14.6 RAINDROP SHAPES AS SEEN BY A POLARIMETRIC DOPPLER RADAR

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1. INTRODUCTION

Direct observations of effective raindrop shapes from dual-polarization radar measurements, are not straightforward. In order to interpret dual-polarization radar measurements one would need to know not only raindrop size-shape relation but also drop-size distribution (DSD). Goddard et al. (1982); Thurai and Bringi (2005) have used disdrometer data to address this problem. Recently Gorgucci et al. (2000) have introduced an additional shape parameter, β, into polarimetric rain rate retrieval algorithm that represents the equivalent slope of the size-shape relationship. This approach uses reflectivity (Z_h), differential reflectivity (Z_{dr}) and specific differential phase (K_{dp}) measurements to estimate β. Use of K_{dp} restricts applicability of the proposed technique to moderate to heavy rain events. Moreover, use of K_{dp} implies reduction of the range resolution, and therefore results in range averaged estimates of β.

Recently, Williams (2002) has shown that non-linear least square optimization procedure can be used for a joint retrieval of three parameters of gamma DSD, vertical air motion and spectral broadening from vertical incident profiler Doppler measurements. Therefore, by combining Doppler spectra observations taken at a sufficiently high elevation angle with dual-polarization measurements of precipitation one should be able to retrieve both DSD information and raindrop size-shape relation on a short time and spatial scales.

To facilitate investigation of β on small time and spatial scales in this study we introduce a new retrieval method of raindrop size-shape relation that is based on dual-polarization spectral measurements of precipitation. In this method the drop-size distribution information, spectrum broadening and ambient air velocity are obtained from Doppler power spectra measurements and given this information the β parameter is obtained from differential reflectivity measurements. The measurements should be carried out at a sufficiently high elevation angle for correct DSD retrieval and at a rather low elevation angle for accurate dual-polarization measurements. Based on simulations we have studied sensitivity of the proposed retrieval method to the radar antenna elevation angle and have determined the window of suitable elevation angles. Moreover, the sensitivity analysis of the proposed technique to spectrum broadening is also performed. The performance of the method is illustrated ondata collected by the CSU-CHILL radar (Brunkow et al. 2000).

2. METHODOLOGY

2.1. *Model*

For radar observations at non zero elevation angle the radial velocity of raindrop can be written as a function of raindrop equivolume diameter (Atlas et al. 1973)

$$v(D) = \left(\frac{\rho_0}{\rho}\right)^{0.4} [9.65 - 10.3\exp(0.6D)] \cdot \sin\theta + v_0, \quad (1)$$

where ρ_0 and ρ are the air densities at the sea level and the altitude of a considered range gate respectively, v_0 is the ambient air radial velocity, D is the equivolume raindrop diameter given in mm, and θ is the antenna elevation angle. Quite air Doppler spectrum measured at a sufficiently high elevation angle can be written as

$$S_{hh}(v)dv = \frac{\lambda^4}{\pi^5|K_r|^2}\sigma_{hh}(D(v))N(D(v))\frac{dD(v)}{dv}dv \quad (2)$$

where λ is the radar wavelength, K_r is the dielectric factor, D is the equivolume diameter, θ is the radar elevation angle, σ_{hh} is the backscattering cross-section, v is the radial velocity of a raindrop and $N(D)$ is the drops-size distribution. In this study, the drop-size distribution, $N(D)$, is assumed to be described by a gamma distribution (Bringi and Chandrasekar 2001).

In reality Doppler spectrum of precipitation is always broadened by turbulence, raindrop oscillations,

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cross-wind or wind shear (Doviak and Zrnic 1993). It is common to model the effect of spectral broadening as a Gaussian shaped convolution kernel (Doviak and Zrnic 1993), therefore the observed spectrum can be written as

$$S_{hh,mod}(v) = S_{broad}(v) * S_{hh}(v) = \\ = \frac{1}{\sqrt{2\pi}\sigma_b} \int \exp\left[-\frac{(v-\widetilde{v})^2}{2\sigma_b^2}\right] S_{hh}(\widetilde{v}) d\widetilde{v}, \quad (3)$$

where $*$ is the convolution operator, and σ_b is the broadening kernel width.

At the S-band frequencies the radar cross-section of raindrops can be estimated using Rayleigh-Gans calculations for oblate spheroids (Bringi and Chandrasekar 2001). Following (Gorgucci et al. 2000; Matrosov et al. 2002; Ryzhkov and Schuur 2003) drop axis ratios can be defined as:

$$\frac{a}{b} = (1 + 0.05\beta) - \beta D. \quad (4)$$

Changes in canting angle distribution of raindrops have a similar effect on the co-polar backscattering cross-section as changes of the magnitude of β. Therefore changes in β would correspond to both changes in canting angle distribution and to oscillation of raindrops. Since raindrop size-shape relations are generally nonlinear, β values also depend on DSD parameters. In order to study whether observed raindrops have equilibrium shapes we also have used Beard and Chuang (1987) relation to compare our retrievals to, as discussed further on.

