14.3 CRITICAL ANALYSIS OF RESULTS CONCERNING DROPLET COLLISIONS IN TURBULENT CLOUDS

A. Khain* (1), M. Pinsky (1) , T. Elperin(2), N. Kleeorin (2) and I. Rogachevskii(2)
(1)The Hebrew University of Jerusalem, Israel
(2)Ben-Gurion University of Negev, Israel

1. INTRODUCTION

It is widely accepted now that turbulence enhances the rate of particle collisions (see overviews by Pinsky et al 2000; Vaillancourt and Yau 2000; Shaw 2003).

There exist three major mechanisms that turbulence affects the collision rate: a) increase in the relative particle velocity (or increase in the swept volume); this effect is also known as turbulent transport effect; b) formation of concentration inhomogeneity (particle clustering), and c) turbulence effect on the hydrodynamic drop interaction (HDI) that leads to an increase in the collision efficiency.

Several main methods of investigation of the turbulent effects are used: analytical studies considering droplet motion in an idealized turbulent flow, direct numerical simulations (DNS), utilization of statistical models of turbulent flow and hybrid methods.

* *Corresponding author address: Alexander P. Khain, The Hebrew University of Jerusalem, Dept. Atmos. Sciences, Israel,*

e-mail: Khain@vms.huji.ac.il

During the past ten years numerous studies dedicated to the problem of collisions of inertial particles in turbulent flows have appeared (some of them are presented in the reference list). A wide range of the turbulence-induced collision enhancement factor was reported in these studies: from a few percent to several hundred.

Authors of majority of the studies apply their results to explanation of rain formation in atmospheric clouds. At the same time many investigations have been performed under conditions quite different from those in real clouds. Thus, a special analysis is required as concerns the applicability of one or another result to actual clouds.

The typical discrepancies between conditions assumed in most of the studies and conditions in the actual clouds are the following:

a) In many studies particle behavior within high concentration monodisperse suspensions was analyzed. High concentrations were assumed for simulations of both cloud droplets with radii below ~20 μm, as well for drops with radii larger 20 μm, including small rain drops with radii 40-70 μm. The mass loading in the simulations assuming high concentration of small rain drops often attains one that corresponds to the liquid water content of 1000 gm^{-3}. In contrast, real clouds represent very low concentration suspensions of droplets having a wide range of sizes. Majority of cloud droplets have radii below 20 μm. Droplet concentration ranges from 50 cm^{-3} in maritime clouds to 1000 cm^{-3} in very continental clouds, so that mean separation distance between droplets usually exceeds 1 mm. For concentration of 40-70 μm-radii drops is a few cm^{-3}, the mean separation distance between such drops usually exceeds 1 cm. The maximum liquid water contents are observed in convective clouds and do not exceed 4 gm^{-3}, so that mass loading is relatively low, of the order of 1-4x10^{-3}.

b) In most of theoretical studies gravity-induced sedimentation is neglected. At the same time the range of drop sedimentation velocities in clouds is quite wide, being proportional to the square of droplet radius. This neglecting is particularly anappropriate and invalid for raindrops, for which the role of gravitation may be dominant.

c) Most theoretical studies and laboratory experiments were performed in turbulent flows characterized by the Taylor microscale Reynolds numbers $\mathrm{Re}_\lambda < 10^2$, while Re_λ varies in the range from $\sim 5\cdot 10^3$ in stratiform clouds to $\sim 2\cdot 10^4$ in strong deep convective clouds (see, e.g., Pinsky et al 2006a).

Application of the results obtained under the conditions quite different from those in real clouds to investigation of cloud evolution may lead to a wrong conclusions on the role of turbulence in clouds and may hinder the correct understanding of cloud physics. As an illustration of such situation, we present results of simulations of development of a deep continental cloud typical of Texas during summertime (Rosenfeld and Woodley 2000; Khain et al 2001; 2004). Vertical velocities in such clouds often exceed 25-30 m/s, the cloud top heights are of 12-14 km. The turbulence in these clouds is quite strong, with dissipation rate $\sim 10^3\ cm^2 s^{-3}$. Droplet concentration in such clouds is about $10^3\ cm^{-3}$, the mean droplet radius is about 6-7 μm. The droplets ascend within strong updrafts to high levels, where they give rise the formation a large amount of ice crystals, small graupel and snow that spread over large areas in cloud anvils. As a result, these clouds produce as a rule quite small precipitation at the surface, mainly due to melting of cloud ice. These clouds do not produce warm rain whereby the raindrops fall to the surface without freezing.

Two simulations were performed using the Hebrew University cloud model with spectral (bin) microphysics (Khain et al, 2004). These simulations differ by the collision enhancement factors indicating the increase in the collision kernel. In the first simulation (run 1) the collision enhancement factor varied within the range 1-5 for cloud droplets, as it was evaluated by Pinsky and Khain (2004). For larger drops the magnitude of the enhancement factor was assumed 1.2 (Pinsky and Khain 1997b). In the second simulation (run 2) the collision enhancement factor was calculated using the results obtained by Riemer and Wexler (2005) (**Figure 1**)

Figure 2 indicates the rain water mass content obtained in run 1 (left panel) and run 2 (right panel).

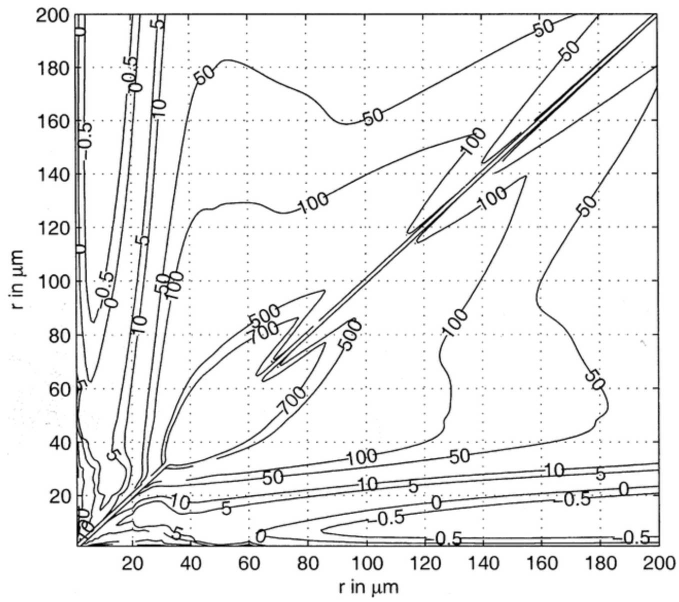

Figure 1: The dependence of the ratio $(K_{turb}-K_{grav})/K_{grav}$ on droplet size (after Riemer and Wexler 2005). K_{turb} is the collision kernel in a turbulent flow, and K_{grav} is the collision kernel in still air. One can see that within a wide range of the drop sizes the enhancement factor exceeds 50 and reaches 700 in the maximum for ~50 μm-radii drops .

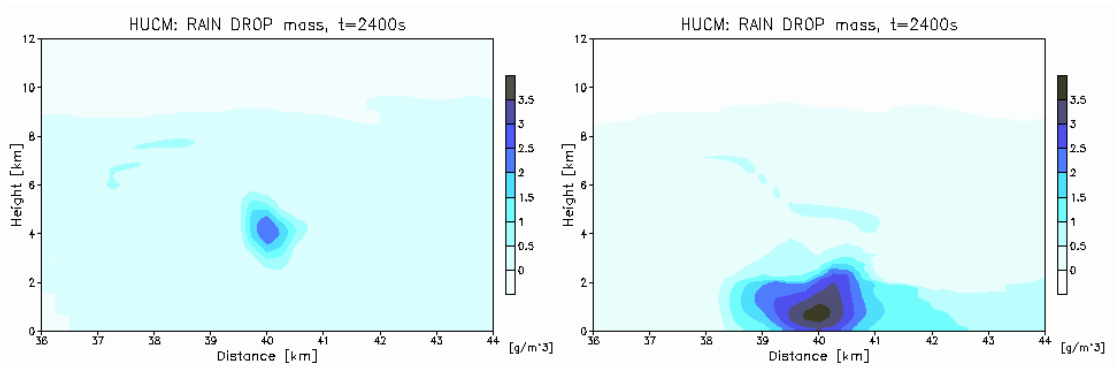

Figure 2. Rain water mass content distributions obtained in simulations with the collision enhancement factor adopted from Pinsky and Khain (1997b, 2004) (left panel) and with the collision enhancement factor adopted Riemer and Wexler (2005) at t=40 min.

