Modeling of Mountain Waves in T-REX

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1. Introduction

The main objective of the Terrain-induced Rotor Experiment (T-REX) is to understand the nature of coupling of mountain-induced rotor circulations to the structure and evolution of overlying mountain waves and to the underlying boundary layer (Grubišić et al. 2004). T-REX also seeks to gain a better understanding of mountain wave dynamics including the characteristics of wave generation, propagation, and breakdown. Field activities took place in March and April 2006 in the Owens Valley region both upwind and directly east of the southern Sierra Nevada Mountains, which is the tallest, steepest, nearly two-dimensional topographic barrier in the contiguous U.S. Of the many ground-based and airborne instrumentation systems employed in the field, the structure of mountain waves are best revealed with in situ and dropsonde data from two aircraft - the High-performance Instrumented Airborne Platform for Environmental Research (HIAPER) and the Wyoming King Air - and the boundary layer wind profilers on the Integrated Sounding System (Cohn et al. 2006).

Several high-resolution groups ran nonhydrostatic models over similar domains in support of either real-time decision-making or the research objectives of T-REX. Of particular interest is the ability of the Weather Research and Forecasting (WRF) model to correctly predict mountain waves. WRF is running operationally at 5-6 km resolution in the NCEP High-Resolution Window domain and at 24-km resolution in the NCEP Short-Range Ensemble Forecast system. The WRF-NAM recently replaced the 12-km Eta model, and the WRF-Rapid Refresh will replace the Rapid Update Cycle (RUC) model in 2008.

The Advanced Research WRF (ARW) and the WRF-Nonhydrostatic Mesoscale Model (NMM) models were run during the T-REX field exercise twice daily as a joint effort between the NOAA Earth Systems Research Laboratory (ESRL) Global Systems Division and the Developmental Testbed Center (DTC). The ARW and NMM were both configured on a 450 km X 450 km domain with 2-km grid spacing and 51 vertical levels. The model cores both made use of the "Eta physics" (consisting of the Ferrier microphysics, Mellor-Yamada-Janiic boundary layer scheme, the NOAH Land Surface Model, Janjic Eta surface layer physics, and the GFDL radiation package). Given the small grid spacing, cumulus parameterization was not invoked. The ARW model configuration includes a 1.5-order TKE closure scheme. Both models were initialized with analyses from the operational 13-km RUC model and used the same boundary conditions. No data assimilation was The use of an identical set of performed. boundary and initial conditions and interoperable physics in both cores allowed us to isolate the sensitivity of the predictability and structure of simulated mountain waves and wave breaking to the model numerics alone.

The main purpose in running the two WRF models was to investigate a variety of numerical issues relevant to the prediction of mountain waves (the 2-km model resolution was not guite fine enough to simulate rotors). One of the issues we are studying is how "internal divergence damping" used in the NMM model affects the simulated wave structures and energy spectra, namely whether better or worse agreement with the T-REX observations occurs with this technique activated. The divergence damping is calculated at each NMM model level to reduce the vertical propagation of gravity wave energy, whereas the ARW uses three-dimensional а damping The expected difference is that mechanism.

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internal gravity waves should be progressively minimized with increasing height using the NMM approach, whereas only the external wave mode would be treated in the ARW approach. The NMM method eliminates the need for any other mechanism to absorb downwardly propagating gravity waves. By contrast, a diffusive damping sponge layer was used in the upper 5 km (33%) of the ARW model. A second issue we are investigating concerns how changes made to the depth of the diffusive sponge layer and the damping coefficient impacts the ARW solutions. A third matter being studied concerns the effect of different terrain smoothing approaches upon the numerical representation of the steep topographic slopes, and whether spurious accelerations may have arisen as the result of truncation errors in the computation of the horizontal pressure gradient in terrain-following coordinates.

This paper begins with a brief summary of understanding of mountain wave present generation and wave breaking in section 2, including a discussion of the primary controlling parameters for mountain waves from theory. The relationships of these computed parameters to the simulated wave structures are examined in our numerical simulations. We then describe some initial findings from our real-time model simulations in section 3. We present results in section 4 that were obtained in experiments run recently since the end of the field phase that relate to some of the numerical issues discussed above.