2.2. *Retrieval approach*

The model of a Doppler power spectrum depends on six parameters, i.e. three DSD parameters, spectrum broadening, ambient air velocity and slope of the drop size-shape relation. It can be seen that β has a negligible effect on a Doppler spectrum and therefore can be omitted from our considerations at this stage of the retrieval. Moreover, one can observe that changes in μ and D_0 would result in changes of the spectrum shape. The spectrum broadening widens and smooths the spectrum. The effect of v_0 and N_w on Doppler spectra is rather easy to imagine. The ambient air velocity, v_0, shifts the velocity axis according to (1). And changes in N_w would result in scaling along reflectivity axis.

Therefore, given a microphysical model of the Doppler spectra observations one can formulate the DSD parameters retrieval procedure as an optimization problem of fitting modeled spectra to the observed ones, that should result in minimization of the following expression (Moisseev et al. 2006):

$$\sum_{v=v_a}^{v_b} \left(\begin{array}{c} \log(S_{hh,mod}(v, N_w, D_0, \mu, v_0, \sigma_b)dv) - \\ -\log(S_{hh,meas}(v)dv) \end{array} \right)^2, \quad (5)$$

Given dual-polarization observations of precipitation and knowing corresponding DSD parameters, one can infer information about drop size-shape relation. For this purpose measurements of differential reflectivity, Z_{dr}, that is ratio of hh and vv reflectivities, can be used. The Z_{dr} values depend on DSD parameters, μ and D_0, as well as on β. Therefore, given D_0 and μ one can retrieve β value by solving the following minimization problem (Moisseev et al. 2006)

$$\min_{\beta} \left(Z_{dr}^{mod}(\beta)|_{D_0,\mu} - Z_{dr}^{meas} \right)^2, \quad (6)$$

where Z_{dr}^{mod} and Z_{dr}^{meas} are respectively modeled and measured differential reflectivities.

2.3. *Elevation angle dependence*

Dependence of the DSD parameters retrieval on the elevation angle is caused by two opposing effects. Firstly, lower the elevation angle smaller the difference between fall velocities radial projections of differently sized raindrops. And therefore spectrum broadening would have a larger effect on the retrieval at smaller elevation angles. Secondly, due to the same reason, a number of spectral lines available for the retrieval would be smaller at lower elevation angles if the same number of samples is used to calculate Doppler spectra. On the other hand, the influence of raindrop shapes on the Z_{dr} measurements is larger for smaller elevation angles. Therefore, accuracy of the β estimate for different elevation angles would be a trade off between accuracy of the retrieval of DSD parameters and influence of raindrop shapes on the Z_{dr} measurements. This effect is shown in the Figure 1. From this figure we can observe that useful elevation angles for this retrieval belong to the interval between 30 and 70 degrees.

3. Results

The CSU-CHILL data were collected during a stratiform rain event on July 23, 2004. The observed reflectivities were ranging between 30 and 35 dBZ. The antenna elevation angle was 30 degrees for this measurement. The range resolution was 50 m. Two minutes of time-series data were collected during this event. The measurements were carried out in alternating mode (Bringi and Chandrasekar 2001). The pulse repetition time of 1 ms was used for this

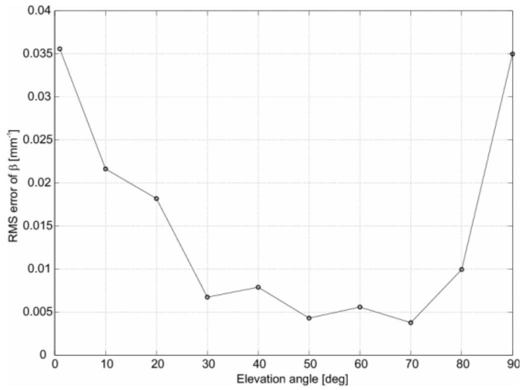

Figure 1: RMS error of β estimate as a function of elevation angle.