In the case when magnitude of the collision enhancement factor is equal to the estimate by Riemer and Wexler (2005), precipitation at the surface starts 7 min after the cloud formation and precipitation rate attains ~40 mm/h. This heavy precipitation is caused by large liquid rain drops and by melted frozen drops. Such fast rain development is typical of extremely maritime clouds and never has been observed in continental clouds. At the same time, no surface precipitation occurs during 40 min in run 1. In agreement with the observations, the evolution of the cloud in run 1 is typical of highly continental cloud without warm rain at the surface. Therefore, employment of the collision enhancement factor calculated by Riemer and Wexler (2005) leads to an unrealistic description of warm and ice clouds microphysics.

In this review we evaluate the applicability of the results obtained in different studies to actual clouds. We discuss the turbulence effects on a) turbulence induced relative droplet velocity; b) droplet concentration fluctuations (droplet clustering) and c) hydrodynamic interaction of droplets. Finally we discuss some problems related to parameterization of collision kernel for cloud modeling.

2. TURBULENCE-INDUCED RELATIVE VELOCITIES BETWEEN DROPLETS (TURBULENT TRANSPORT EFFECT)

An important characteristic of a drop's inertial response to a turbulent flow is Stokes number $St = \varepsilon^{1/2}\nu^{-1/2}\tau$ (ε is the dissipation rate, τ is drop relaxation time, ν is the kinematic viscosity of air), which is normally below 0.3 for cloud droplets. Small ~50 μm-radii rain drops (or drizzle particles) are characterized by $St \sim 0.5 - 1.0$. Since the behavior of cloud droplets and small rain drops in a turbulent flows is quite different, as well as the levels of understanding of this behavior, we will consider them separately.

It is a usual practice in the state-of the art studies in Cloud Physics (both theoretical and numerical) to calculate drop collisions in clouds under pure gravity conditions (a still air assumption). Thus, to evaluate effect of turbulence on cloud evolution, it is necessary to calculate the collision enhancement in a turbulent flow as compared to still air conditions.

2a. Cloud droplets

Behavior of relative velocities between droplets in turbulent flows was investigated in numerous analytical and DNS studies. In most theoretical and DNS studies the effect of differential sedimentation was neglected. The relative velocities between droplets in turbulent flow in these studies are normalized by the standard air velocity fluctuations, or compared with those in zero-inertia case. The results of these studies do not allow one to evaluate the collision enhancement factor caused by the turbulent transport effect as compared to that in still air. There are a few studies where the gravitation sedimentation is taken into account that allow such comparison. Saffman and Turner (1956), Wang et al (1998) and Dodin and Elperin (2002) presented analytical formulas accounting for sedimentation effect. This effect was taken into account by Pinsky et al (2006a) using a statistical model of turbulent flow and by Franklin et al (2005) in DNS simulations.

Figure 3 presents a comparison of the results obtained in some of these studies. The difference in the results does not exceed 10-

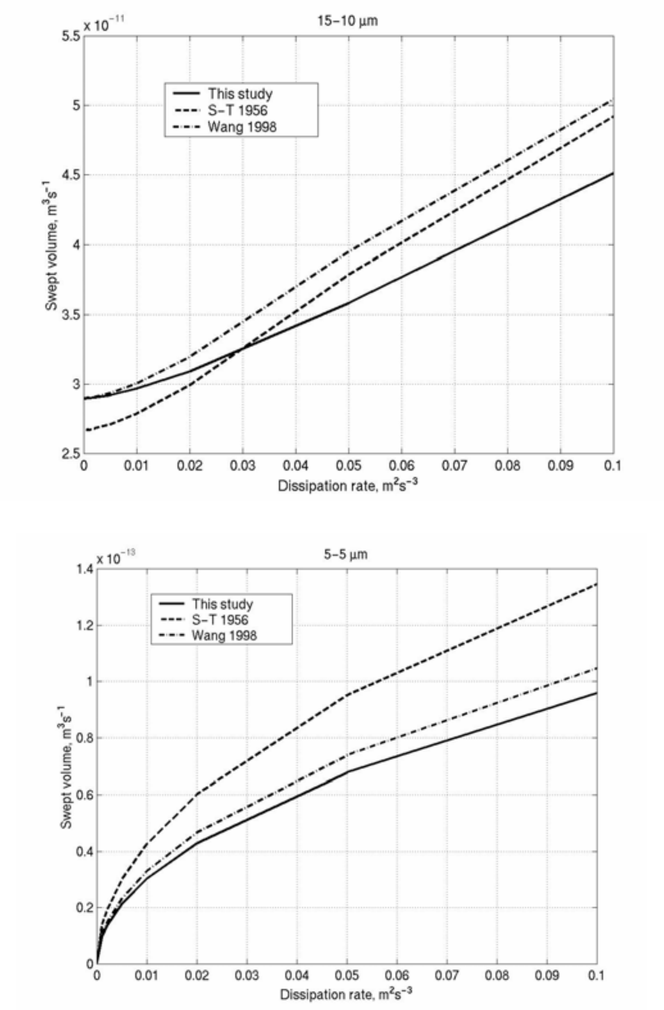

***Figure 3**: Dependence of the turbulent induced swept volumes (which are proportional to the relative velocities between droplets) on the dissipation rate according to results of* Saffman and Turner (1956), Wang et al (1998) and Pinsky et al (2006a) (marked as "this study") *for 10 μm and 15 μm –radii droplet pair (first panel); and 5 μm-5 μm radii droplet pair second panel).* $\mathrm{Re}_\lambda = 2 \cdot 10^4$ *(after Pinsky et al 2006a).*

15% and can be attributed to different formulation of collision kernels (cylindrical vs spherical formulation) and to Gaussian/non-Gaussian assumptions about the velocity distributions.

Results obtained by Dodin and Elperin (2002) agree well with those obtained by Wang et al (1998).

Figure 4 shows the dependence of the mean normalized swept volume on turbulent dissipation rate ε for the 15-10 μm -radii droplet pair at different Re_λ according to the results obtained by Pinsky et al (2006a). As seen from Figure 4, the increase in the relative velocities between cloud droplets does not exceed 60% even under very strong turbulence. These results agree well with those obtained in DNS by Franklin et al (2005), who evaluated the increase in the relative velocity as compared to the difference in gravity-induced terminal velocities by 1.2-1.4 times for 10-15 μm -radii droplet pair. Smaller enhancement factors obtained by Franklin et al (2005) as compared to those obtained by Pinsky et al (2006a) can be attributed to small Re_λ used in the DNS.

Thus, the enhancement factor for cloud droplets caused by turbulent transport effect can be estimated to vary in the range from a few to several tens of percents depending on the turbulence intensity and droplet size.

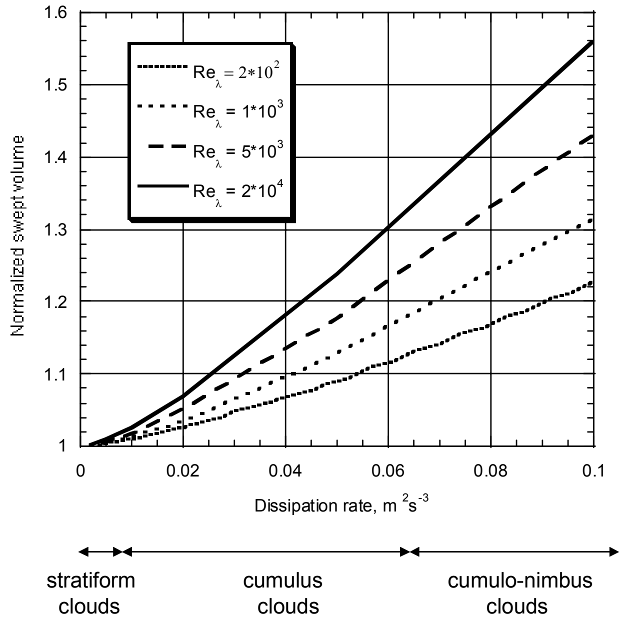

Figure 4. Dependence of mean normalized swept volume on turbulent dissipation rate ε for the 15-10 μm -radii droplet pair at different Re_λ (after Pinsky et al 2006a).

2b. Small rain drops (40-70 μm radii)

Motion of small rain drops in a turbulent flow differs significantly from that of air parcels because both large inertia and large gravity induced sedimentation velocities. Small rain drops response to turbulent vortices of scales larger than those for cloud droplets. **Figure 5** shows spectra of relative velocities for three 10μm-30μm, 10μm-50 μm and 10μm-100 μm radii drop pairs. The calculations are performed following Khain and Pinsky (1995) assuming Kolmogorov's spectral density in the inertial subrange and taking gravity effects into account.

