagonal prism, one of the most abundant ice crystals form.

2. STOKESIAN PARTICLES COLLISION

The rate of collisions is determined by collision kernel that describes the probability of collisions between two particles per unit time. It is accepted to represent this probability as the product of collision probability without hydrodynamic interaction (hereafter HDI) between particles and collision efficiency that takes into account the effects of HDI. The rate of collisions with and without HDI is determined by the volume, swept (hereafter referred to as SV) by colliding particles in their relative motion per unit time correspondingly with and without HDI. In this study we limit ourselves by the case without HDI.

For turbulent field representation an approach, elaborated by Pinsky et al. (2004, 2005), was employed. According to this approach, turbulent field is represented by a set of non-correlated samples of turbulent field. Each sample (hereafter, elementary volume) can be assigned to a certain point of a turbulent flow at a certain time moment. The scales of elementary volume are determined as those within which Lagrangian acceleration of air velocity, $A_J$, and tensor of velocity strain, $S$, can be considered uniform in space and invariant in time. As was shown by Pinsky et al. (2005), these scales are of order Kolmogorov length and time scales. Similar statistical representation of a turbulent flow as a set of small volumes (packets) within which energy dissipation rate, velocity shears and particle concentration were assumed uniform was used recently by Koch and Pope (2002).

Let us introduce the volume, within which HDI is important. Simple scale considerations show that the scales of HDI volume lie within elementary volume for mass equivalent radius $1\mu m \leq r_{eq} \leq 20\mu m$. This is the particles size range for which one can regard collisions as occurring at constant $A_J$ and $S$.

To simulate large number of collisions long series (50,000) of $S$ and $A_J$ pairs (field realizations) were produced. These pairs were generated by means of statistical generators, reproducing Lagrangian accelerations and shears with probability distribution functions (PDFs) found in cloud measurements and laboratory
experiments for high Reynolds numbers (Antonia et al., 1981; Belin et al., 1997; Hill and Thoroddsen, 1997; Kolmyansky et al., 2001; La Porta et al., 2001; Voth et al., 2002).

Another important feature of our approach is that we approximate particle motion in elementary volumes with the help of approximate analytical solution. This solution allows obtaining particle orientation and velocities (translation and angular) probability distribution functions (PDFs). This is a generalization of an approach used by Pinsky et al. (2005) for investigation of small cloud droplets collisions. The knowledge on these PDFs allows in its turn calculation of SV as the integral over all possible collision variants properly weighted.

Estimation of SV for generated series of $A$ and $S$ produce SV series. Mean SV was estimated by averaging over these series. SV PDF was obtained by calculation series histograms.

3. **ANALYTICAL SOLUTION**

Equations, describing spheroid motion in the creeping flow approximation, are (for, example, Broday et al., 1998):

\[ m \frac{d\mathbf{v}}{dt} = K (\mathbf{u} - \mathbf{v}) - mg(1 - \frac{\rho_f}{\rho_p}) \mathbf{e}_y, \]  

\[ l \frac{dw}{dt} + d\mathbf{w}_q \cdot \mathbf{w} = \mathbf{Q}(\mathbf{R} \times \mathbf{u}) - \Omega \mathbf{w}. \]  

$\mathbf{u}$ is a fluid velocity; $K$, $\mathbf{Q}$ and $\Omega$ are tensors of particle resistance to translation and rotation. These equations imply: a) small particle Reynolds number approximation; b) assumption on constant $S$. We apply the method of successive approximations (Korn and Korn, 1968), using solution in the non-inertial limit ($m \to 0$) as a starting point.

3.1 **Translation velocity**

The approximate solution is:

\[ \mathbf{v} = \mathbf{u} + K_v(\mathbf{e}) \cdot \sum_{i=1}^{\infty} G_i(\mathbf{e}), \]  

Matrix $K_v$ and vectors $\{G_i\}$ are functions of spheroid orientation, $\mathbf{e}$. Consequently spheroid velocity deviation from air velocity, $\mathbf{v}' = \mathbf{v} - \mathbf{u}$, is also function of its orientation only. This solution provides relatively simple recurrent relation for vectors $\{\mathbf{G}_i\}$.