2. Background on mountain waves

Favorable conditions for the occurrence of mountain waves are believed to consist of a stable layer or inversion and relatively strong crossmountain winds at the mountaintop level. Using linear theory, Scorer (1949) showed that vertical variations in stability and wind shear could produce trapped lee waves if the <u>Scorer parameter</u>

$$l^{2} = \frac{N^{2}}{U^{2}} - \frac{U_{zz}}{U} - k^{2}$$

decreases sufficiently rapidly from the lower stable layer to the less stable upper layer, i.e., if

$$l_L^2 - l_U^2 > \frac{\pi^2}{4H^2},$$

where *H* is the depth of the lower layer, *N* is the Brunt-Väisälä frequency, *U* is the incident wind speed, U_{zz} is the curvature in the wind profile, and $k = 2\pi/\lambda_x$ is the horizontal wavenumber. Trapped waves may help to promote the development of rotors (Grubišić et al. 2004).

The width of the mountain in a twodimensional framework also plays a role in determining whether the waves are evanescent (trapped modes lacking any vertical tilt) or freely propagating (phase lines tilting upstream). As shown by Queney (1948), for a very narrow mountain with half-width $a \ll U/N$, the mountain primarily forces evanescent waves - characteristic of nonhydrostatic trapped lee waves, whereas vertically propagating modes dominate for wide mountains with a >> U/N (for $U \sim 10$ m s⁻¹ and $N \sim$ 10^{-2} s⁻¹, we have U/N = 1 km). Mountains with half-widths in the range of 1 - 10 km can produce both kinds of wave modes. As the parameter Na/U increases from a value of 1 to 10. one gradually progresses from the nonhydrostatic to the hydrostatic regime, and for this reason, we will refer to Na/U as the hydrostatic parameter.

Another parameter important for determining the nature of mountain waves is the <u>Froude</u> <u>number</u> (also referred to as the <u>nonlinearity</u> <u>parameter</u>), defined as

$$F = Nh/U$$
,

where *h* is the mountain height. As the Froude number increases from 0 to 1, the waves become increasingly nonlinear, causing the wave fronts to become steeper and perhaps to break. In a study of the partial reflection of vertically propagating mountain waves in a mean state with constant wind speed and a two-layer stability structure, Durran (1986) found that the most rapid transition from linear to nonlinear (characterized by large drag) occurred as the Froude number in the lower layer increased from 0.2 to 0.6. The Froude number has other influences on the nature of the gravity wave response. For 0 < Nh/U < 0.5, lee waves occur downstream of the obstacle similar to the prediction from linear theory, whereas for 0.5 <Nh/U < 2.0, upstream-propagating waves are observed in the steady-state, hydrostatic case (Baines and Hoinka 1985)

If wave energy can leak upwards and propagate to higher altitudes, this can result in

wave breaking aloft, which occurs as vertically propagating gravity waves amplify, in part due to the decrease of density with altitude and nonlinear processes. One consequence of mountain wave breaking is that the waves lose energy due to turbulent breakdown. Klemp and Lilly (1975) showed from linear gravity wave theory that the waves can become intense if an inversion is present near the mountain-top level upstream of the obstacle and the layers beneath the inversion and above it both have a thickness equaling 1/4 the vertical wavelength of a hydrostatic gravity wave in the respective layer ($\lambda_z = 2\pi U/N$). Since the vertical wavelength is shorter in the more stable layer near the surface, the predicted optimal structure is one in which there is a relatively thin inversion near the mountain top level and a thicker. less stable laver aloft. This structure supports waves that are not fully reflective, i.e., they do not lead to a resonant mode, as in the case of the lee waves studied by Scorer.

Peltier and Clark (1979) showed that flow overturning and consequent dramatic acceleration of the low-level flow in the lee of a mountain results when the Froude number F is slightly supercritical, i.e., when F > 1. A trapped resonant mode develops in the low wind speed "cavity" that occurs between the ground and the region of overturning (Laprise and Peltier 1989). If the wind speed goes to zero and the Richardson number (Ri) is smaller than 0.25 at such a "critical level" due to the development of local shear instability, then "wave over-reflection" may even occur. However, Clark and Peltier (1984) demonstrated that only if the height of the critical layer above the topography is 34 of the vertical wavelength for a hydrostatic gravity wave or some integral number of such wavelengths in excess of this, would the upwardly propagating and reflected waves interfere constructively in such a way as to result in a large amplitude resonant response.

The results of Smith (1985), in which a "dividing streamline" separates the laminar flow beneath the wave breaking region from the turbulent flow above, showed that high amplitude gravity waves occur if the critical layer is located *anywhere* between ¼ and ¾ vertical wavelengths. This prediction was supported by Durran and Klemp (1987), who found that, as the depth of the lower stable layer increased for a fixed mountain height, the flow evolved from one of "supercritical" (decelerating flow), to a propagating hydraulic jump, a stationary hydraulic jump, and finally, to

subcritical flow everywhere (accelerating restricted flow). These results are qualitatively similar to the behavior of a free surface in hydraulic theory.