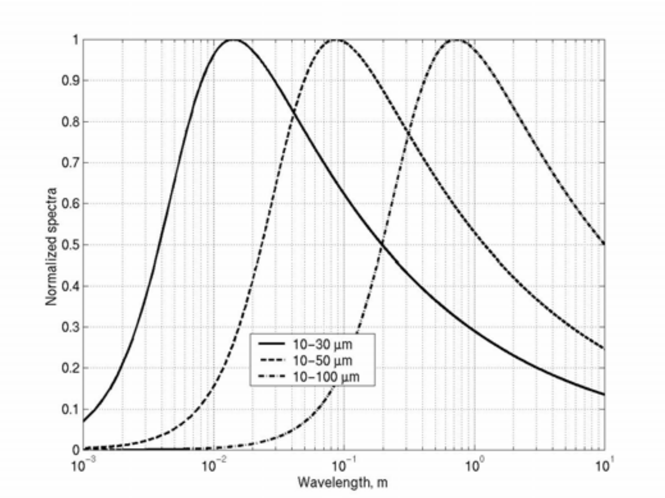

Figure 5. Relative velocity spectra (normalized by their maxima) for 10μm-30μm, 10μm-50 μm and 10μm-100 μm-radii drop pairs (after Khain and Pinsky 1995).

One can see that the shape of spectrum is determined by the larger drop in a drop pair. The maxima of these spectra are located at the wavelengths of 1.5 cm, 8 cm and 70 cm, respectively. These maxima represent the characteristic sizes of the turbulent vortices that affect the relative velocity between the drops in these pairs. The relative motion between drops in drop pairs containing small rain drops are affected by turbulent vortices with scales from several to a few tens of centimeters. Since the linear scales of the computation areas in most LES models are several cm, these models are incapable to describe these vortices adequately, and, consequently, they are unable to describe the motion of small rain drops appropriately.

Because of these reasons there are significant difficulties in theoretical analysis and DNS for investigating transport effect related to small rain drops. Some theoretical evidence of the "unusual" behavior of small rain drops can be seen from the following considerations. As it was shown by Pinsky et al (2006a) the equation for turbulence-induced particle velocity deviation along the particle trajectory $V_i^{'}$ can be written as follows:

$$\frac{dV_i^{'}}{dt} = -V_j^{'}\left(\frac{1}{\tau}\delta_{ij} + S_{i,j}(x_i,t)\right) - \left(A_i(x_i,t) + V_t S_{i,3}(x_i,t)\right) \tag{1}$$

where $V_i^{'} = V_i - W_i(x_i,t) - V_t\delta_{3i}$ is the relative fluctuating particle velocity, V_i and W_i are particle and air flow velocities, respectively; V_t is gravity induced sedimentation velocity, $A_i(x_i,t)$ is the Lagrangian acceleration of the turbulent flow in the point of particle location, $S_{ij}(x_i,t)$ is the turbulent shear tensor.

The character of the solution of Eq. (1) depends on the eigenvalues of the tensor $\left(\frac{1}{\tau}\delta_{ij}+S_{ij}\right)$. In the case when all real parts of the eigenvalues are positive, the velocity $V_i^{'}$ tends to the quasi-stationary value, which leads to a comparatively small increase in the swept volume (Pinsky et al, 2006a). In the case when the real part of any eigenvalue is negative, the relative local velocity grows exponentially. These cases will be referred to as non-stationary (or unstable) ones. The condition of the growth is

$$\lambda_{\min} < -1/\tau \qquad (2)$$

where $\lambda_{\min}$ is the minimum real part of the eigenvalue of $\left(\frac{1}{\tau}\delta_{ij}+S_{i,j}\right)$.

Figure 6 shows a fraction of the non-stationary (unstable) cases for drops of different size under turbulent conditions typical of three types of atmospheric clouds: stratiform ($\mathrm{Re}_\lambda = 5\cdot 10^3, \varepsilon = 0.001\ m^2 s^{-3}$), cumulus ($\mathrm{Re}_\lambda = 2\cdot 10^4, \varepsilon = 0.02 m^2 s^{-3}$) and cumulonimbus ($\mathrm{Re}_\lambda = 2\cdot 10^4$, $\varepsilon = 0.1\ m^2 s^{-3}$). For estimating turbulent shear tensor used to calculate the results showed in Fig. 6, we used a statistical model of turbulent flow suggested by Pinsky et al (2004).

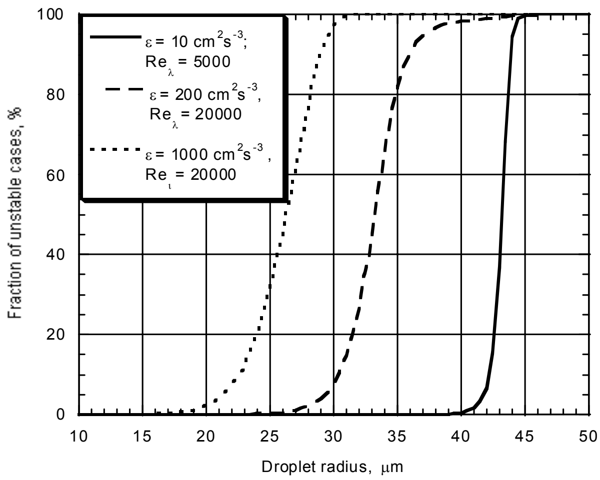

Figure 6. Dependence of the fraction of the non-stationary (unstable) cases on droplets size under turbulent conditions typical of stratiform, cumulus and cumulo-nimbus clouds.

Inspection of Figure 6 shows that motion of cloud droplets with the radii below ~20 μm obeys the quasi-stationary condition in all cloud types. At the same time, droplets with the radii larger than 40 μm tend to deviate strongly from the air flow in all cloud types.

Note that Figure 6 indicates the local growth of the relative velocities of small rain drops during small periods of time (or at small spatial scales), where shears and accelerations can be assumed nearly constant. Rapid sedimentation of the drops must destroy such "resonance" response of small rain drops to local turbulent vortices.

An attempt to overcome these difficulties has been undertaken by Pinsky and Khain (1997a,b) who performed matching of two asymptotic regimes of drop motion when $V_t << u'$ and $V_t >> u'$, where u' is the characteristic turbulent velocity. The results reported in these studies indicate rapid decrease in the turbulence-induced relative drop velocity at drop radii $a > 30 \mu m$. The conclusions obtained in these theoretical studies are supported by results obtained using the simplified turbulence mode by Pinsky and Khain (1996).

We assume that the enhancement factor obtained in these studies for cloud droplets is somehow overestimated because of some approximations concerning the turbulent velocity field (e.g., assumption of the statistically stationary velocity field). Nevertheless, these results (e.g., suppression of the drop "resonance" response) indicate a dramatic influence of sedimentation velocity on the behavior of small rain drops.

One should mention that relative velocities between drops with large St were calculated in numerous DNS studies (e.g., Wang et al 2000; Zhou et al 2001). Numerous parameterization formulas for calculation of swept volumes were proposed. Unfortunately, in the DNS gravitational sedimentation was neglected. Besides, small computational volumes used in DNS do not allow one to describe vortices which affect the motion of small rain drops. Thus, the results obtained in DNS can be hardly applied to cloud conditions.

Taking into account some limitations in the analytical approach and the numerical simulations, further investigations are required to obtain more accurate quantitative (and, may be, qualitative) evaluation of the transport effect for small rain drops.

3. DROP CLUSTERING

While droplet clustering is discussed, two main question arise: a) does small scale clustering occur in real clouds? and b) if the answer to the first question is positive what is the effect of clustering on droplet collisions in clouds?

Numerous analytical and DNS studies reported effects of inertial particle clustering (see the list of references). The existence of small-scale droplet clustering in real clouds during some period of time was in doubt (e.g. Chaumat and Brenguier, 2001). Pinsky and Khain (2001; 2003); and Kostinski and Shaw (2001) found centimeter scale droplet concentration fluctuations caused by turbulent-inertia effect.

Small scale fluctuations of liquid water content were found by Gerber et al (2001).

Figure 7 presents the results obtained by Pinsky and Khain (2003) in statistical analysis of long series of drop arrival times in about 60 cumulus clouds.