Let’s define Stokes number to be $St = \tau_t/\tau_f$. Here $\tau_t \approx 10^{-3}$ s characterizes particle adjustment time to fluid translation, and $\tau_f \approx 1/Sh$ is a characteristic time of fluid velocity variation ($Sh$ - characteristic shear). The truncation error is of order $(St)^k$, $k$ being first neglected term in (8). For example, omitting terms of order $i \geq 2$ for $St = 0.04$ (corresponds to $r_{eq} = 20 \mu m$) one obtains truncation error less than 0.2%. The important consequences of (8) are that $\mathbf{v}'$ is defined basically by the Lagrangian acceleration of the fluid and increases approximately as $a^2$. Both these conclusions imply $St \ll 1$.

3.2 **Angular velocity**

The approximate solution is:

\[ \mathbf{w} = K_w(\mathbf{e}) \cdot \sum_{i=1}^{\infty} \mathbf{R}_i(\mathbf{e}) \]  

$\mathbf{\xi}$ is the spheroid angular velocity in the non-inertial limit. Matrix $K_w$ and vectors $\mathbf{\xi}$ and $\{\mathbf{R}_i\}$ are all functions of spheroid orientation. Consequently spheroid angular velocity is also determined by orientation. Again there exists recurrent relation for vectors $\{\mathbf{R}_i\}$. The truncation error in (9) scales again as $(St)^k$.

If exact solutions of (6) and (7) are bounded (stable behavior), then any arbitrary initial velocity approaches quickly (on time scale of $\tau_t$) to a solution, shown schematically in fig. 1 by a dashed line (limit solution). Solutions (8) and (9) do not describe transient processes but converge directly to this solution. We therefore refer to them as limit solutions. Main inaccuracy when using (8) and (9) arises rather due to the difference between limit and exact solutions than due to truncation procedure. This difference however is small for small $St$. If exact solutions of (6) and (7) are unbounded (unstable behavior), solutions (8) and (9) diverge. It means that they can not describe the so-called sling effects. Such events however are rare for Stokesian particles in atmosphere (Pinsky et al., 2005).

To check solutions (8) and (9), we compared them with numerical solution of (6) and (7) in several situations: a) simple shear flow; b) Poiseuille flow; c) realization of turbulent filed. In all the cases the difference did not exceed 0.1%.
4. RESULTS

We calculated mean SV and SV PDF for a wide range of turbulent field parameters: $5 \times 10^3 \leq \text{Re}_a \leq 20 \times 10^3$ and $0.002 \leq \varepsilon \leq 0.1$ ($\text{Re}_a$ - Taylor scale based Reynolds number; $\varepsilon$ - energy dissipation rate). This range covers cloudy turbulence from stratiform up to deep cumulus clouds. The calculations were performed for spheroids of different sizes ($1 \mu m \leq r_{eq} \leq 20 \mu m$) and different aspect ratios ($0.05 \leq \beta \leq 0.2$). Here we present some of the results.

Fig. 2 shows SV PDF (histogram) for pair of oblate spheroids with radii $15 \mu m$ and $10 \mu m$ ($\text{Re}_a = 20 \times 10^3, \varepsilon = 0.05 m^2 s^{-3}$). Fig. 3 shows mean SV for the pair of prolate spheroids with equivalent radii $15 \mu m$ and $10 \mu m$ ($\text{Re}_a = 20 \times 10^3$). Fig 4 shows the same results for oblate spheroids of the same mass. SV is normalized everywhere by SV in a pure gravity case. Figures 5 and 6 give analogous results for spheroids with equivalent radii $2 \mu m$ and $1 \mu m$. Fig. 7 and 8 show SV for prolate and oblate spheroids of similar size: $r_{eq1} = 15 \mu m$, $r_{eq2} = 15 \mu m - \Delta r_{eq}$. Reynolds number in this case is $20 \times 10^3$, dissipation rate $0.05 m^2 s^{-3}$. $\Delta r_{eq}$ varies between $0.2 \mu m$ and $1 \mu m$. The results manifest that:
- mean SV for spheroids is higher than that for droplets of the same mass; the effect enlarges with aspect ratio deviation from unity, being more pronounced in case of elongated spheroids.
- influence of turbulence on mean SV becomes especially large for small particles (of order $1 \mu m$) and particles of similar size.