Numerical models have become sophisticated tools for study of the structure and dynamics of mountain waves and rotors. Doyle et al. (2000) conduced experiments of the infamous Boulder windstorm of 11 January 1972 using twodimensional nonhydrostatic models with a horizontal grid spacing of 1 km for a "Witch of Agnesi" mountain with a half-width a = 10 km. The models differed in many respects, including the finite differencing techniques, though all the models employed terrain-following vertical Some of the models used the coordinates. equations while others anelastic were compressible. Some used fourth-order smoothing while others used second-order. Some had explicit prediction equations for turbulent kinetic energy (TKE) while others represented turbulence with simple first-order closure techniques. Despite these many differences, all of the models tested (WRF was not one of them) produced upper-level wave breaking in similar regions just downstream and above the hydraulic jump in the lower immediately of the troposphere downwind The damping effect of the wavemountain. induced critical layer on the vertical extent of the wave breaking was apparent in plots of the Richardson number fields. On the other hand, the structure of the wave breaking was greatly affected by the numerical dissipation, the numerical representation of horizontal advection, the lateral boundary conditions, and especially by the vertical resolution. In particular, models that used less horizontal eddy diffusivity tended to exhibit smaller-scale wave breaking substructures relative to the other models. These sensitivities help to motivate the current study.

3. Results of real-time simulations

We show a number of vertical cross-sections obtained from the model simulations for various cases. Most of these displays are oriented along the cross-mountain flight track option B for the HIAPER ($245^{\circ} - 65^{\circ}$), since the prevailing flow pattern aloft was southwesterly in most cases. The span of cross section B is 275 km (Fig. 1). For the purpose of calculating the governing flow parameters from theory for this cross section, we used the following values for the dominant mountain range: mountain half-width *a* = 65 km and mountain height *h* = 3.6 km.



Fig. 1. Terrain height (m) for the Sierra Nevada region in T-REX used in the WRF 2-km model simulations. Model domain shown is 450 km on each side. Also shown are the three cross sections corresponding to the HIAPER tracks.

According to Doyle et al. (2006), the largest amplitude mountain waves observed by HIAPER occurred during the flights of IOP 4 (14 March), IOP 6 (25 March), and IOP 13 (16 April), when the maximum vertical velocity observed by the aircraft was 6, 12, and 10 m s⁻¹, respectively. We show results here for IOP13 and IOP10 (8-9 April).

Analysis of the real-time WRF model forecasts indicates that the NMM divergence-damping effect reduced the amplitude of vertically propagating waves considerably more than did the ARW upper-layer sponge scheme. NMM forecast gravity waves in the lower stratosphere were much smoother, weaker (or even non-existent), and displayed shorter horizontal and vertical wavelengths than in the ARW model. These characteristic differences are illustrated in the case of IOP10 in Fig. 2. Note that the horizontal wavelength in the NMM case is only approximately one-half of the 18-km wavelength characterizing the hydrostatic mountain wave in the ARW solution. Such a 9-km wavelength is barely resolvable by a 2-km grid model and suggests problems needing to be addressed.



Fig. 2a. Horizontal wind speed (m s⁻¹, see color bar at bottom) and isentropes (4K interval contours) in section B from a 9-h ARW forecast valid at 0900 UTC 8 April 2006. Note vertically propagating hydrostatic mountain wave in the immediate lee of the Sierra Nevada and downstream lee wave train. Ordinate is pressure (hPa).



Fig. 2b. As in Fig. 2a, except from the 9-h NMM forecast. Note the much weaker mountain wave in the immediate lee of the Sierra Nevada, and its shorter horizontal and vertical wavelengths relative to that in the ARW forecast.

An even more startling difference in the prediction of mountain waves between the two WRF models is shown in Fig. 3 for another model run made 24h later during IOP10. In this case, the NMM failed altogether to produce either the hydrostatic mountain wave or the train of nonhydrostatic lee waves downstream of the Sierra Nevada mountain chain.



Fig. 3a. As in Fig. 2a, except for an 18-h ARW forecast valid for 0600 UTC 10 April 2006. Note the vertically propagating mountain wave in the immediate lee of the Sierra Nevada, and a train of lee waves downstream of the mountains.