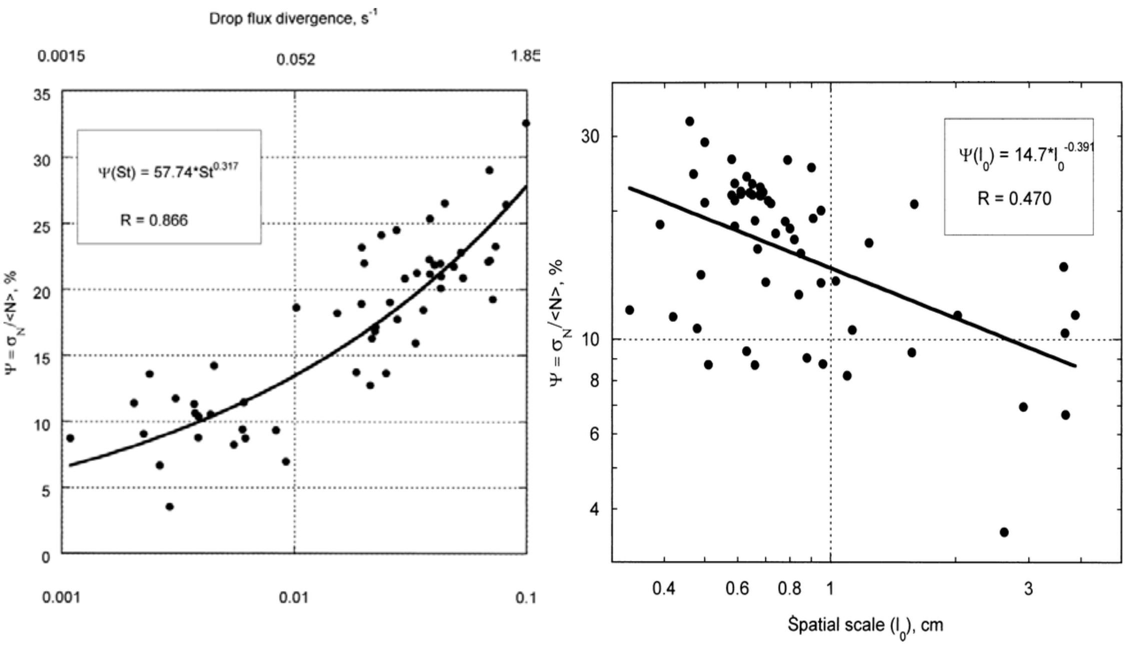

Figure 7. Normalized fluctuations of droplet concentration in clouds as the function of St number (left) and on spatial scales (right) (after Pinsky and Khain 2003)

This study indicates that droplet clustering exists in clouds and the rate of the clustering was found to increase with St, in correspondence with the theoretical predictions. One can see also the increase in the clustering rate with the decrease in the spatial scale.

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Mechanism of small-scale inertial particle clustering in turbulent flow due to localization of the second-order correlation function of particle number density was suggested first by Elperin et al (1996). Effects of droplet clustering on the collision rate between droplets are usually characterized by correlation functions G_{11} (for monodisperse suspensions) and by function G_{12} (bi-disperse suspensions) introduced by Read and Collins (1997) and since that widely used everywhere (e.g., Wang et al 2000; Zhou et al 2001; Chun and Koch 2005; Chun et al 2005). These functions can be defined as follows: $G_{11}(2a) \approx 1 + \frac{\sigma_N^2}{\langle N \rangle^2}$

and

$$G_{12}(a_1 + a_2) \approx \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle},$$

where N_1 and N_2 are instantaneous number densities of droplets with radii a_1 and a_2, respectively. In order to characterize droplet collisions upon contact, functions G_{11} and G_{12} are calculated at separation distances $2a$ *or* $a_1 + a_2$ (i.e. upon contact between drops) in monodisperse and bi-disperse suspensions, respectively.

3a. Cloud droplets

Figure 8 shows the dependence of G_{11} on St obtained by different authors in theoretical analysis, and DNS. One can see that the difference between the curves is small for St<0.1. The differences between the results can be attributed both to differences in approaches utilization of small St in DNS, etc. In general, G_{11} is below 1.3 for droplets with St<0.25 remaining, however, larger than it follows from analysis of measurements (Fig. 7).

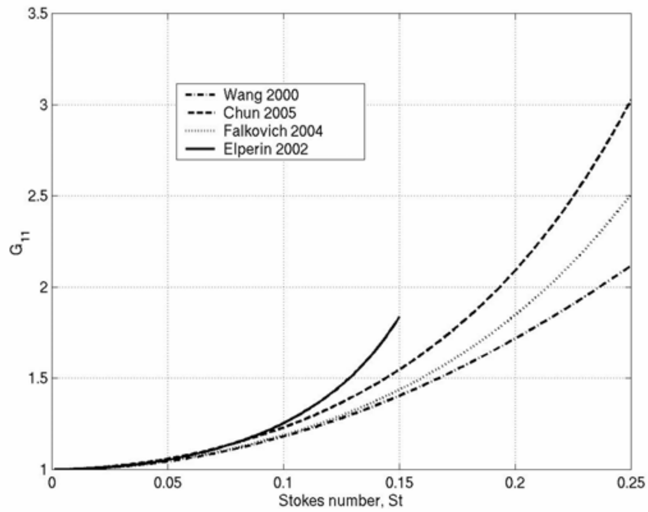
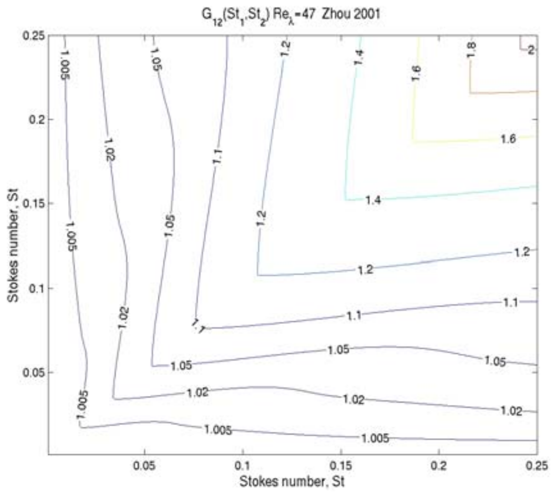
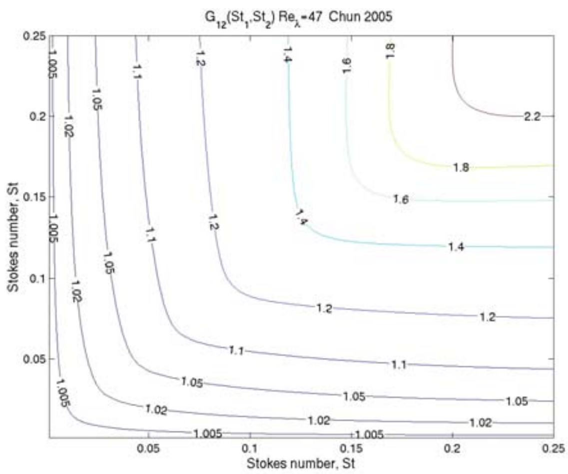


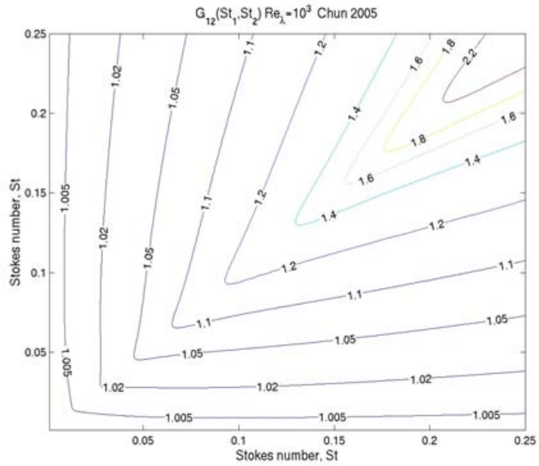
Figure 8. Dependencies of G_{11} on St obtained by different authors (Wang et al 2000; Chun and Koch 2005; Falkovich and Pumer 2004; Elperin et al 2002).