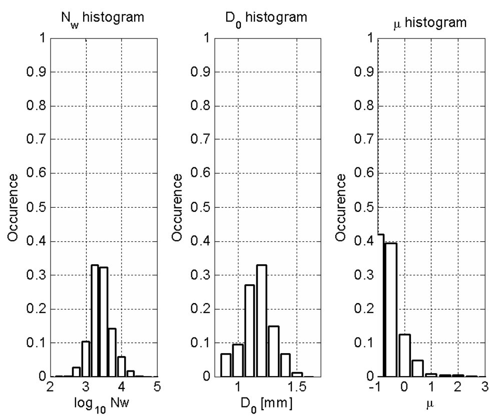

Figure 2: Histogram of the retrieved DSD parameters.

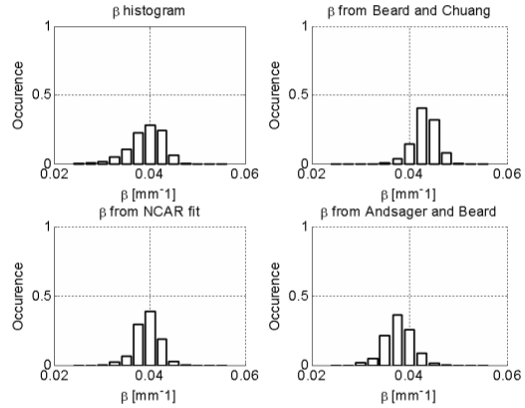

Figure 3: Hist of the retrieved β values.

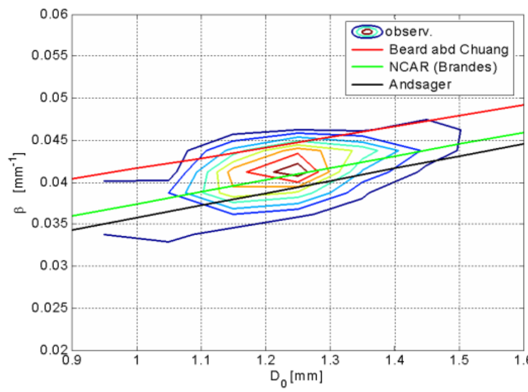

Figure 4: Scatterplot of retrieved β values vs. D_0 values.

measurement. The Doppler power spectra were estimated by averaging 15 spectra, where each spectrum was estimated from 64 samples using the periodogram approach with the hamming window. The resulting velocity resolution was 0.39 m/s. The prposed method was applied to the observations, in Figure 2 the retrieved DSD parameters are given.

Then, given the DSD parameters, the β values were retrieved by matching modeled and observed Z_{dr} values. The histogram of the resulting β values is shown in Figure 3. Moreover, using retrieved DSD parameters and Beard and Chuang (1987) size-shape relation we have calculated Z_{dr} values, as if the raindrops have had equilibrium shapes. Applying β retrieval procedure (6) on these Z_{dr} values, instead of measured ones, we estimated effective slope of Beard and Chuang (1987) size shape relation for a given DSD. The results of these calculations is also shown in Figure 3.We can observe that retrieved β values are smaller than equilibrium ones, as one would expect for oscillating raindrops (Chandrasekar et al. 1988). In the Figure 4 the scatterplot of retrieved β values as function of the retrieved D_0 is shown. It should also be noted that both definitions of β confirm this conclusion. This precipitation event is characterized by mean $\log N_w$ value of 3.63, mean D_0 value of 1.3 mm and mean μ value of 0.4, as shown in the Figure 2.

4. Conclusions

It was shown that the slope of raindrop shape size relation can be retrieved from dual-polarization spectral analysis of time-series radar data. As a result a fine scale analysis of the precipitation microphysical properties can be carried out. It is shown that DSD parameters retrieval method, that usually requires solution of five parameter nonlinear optimization problem, can be simplified to a three parameter nonlinear least square problem.

The effect of different radar elevation angles on the errors in the retrieved parameters was also studied. It was shown that elevation angles lying in the range between 30 and 70 degrees are optimal for proposed study.

Since DSD retrieval is the most critical part of this method, it would advantageous to validate the retrieved parameters. The measurement setup during the observations used in this study, however, does not allow us to compare retrieved parameters to measurements taken using some other instrument. An indirect validation of the retrieved DSD parameters, nonetheless, is possible. By using retrieved DSD parameters and equilibrium raindrop shapes (Beard and Chuang 1987) we have calculated Z_{dr} values. Goddard et al. (1982) have observed that in this case calculated Z_{dr} values would be 0.1-0.3 dB larger than observed ones. That was confirmed by our observations and calculations, that indicates that our retrieval has resulted in reasonable values of DSD parameters.

To illustrate the performance CSU-CHILL measurements of stratiform precipitation event with light to moderate rain intensities was used. In this case we have observed that raindrops appear more spherical than raindrops in equilibrium. As was previously observed by Chandrasekar et al. (1988).

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