Fig. 3b. As in Fig. 3a, except for an 18-h NMM forecast valid for 0600 UTC 10 April 2006. Note the total absence of either a hydrostatic mountain wave or a train of nonhydrostatic lee waves in this model forecast.

4. Sensitivity experiments

Since the field phase of T-REX ended only a couple of months ago, only a small number of sensitivity experiments have been performed so far to examine a number of numerical issues. The primary experiments completed to date concern the vertical CFL (timestep) condition, and tests of the sponge damping layer depth and damping coefficient.



Fig. 4a. Omega vertical velocity (color-fill, Pa/s) and 2K isentropes from a 24-h ARW model forecast valid at 00 UTC 10 April 2006 using a 7.5-sec timestep.



Fig. 4b. As in Fig. 4a, except from a 24-h ARW model forecast valid at 00 UTC 10 April 2006, except using a 5.0-sec timestep.

a. Vertical CFL instability and other issues

Normally, the presence of an undesirable numerical instability, known as the "violation of the vertical CFL criterion" can be easily identified and this problem corrected by shortening of the model timestep. This instability developed in the NMM real-time model simulation of the IOP13 case, but was easily corrected by reducing the timestep from 4 sec to 3 sec. In other cases, although the timestep choice did not produce any apparent instability, it did nonetheless affect the model solution in subtle and unanticipated ways. An example of this behavior is demonstrated in Fig. 4. In this case, the timestep for the ARW model was varied from its default value used in real-time (7.5 sec, see Fig. 4a) to smaller values of 6.0 sec and 5.0 sec (Fig. 4b). No significant difference in the simulations should have occurred, yet careful inspection reveals subtle differences in the vertical velocity field, even in the planetary boundary layer. This suggests that the solution had not converged.

b. Diffusive sponge layer depth and damping coefficient tests

The upper boundary condition is a non-trivial problem for prediction of mountain wave structure, as it should act to greatly reduce the spurious reflection of upward-propagating gravity waves to avoid fictitious sources of vertical fluxes of energy and momentum. Special care must be taken since the upper boundary condition will fundamentally affect the entire wave solution. The design of the upper dissipative "sponge" layer in the ARW model follows Klemp and Lilly (1978), which uses a smoothly-varying (sin² z) function to represent the increase of eddy viscosity with height. Insufficiently weak maximum viscosity causes large reflection, however reflection can also arise when the maximum viscosity is chosen too large, as then its vertical gradient can create problems. As the viscous sponge layer depth decreases, the sensitivity to varying viscosity increases. We performed several tests of varying viscous layer depth (from 5 km to 10 km) and damping coefficient (from 0.0 to 0.04 and 0.06). Our results show a significant sensitivity to the choice of these variables (Fig. 5). While this is not unexpected, it was surprising to see how much influence the viscous sponge properties had upon vertical motions in the lower troposphere. We are continuing to conduct experiments to arrive at a more optimal solution.

5. Conclusions

Despite these many numerical issues regarding appropriate timesteps to use in both WRF models, the viscous sponge layer properties in the ARW model, and the NMM divergencedamping impacts, there were cases of upper-level wave breaking in both the ARW and NMM forecasts that were verifiable in the HIAPER aircraft measurements. The perturbations were very large on a number of occasions, the largest being for the IOP13 case (16 April 2006), where horizontal wind speed perturbations of 50 m s⁻¹ and 15 K potential temperature perturbations over a distance of just 30 km were forecast. The vertical motions associated with the mountain waves forecast by the ARW model in this case were as large as 7.5 m s⁻¹ at 11.3 km and 3.8 m s⁻¹ at 13.1 km, compared to the values observed by HIAPER of 10 and 6 m s⁻¹, respectively (see Fig. 12 in Doyle et al. 2006).



Fig. 5a. As in Fig. 4a, except from a 24-h ARW model forecast valid at 00 UTC 17 April 2006, using no upper-level viscous damping layer. Model timestep is 7.5 sec.



Fig. 5b. As in Fig. 5a, except from a 24-h ARW model forecast valid at 00 UTC 17 April 2006, using a damping coefficient of 0.06 for a 5-km deep viscous damping layer. Note reduction of wave reflection in uppermost layer, but also other effects unexpectedly occur at low levels.

At the conference, we will discuss the results of additional sensitivity experiments being run for selected IOP events after the field phase to investigate such issues as the method used for reducing spurious gravity wave reflection off of the top boundary, the NMM divergence damping method, and the handling of the terrain insofar as its smoothness and effects on the surface pressure gradient force are concerned. We also hope to be able to show actual aircraft measurements needed to verify the model simulations.

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