Figure 9 compares the function G_{12} for bi-disperse suspensions of small droplets obtained by Zhou et al (2001) in DNS (9a), and theoretical results of Chun et al (2005) (9b) calculated for the same values of $\mathrm{Re}_\lambda = 47$. One can see a very good agreement between these results. In the model by Chun et al (2005) function G_{12} depends on the Largangian acceleration that, in its turn, depends on Re_λ. Figure 9c shows function

G_{12} in case when the results of measurements by La Porta et al (2001), Voth et al (2002) for $\mathrm{Re}_\lambda = 10^3$ were used. According to La Porta et al (2001) further increase in Re_λ does not result in increase of the fluctuations of Lagrangian acceleration, and, respectively, G_{12}. According to parameterization proposed by Hill (2002) fluctuations of the Lagrangian acceleration continue increasing with Re_λ also for $\mathrm{Re}_\lambda > 10^3$. Function G_{12} calculated using the parameterization proposed by Hill is shown in Fig. 9d. It is interesting that increase in Re_λ leads to a strong decrease in G_{12} for drops of different size. This effect is attributed to the fact the Lagrangian accelerations tend to decrease the spatial correlation of concentration fluctuations of droplets of different size.







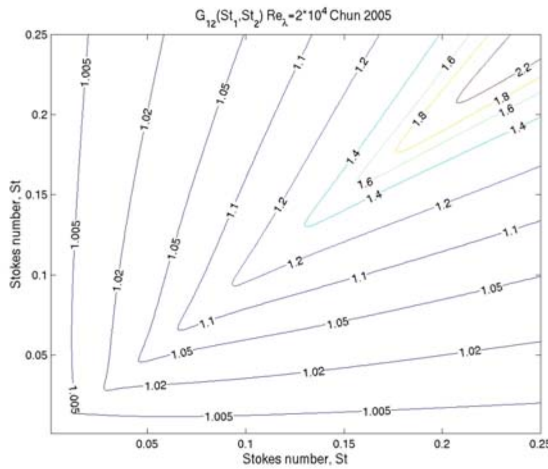


Figure 9a-d ***T****he function* G_{12} *for bi-disperse suspensions of small droplets. (a) Zhou et al (2001),* $\mathrm{Re}_\lambda = 47$ *(b) Chun et al (2005b) ,* $\mathrm{Re}_\lambda = 47$ *; (c) Chun et al (2005b),* $\mathrm{Re}_\lambda = 10^3$ *; (d) Chun et al (2005b),* $\mathrm{Re}_\lambda = 2 \times 10^4$

The collision enhancement factor is maximum for droplets of close size St~0.25 (G_{12}~*2*). Note that for St>0.1 the difference in drop size causes sharp reduction of function G_{12} because of the growth of the Lagrangian acceleration. Note that these results were obtained neglecting droplet sedimentation. The differential drop sedimentation being taken into account may further decrease the clustering rate.

Therefore, the values G_{12} *should be recalculated to take into account the sedimentation effect. Taking into account differential sedimentation is of particular importance important for droplets with St>0.1.*

3b. Small raindrops

Wang at al (2000), Reade and Collins (2000), Zhou et al (2001), Elperin et al (2002) , Falkovich et al (2002) reported a dramatic increase in G_{11} and G_{12} for St>0.3 (see Fig. 1 and Fig. 10). Elperin et al (2002) distinguished two regimes of clustering, weak clustering and strong clustering. In the latter regime strong increase in the clustering rate occurs for Stokes numbers above the critical threshold St~2/9.

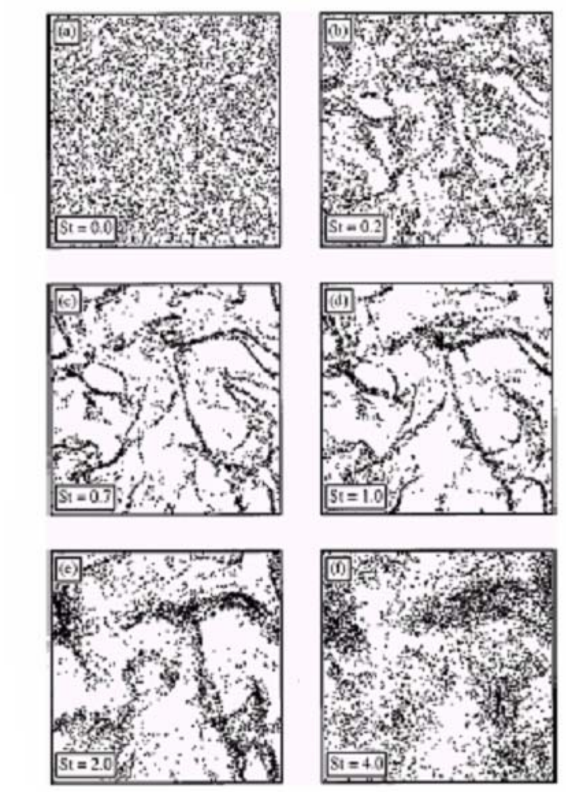


Figure 10. *Slices of ghost-particle simulations at: (a) St=0.0; (b) St=0.2; (c)St=0.7; (d) St=1.0; (e) St=2.0; and (f) St=4.0/ dots correspond to particle center locations (after Reade and Collins 2000)*

DNS simulations (e.g. Reade and Collins 2000) indicate that zones of the enhanced concentration are narrow elongated structures with the width that depends on the St (Fig. 10).

At St~0.7-1 the width of the regions with enhances concentration is less than the Kolmogorov microscale, so that droplet concentration fluctuations increase by one-two orders of the magnitude with the decrease of the scale from the Kolmogorov microscale down to the drop size scale. Spatial redistribution of droplets and formation of these regions with the enhanced concentration is often interpreted as the formation of fractal structures (Falkovich and Pumir 2004).

We believe, however, that two main factors that are not taken into account in these studies should dramatically decrease clustering effect of small rain drops, as well as its effect on the raindrop growth in real clouds. These factors are: significant difference in the gravity induced sedimentation velocities and small concentration of these drops.

Effect of differential drop sedimentation

Figure 11 illustrates spatial separation of two zones of preferential concentration in initially well mixed bi-disperse suspension. The spatial decorrelation is especially significant if one of the species is represented by small rain drops with significant (several tens cm/s) sedimentation velocity.

Another illustration of effect of differential sedimentation on droplet clustering is presented in **Figure 12**, where trajectories of droplets with radii of 10 μm (blue), 30 μm (green) and 50 μm (red) are plotted. These trajectories were calculated using the "cheap" turbulent model (Pisnky and Khain 1996)

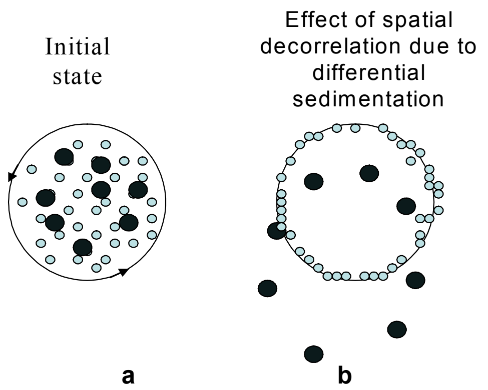


Figure 11. Schematic illustration of the effect of differential drop sedimentation

One can see that in the presence of gravity, the angle between trajectories of droplets increases with difference in drop size. The increase in the angles decreases the common areas, where droplets of different size can be located simultaneously.

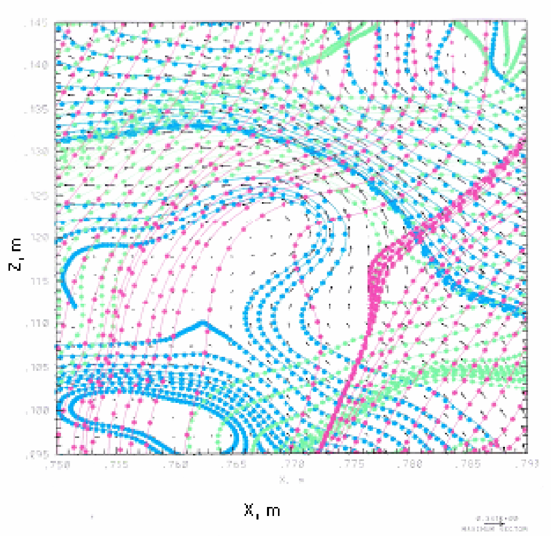


Figure 12. Trajectories of droplets with radii of 10 μm (blue), 30 μm (green) and 50 μm (red). These trajectories were calculated using the approximate turbulence model (Pisnky and Khain 1996). The area 4cm x 4 cm is shown.