The proposed method incorporates three main approximations: a) small particle Reynolds number approximation; b) assumption on constant $\bar{A}$ and $S$ during collision; c) approximation of motion equations solution by means of the limit solution. While it is not possible to evaluate error due to the second approximation, one can check the validity of the first and third approximations.

To check small Reynolds approximation we evaluated characteristic value of spheroid Re in all 50,000 realizations and calculated the part, in which Re turned out to be larger than 0.1. While for $r_{eq} = 15 \mu m$ spheroid accuracy turned out to be sufficient in all cases, for $r_{eq} = 20 \mu m$ error becomes large for intensive turbulent field. For $r_{eq} = 2 \mu m$ all realizations resulted in $\text{Re} < 0.1$.

To check the third approximation we compared numerical and limit solutions and calculated the part of realizations, in which this difference exceeded 5%. Again $r_{eq} = 15 \mu m$ spheroid provided sufficient accuracy. Accuracy for $r_{eq} = 20 \mu m$ spheroid was sufficient in the case of oblate spheroid, but in the case of elongated form and intensive turbulent field it became low. For $r_{eq} = 2 \mu m$ spheroid accuracy was again excellent in all cases.
Fig. 3  Mean SV for the pair of prolate spheroids \((r_{eq} \text{ are } 15\mu m \text{ and } 10\mu m)\) as a function of dissipation rate.

Fig. 4  The same as in fig. 3 for oblate spheroids of the same masses.

Fig. 5  The same as in fig. 3 for prolate spheroids with \(r_{eq} 2\mu m \text{ and } 1\mu m\).

Fig. 6  The same as in fig. 3 for oblate spheroids with \(r_{eq} 2\mu m \text{ and } 1\mu m\).

Fig. 7  Mean SV for the pair of prolate spheroids \((r_{eq} \text{ are } 15\mu m \text{ and } 15\mu m - \delta r_{eq})\) as a function of \(\delta r_{eq}\).

Fig. 8  The same as in fig. 7 for oblate spheroids of the same masses.
As a whole, together with the restriction due to assumption on constant field parameters, \( r_{eq} \approx 20 \mu m \) may be regarded as the upper limit for the validity of the method proposed. It is worthy noting, however, that the errors due to both small Reynolds and employment of limit solution reduce quickly with decreasing \( \varepsilon \) and for \( \varepsilon = 0.01 m^2 s^{-3} \), for example, main limitation comes from the assumption on constant \( A \) and \( S \).

5. CONCLUSIONS

A novel approach is elaborated for investigation of Stokesian non-spherical particles collisions in a turbulent field, characteristic for cloudy conditions. Turbulent field is represented as a large set of small field samples (elementary volumes), within which field parameters may be regarded constant in space and time. Approximate analytical solution of spheroids motion equations in a general shear flow is found and employed for description of spheroid motion in elementary volumes. This solution allows obtaining spheroid orientation and velocities PDFs. The statistics of SV (PDF and mean value) is estimated by means of averaging over SV series, calculated for generated series of elementary volumes. This approach may be applied for estimation of collision statistics both with and without HDI. In the present, first, stage the case without HDI was regarded.

SV PDF and mean value were calculated for the wide range of turbulent field parameters, representing turbulence intensity from stratiform up to deep cumulus clouds, and for different spheroids sizes and aspect ratios (from strongly elongated up to plate-like forms). The results indicate that turbulence enhances SV significantly comparing with the pure gravity case. The effect increases with turbulence intensity and aspect ratio deviation from unity, and becomes especially pronounced in case of similar and micron size spheroids.

Analysis shows that for spheroids mass equivalent radii \( 1 \mu m \leq r_{eq} \leq 20 \mu m \) the approach proposed may be regarded valid. It is worthy noting however that this range depends on turbulent field intensity and for calm enough turbulence (e.g. \( \varepsilon = 0.01+0.02 m^2 s^{-3} \)) it may be wider.

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