Since different drops response to the vortices of different size (Fig.5), the structures and locations of enhanced concentration regions formed by drops of different size should be quite different and well separated (simple evaluations indicate that distance between zones of preferential concentration of cloud and small rain drops can easily exceed several centimeters of even tens of centimeters. Note that rain drops grow largely by collisions with cloud droplets. Taking into account a significant difference in sedimentation velocities it can be concluded that the locations of regions with the enhanced concentration for cloud droplets and rain drops are completely uncorrelated.

Thus, the gravitation sedimentation dramatically decreases the effect of turbulent-induced drop clustering on the growth of small (as well as large) raindrops.

Effects of smallness of the drop concentration

As it was mentioned above, the concentration of drops with radii 40-70 μm in clouds is small, so that the mean distance between such drops is of the order of 1 cm. Small drops with approximately the same sizes may be separated by the distance easily exceeding 10 cm. This implies that the drops with equal sizes may be located inside different turbulent vortices and can be located within different regions with preferential droplet concentrations. In this relation the question arises: does the mean distance between drops always decrease as a result of droplet accumulation in the regions with the preferential concentrations? The problem is illustrated schematically in **Figure 13** and **Figure 14**.

Figure 13. Illustration of the effect of the low drop concentration on the mean distance between the drops in case when the drops with similar sizes belong to the same region with preferential droplet concentration (the same attractor).

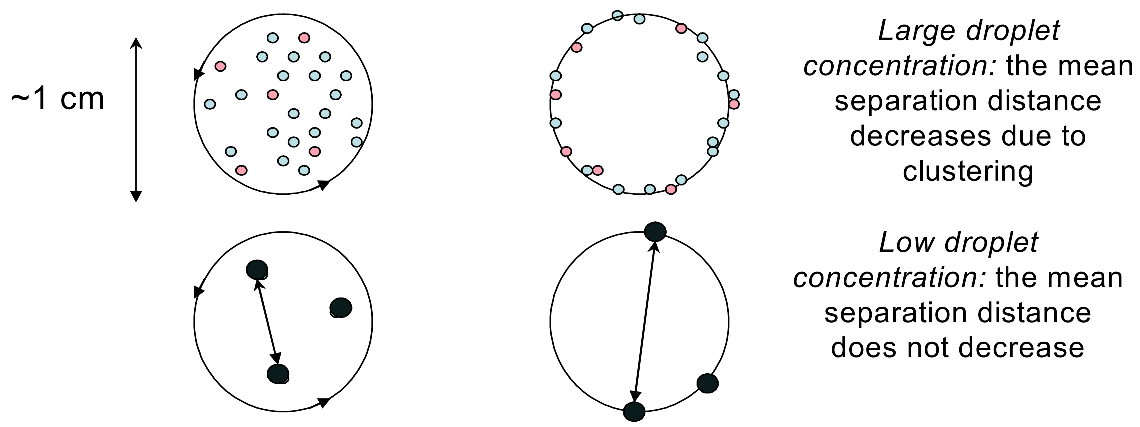


Figure 14 shows regions of enhanced concentration of drops characterizing by St=0.7 as seen from DNS performed by Read and Collins (2000). Red dots indicate probable location of such drops when their concentration is low (like in real clouds) and the mean separate distance is several cm. The drops may belong to different clusters (different regions of preferential concentration). The decrease in the distance between such drops as a result of their shift to the preferred concentration regions is not obvious.

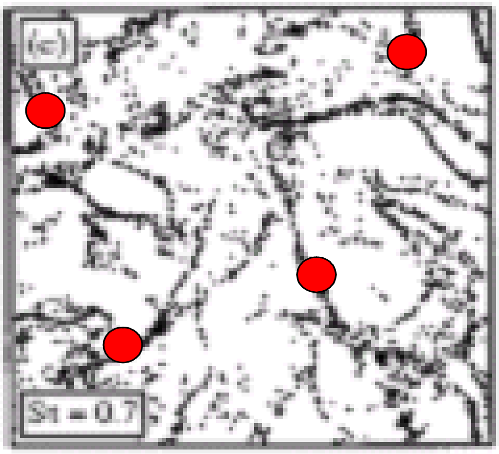


Figure 14. Regions with the enhanced concentration of drops characterized by St=0.7 as seen in DNS performed by Read and Collins (2000). Red dots indicate probable location of drops in a real cloud. The drops belong to different regions with the preferential concentration.

For crude evaluation of the effects of clustering on the mean distance between drops, let us assume that due to turbulence effects droplets are redistributed in space and accumulate in the regions characterized by fractal dimension α. One can evaluate the decrease of the mean distance between the drops by factor $G = (Nl^3)^{(1/\alpha - 1/3)}$, where N is the drop concentration in the cluster, l is the linear scale of the cluster chosen ~1 cm as can be seen in Figure 10. According to Falkovich and Pumir (2004) the fractal dimension α is the function of St. Table 1 presents the results of the evaluations when parameters are chosen close to those typical for real clouds:

Table 1

Drop radius r, μm	10	20	50
St	0.1	0.3	0.7
N, cm^{-3}	300	10	1
α	2.9	2.5	1.4
G	1.07	1.16	1

Table 1 shows that the decrease in the distance occurs for 20 μm droplets and it is of the order of 16%. The mean distance between 50 μm droplets practically does not change. Therefore low concentration of drops with St~1 renders the clustering effect virtual since turbulence-induced droplet spatial redistribution does not lead to the actual decrease in the separation distance.

In summary, the results of theoretical and numerical analyses (DNS) performed by neglecting the differential sedimentation and low raindrop concentrations are irrelevant for actual clouds. It seems that the effect of clustering of such drops on collisions in real clouds is much weaker (if any) than that reported in many studies.

4. EFFECTS OF TURBULENCE ON HYDRODYNAMIC DROPLET INTERACTION

4a. Cloud droplets

The number of studies that take into account the turbulence effect on the hydrodynamic drop interaction (HDI) is quite limited (Pinsky et al 1999, Pinsky and Khain 2004; 2006; Franklin et al 2004; and Wang et al 2005).

Small number of these studies is surprising since the collision efficiencies for sedimentation velocities are quite small (0.001-0.1) only turbulence can increase the collision efficiency and the collision rate. All these studies employ a superposition method for calculating hydrodynamic interaction between approaching droplets. Pinsky et al (1999) calculated collision efficiencies between drops moving within a turbulent flow field generated by an approximate turbulence model in which the distribution of velocities was assumed Gaussian. Pinsky and Khain (2004) calculated the collision efficiencies and collision kernels in a turbulent flow with high Re_{λ} typical of atmospheric clouds.

In the latter study, however, only the Lagrangian accelerations were taken into account, while the effect of turbulent shear was disregarded. Franklin et al (2004); and Wang et al (2005) calculated collision efficiencies between droplets with several selected sizes using the velocity field generated by DNS models. Recently, Pinsky et al (2006b) (see also this issue) used a statistical turbulence model (Pinsky et al 2006a) to produce the PDF of acceleration and shears as measured under high Re_{λ}.They calculated collision efficiencies within the whole range of the cloud droplet sizes under different turbulence intensities typical for clouds of different type. Comparison between the values of collision efficiencies obtained by Pinsky and Khain (2006b) calculated under the conditions similar to those used by Wang et al (2005) ($\varepsilon = 100 cm^2 s^{-3}$, drop collector radius 20 μm) indicates a reasonably good agreement in the results.

Figure 15 shows the collision efficiencies between 15 μm -(above) and 20 μm (low panel) collectors with smaller droplets under different dissipation rates and Re_λ according to results obtained by Pinsky et al (2006b). The pure gravity values of the collision efficiencies are shown as well. Results obtained by Wang et al (2005) for $\varepsilon = 100 cm^2 s^{-3}$ are marked by crosses on the right panel. One can see that strong turbulence increases significantly the collision efficiencies between cloud droplets whereby a significant increase in the collision efficiency occurs for droplets with similar sizes.

Figure 16 shows the dependence of the averaged normalized collision kernel for the 10 μm- and 20 μm droplet pair vs. turbulence dissipation rate ε for different Re_λ.

While the factor of the swept volume increase was found to be 1.6 for very strong turbulence intensity (see Fig.4), the collision kernel increases by the factor as large as 4.8. Therefore the effect of turbulence on the HDI appears to be the main mechanism by means of which turbulence increases the rate of cloud droplets collisions. Note that turbulence enhancement factor attains minimum for the

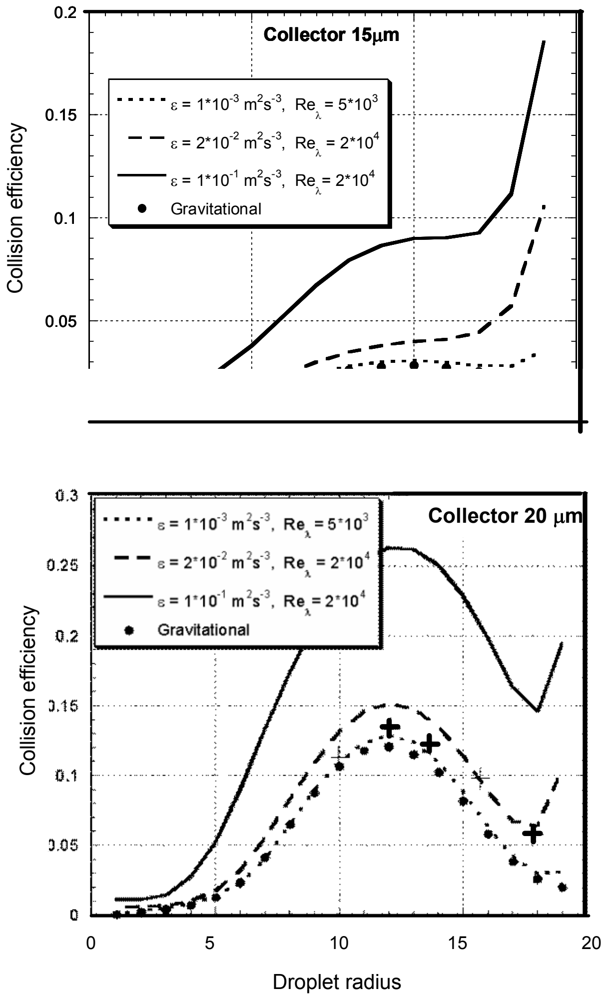


Figure 15. Collision efficiencies between 15- μm (left) and 20- μm radii (right) collectors with smaller droplets under different dissipation rates and Re_λ (after Pinsky et al 2006b). Results obtained by Wang et al (2005) for $\varepsilon = 100 cm^2 s^{-3}$ are marked by crosses on the right panel.

$10\,\mu m$-$20\,\mu m$ radii droplet pair. The increase in the collision kernel of droplet pairs containing droplets of close size or droplets smaller than ~3 μm is much more pronounced.

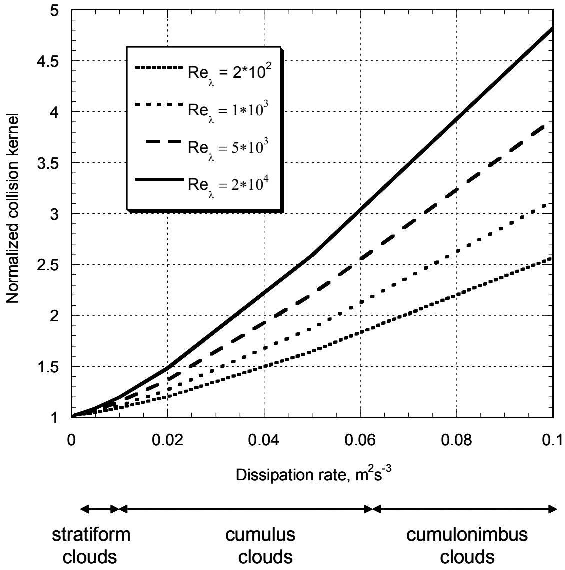


Figure 16. *Dependence of the averaged normalized collision kernel for the 10 μm- and 20 μm droplet pair on the dissipation rate ε under different Re_λ (after Pinsky et al 2006b).*

Inspection of Fig. 16 shows that the collision kernel increases with the increase of ε and Re_λ. Therefore taking into account the effect of Re_λ is of the same importance as accounting for the effect of the dissipation rate ε.

Figure 17 compares contributions of different turbulence mechanisms on the collision rate enhancement. It shows the dependence of the relative increase of the collision efficiency, swept volume and radial correlation function G_{12} on the dissipation rate for the $10\,\mu m$ - 20 μm radii drop pair. The value of Re_λ was set equal to $2\cdot 10^4$. The droplet concentration fluctuations caused by the turbulence-inertia mechanism were evaluated using results of Zhou et al (2001) (Re_λ=47) and Chun et al (Re_λ=1000).

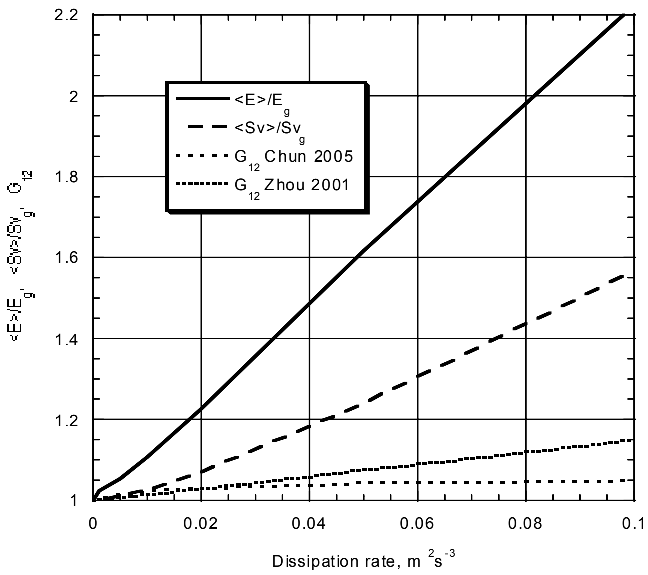


Figure 17. *Dependences of function $G_{12}(a_1+a_2)$, normalized swept volume and normalized collision efficiency on the dissipation rate for the 10 μm- and 20 μm-radii droplet pair. The collision efficiency increases faster than other quantities. The clustering effect is evaluated using results of Zhou et al (2001) and Chun et al (2005)*

Fig. 17 shows that *the growth of collision efficiency is the major factor by means of which turbulence increases collision rate between cloud droplets.* For large dissipation rate, the effect of clustering according to Chun et al (2005) turned out to less than that obtained by Zhou et al (2001) because of stronger decorrelation of spatial location of droplets with different size within a flow with higher Largangian accelerations.

4b. Small rain drops

Collision efficiencies between small rain drops and most of cloud drops is close to one even in the pure gravity case (Pruppacher and Klett 1997). Therefore turbulence cannot increase further the collision efficiency for droplets of these sizes. However, there remains the question, how turbulence influences the collision efficiency between large drops and the smallest 1-5 μm droplets. Laboratory experiments (Vohl et al 1999) indicate that turbulence increases the collision rate between small droplets and small raindrops. In order to match observations the collision kernel has to be increased by ~10-15% as compared to the gravitational one. These values provide the upper limit for the growth of the collision efficiency. However it must be noted that in these experiments a) the 60 μm radius drop collectors were injected into the wind tunnel initially, so that the collector size was larger than 60 μm in course of the experiment and b) Re is much smaller than in the atmosphere. Wang et al (2005) suggested that multiple interactions must be taken into account in order to calculate properly hydrodynamic interaction of droplets. The latter conclusion seems doubtful since it is based upon the long-range behavior of droplet interaction in Stokes flow model. However it is well known that Stokes solution is invalid for large distances where inertial effects prevail. *Clearly, more numerical and theoretical investigations are required to understand the role of turbulence on motion and collisions of drops with St~1.*

5. PARAMETERIZATION OF TURBULENT EFFECTS IN THE STOCHASTIC EQUATION OF COLLISIONS

5a. Averaging and correlations

The stochastic collision equation (3) is in general use in a great numbers of theoretical and modeling investigations of cloud development.

$$\frac{dN}{dt}=\frac{1}{2}\int_0^m N(m')N(m-m',m')S_v(m-m',m')E\,dm'-$$
$$-N(m)\int_0^\infty N(m')S_v(m,m')E(m,m')dm'$$

(3)

Here S_v is the mutual swept volume between colliding droplets; E is the collision efficiency indicating decrease in the collision rate due to HDI. Swept volume is defined as the influx the relative velocity vector through the spherical surface with radius (a_1+a_2) (see. e.g., Pinsky et al (2006a). This equation is usually used in Cloud Physics to describe the process of collisions within significant air volumes with linear scales of several tens to a few hundred meters. Respectively, a proper averaging of terms representing the product $N_1N_2S_vE$ (each of the values is determined at small time and spatial scales) should be performed. The usual practice in state-of-the-art numerical models is to neglect correlation between these values, so that it is assumed that $\langle N_1N_2S_vE\rangle=\langle N_1\rangle\langle N_2\rangle\langle S_v\rangle\langle E\rangle$ (in most cases pure gravitational values of swept volume and collision efficiencies are used). In numerous studies dedicated to effects of clustering (e.g., Zhou et al 2001), where hydrodynamic interaction is neglected (E=1), increase in the collision rate due to clustering effect is expressed using the averaging $\langle N_1N_2S_v\rangle=\langle N_1\rangle\langle N_2\rangle\langle S_v\rangle G_{12}$.

Note, however, that since all of the values in the product $N_1N_2S_vE$ depend on the Lagrangian accelerations and turbulent shears, they are depended. For instance, as it was mentioned above, an increase in the Lagrangian accelerations decreases the droplet clustering effect (decrease in G_{12}), but increases the collision efficiency. At it was shown by Pinsky et al (2006b), positive correlation between S_v and E takes place under strong turbulence (large Lagrangian accelerations). Thus, both positive and negative correlations take place. Strictly speaking the product $N_1N_2S_vE$ should be averaged as a whole, i.e the value of $\langle N_1N_2S_vE\rangle$ should be calculated using an approach that have to take into account differential drop sedimentation.

Basing on study (Elperin et al 2002) and taking sedimentation effects into account, these authors propose the following averaged stochastic collision equation (4):

$$\frac{dN}{dt} = \frac{1}{4}\int_0^m N(m')N(m-m')\left[\ (G(m')+G(m-m')\ \right] S_v(m-m',m')E(m-m',m')dm' - \\ -\frac{1}{2}\int_0^\infty N(m)N(m')\left[\ (G(m')+G(m-m')\ \right] S_v(m,m')E(m,m')dm' \tag{4}$$

Where the enhancement factor G depends on clustering regime as follows:

$$G(St) = \begin{cases} 1+\frac{9}{2}\frac{St}{\sqrt{1-\frac{81}{4}St^2}}, & if\ G<5 \\ 5, & if\ St_{cr} < St \le 0.4 \\ 1+4\exp(-2St+0.8), & if\ St > 0.4 \end{cases} \tag{5}$$

Parameterization (4, 5) was obtained under several assumptions related to the sedimentation effect (reflected by the exponential decrease in the enhancement factor in (5)), the topology of the concentration field and nonlinear interaction between drops and turbulent environment. It should be noted that this parameterization should be justified in future studies.

The problem how to perform such averaging using the simplest and accurate enough approach still waits for its solution.

5b. Problems related to representation of collision kernels

In some studies (e.g. Riemer and Wexler, 2005) the collision kernel is represented as a sum $K_{tot}=K_{sed}+K_{turb}$, where the "sedimentation" kernel K_{sed} is defined by the product of the swept volume in calm air and the gravity collision efficiency (pure gravity collision kernel). The turbulent kernel K_{turb} is obtained from DNS that did not include gravity-induced sedimentation. This simple estimate of the coagulation kernels can be used when different mechanisms act within different ranges of particle size. For instance, Butuirat and Kielkiewicz (1996) used this formulation considering collisions within a mixture of submicron aerosol particles and several micron radius droplets. They represented the collision kernel as a sum of the Brownian coagulation kernel within the size range of the smallest aerosol particles and the coagulation and gravitational coagulation kernel within the range of large particles.

In case when collisions of cloud drops are considered gravitational and turbulent mechanisms are effective within the same range of droplet sizes. Respectively, the assumption of the linear behavior of the total kernel becomes questionable because of non-linear effects.

We illustrate this statement for a simple case, when only transport effect (swept volume) is considered. It is reasonable to write the swept volume in a turbulent flow as: $S_v = \pi(a_1 + a_2)^2 \left\langle \left[\Delta W_x^2 + \Delta W_y^2 + (\Delta W_z - \Delta V_t)^2\right]^{1/2} \right\rangle$ where ΔW_i is the turbulence induced relative velocity in the corresponding direction.

In a linear case the swept volume can be written as

$$S_{vadd} = S_{vsed} + S_{vturb} = \pi(a_1 + a_2)^2 \left[\left|\Delta V_t\right| + \Delta W_{turb}\right]$$

where $\Delta W_{turb} = \left\langle \left[\Delta W_x + \Delta W_y^2 + \Delta W_z^2\right]^{1/2} \right\rangle$.

Figure 18 shows the magnitude of (S_v - S_{vadd})/ S_v as a function of the ratio $\Delta W_{turb}/\Delta V_t$ calculated for 10 - 20 μm -radii droplet pair. The calculations were performed under assumption of the Gaussian distribution of the relative velocities. It can be seen that the relative error $(S_v - S_{vadd})/S_v$ is zero for a gravitational kernel and becomes small at very high dissipation rates when gravity effect can be neglected. Within the range where both factors are important, the error is significant (30% in our case).

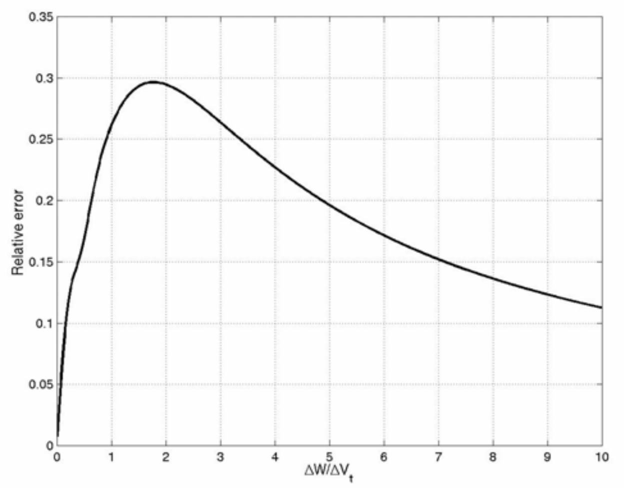


***Figure 18** The value $(S_v - S_{vadd})/S_v$ as a function the ratio $\Delta W_{turb}/\Delta V_t$ calculated for 10 and 20 microns radii droplet pair. The calculations were performed under assumption of the Gaussian distribution of relative velocities. The relative error $(S_v - S_{vadd})/S_v$ becomes significant when both gravity and turbulent effects are important.*

In case when hydrodynamic interaction is taken into account, the validity of the additivity hypothesis becomes even more questionable. Sometimes, the gravitational swept volume is multiplied by pure gravity collision efficiency, while “turbulent” kernel calculated under neglecting sedimentation effect is multiplied by some “turbulent” collision efficiency. Such approach hardly can be physically justified. Besides, as it was shown above, neglecting sedimentation effect, as well as neglecting the fact that drop concentration is small, overestimate the "turbulent" collision kernel.

6. CONCLUSIONS

The validity of direct application of results obtained in numerous studies dedicated to turbulent effects on particle collisions to real clouds is analyzed. It is shown that in many cases conditions under which the collision enhancement was estimated crucially differ from those in real clouds. The main limitations of many theoretical and DNS studies is neglecting effect of differential sedimentation of drops caused by gravity, as well as utilization of much higher concentration of large drops (St~0.3-1) as compared to that can be found in real clouds. Neglecting these effects dramatically overestimates clustering effect on collisions, especially for small raindrops characterized by St~0.3-1. This makes the direct application of the results to real clouds questionable.

During past few years a significant progress in understanding turbulent effects on collisions between cloud droplets is achieved. We believe that comparably few additional efforts are required to get a reasonable parameterization of the turbulent effects on cloud droplet collision for cloud conditions. Sensitivity studies with cloud models indicate that the enhancement factor for cloud droplets falls within the range from few percents for stratiform clouds up to factor of the order of 5 in strong cumulus clouds.

At the same time, the effect of turbulence on collisions of drops within the pairs containing small raindrops remains largely unknown. More studies are required to understand motion and collisions of these particles within turbulent clouds. These studies should take into account the specific features of these drops: high sedimentation velocity and a very low concentration. Note that the problem of collisions with particles characterized by St~1 is especially important for ice cloud microphysics, because many ice particles can be characterized by this St values.

The problem of averaging of the stochastic collision equation with purpose of parameterization of turbulent effects in cloud models remains largely unsolved. This situation is partially related to the fact that effects of turbulence on clustering, on relative velocity and on collision efficiency are not independent. -An appropriate averaging should be performed within an approach that takes into account the effects of differential drop sedimentation.

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