5.4 KATABATIC FLOW OVER LONG SLOPES: VELOCITY SCALING, FLOW PULSATIONS AND EFFECTS OF SLOPE DISCONTINUITIES

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1. Introduction

About 70% of the Earth's land surface is characterized by complex topography (Strobach 1991), so are the terrains of most urban areas of the world. In presence of weak synoptic winds, as in the case of the desert southwest of the United States, the major local circulation in complex terrain is driven by heating/cooling of the uneven topography, known as the thermal circulation (Whiteman 2000). It consists of slope and valley flows, with upslope and up-valley winds occurring during the day and down-valley and downslope winds occurring at night. These flows are in the meso-scale (10 -100 km) realm of the continuum of thermally driven flows on the Earth, which span from convective turbulence in the atmospheric boundary layer (~ 10 m – 1 km) to meridional deep convective (Hadley) cells (10^4 km). The downslope (or katabatic) winds, which are the focus of this paper, are driven by buoyancy forces that arise due to nocturnal radiational cooling above sloping surfaces. The cooling also leads to stable stratification, which inhibits turbulence while suppressing shear stresses near the ground. In complex terrain, stable stratification leads to a rich variety of flow phenomena, for example, pooling of dense fluid in basin-like topography, intrusion of katabatic currents at different altitudes along isopycnals, continuous weak turbulence due to shear existing between intrusions and reflection of internal waves from sloping boundaries (vis-à-vis the flat terrain case where turbulence is highly intermittent in space and time) and hydraulic phenomena that appear at slope discontinuities. This paper presents a summary of the results of a subset of studies that are being conducted at Arizona State University concerning urban meteorology and air pollution with the goal of improving parameterizations for meso-scale models. Basic studies of downslope flows have shown that there are two main types, one of which is unsteady gravity currents determined by the buoyancy flux (and momentum) at higher elevations of the slope (e.g. Hunt *et. al.* 2005) and the other is turbulent quasi-steady currents determined by local buoyancy forces and turbulent stresses (Turner 1973). In Section 2, the layer averaged equations are presented, which are then used to derive velocity scales for some special cases of katabatic winds. The equations show that under highly stable cases the downslope winds can show flow pulsations, corresponding to a frequency where internal waves impinge normal to the slope. Mountain slopes are usually not uniform, and tend to be steeper at higher altitudes. When katabatic flow arrives from steeper slopes to gentle low altitude slopes (e.g. mountain foothills), there can be hydraulic jumps, which are considered in Section 3. Field and laboratory experiments on these phenomena are described in Section 4, followed by conclusions in Section 5.

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2. Layer-Averaged Equations and Velocity Scaling

The well-known layer averaged equations in 2D for velocity and buoyancy (Manins & Sawford 1979), in usual notation, can be written as

$$\frac{\partial Uh}{\partial t} + \frac{\partial U^2 h}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2} S_1 \Delta b h^2 \cos\alpha\right) - S_2 \Delta b h \sin\alpha - C_D U^2 - \left(\overline{u'w'}\right)_H \quad (1)$$

and

$$\frac{\partial}{\partial t}\left(S_2 \Delta b h\right) + UhN^2\left(\sin\alpha - S_3 E\cos\alpha\right) + \frac{\partial}{\partial s}\left(U\Delta b h\right) = B_o - \left(\overline{b'w'}\right)_H \quad (2)$$

where the layer averaging procedure is defined and discussed in Manins & Sawford (1979).

Note that the first two terms in (1) have scales of Uh/T and U^2h/L_H, where L_H and T are the along-slope length and time scale of the flow (Mahrt 1982). The ratio of the two terms becomes L_H/UT, and the unsteady term becomes insignificant when $L_H/UT << 1$, whence the principal balance of forces in (1) becomes the same as that assumed by Businger & Rao (1965), viz.

$$U^2 h / L_H \sim \Delta b h \sin\alpha$$

or

$$U = \lambda_u \left(\Delta b L_H \sin\alpha\right)^{1/2}, \qquad (3a,b)$$

where $L_H \sin\alpha$ is the vertical distance down the slope and λ_u is a constant. Obviously, (3) is valid only when

$$L_H / UT << 1$$

or $$T >> \left(L_H / \Delta b \sin\alpha\right)^{1/2} = T_S, \qquad (4)$$

whence the averaging is done on time scales on the order of an hour, viz. L_H ~ 10 km, Δb ~ 0.02m/s^2 and $\alpha \approx 0.04$ rad (Doran & Horst 1981). The surface shear stress term (~ $0.01U^2/H$) and the Reynolds stress at H are neglected assuming the dominance of the "stress" terms arising from

$$\frac{\partial}{\partial s}\left(U^2 h\right) = Uh\frac{\partial U}{\partial s} + U\frac{\partial Uh}{\partial s} = Uh\frac{\partial U}{\partial s} + EU^2. \qquad (5)$$

If the entrainment stress is dominant at smaller Richardson numbers, say Ri < 0.8, where $Ri = \Delta bh\cos\alpha/U^2$ (Princevac *et al*. 2005), then a major balance of the form

$$EU^2 \sim \Delta bh\sin\alpha \rightarrow U \sim \lambda_{u1}\left(\Delta bh\sin\alpha/E\right)^{1/2}, \qquad (6)$$

can be expected. For very small Ri, E can be approximated as constant (Fernando 1991), and thus the velocity scale becomes

$$U \approx \lambda_u^{*}\left(\Delta bh\sin\alpha\right)^{1/2}. \qquad (7)$$

When considering time scales smaller than T_S, the unsteady term dominates in (1). Also, late into the night, the effects of entrainment is negligibly small (more specifically $E << \tan\alpha$). Therefore, over small slopes, one may expect linear oscillations determined by

$$\frac{\partial Uh}{\partial t} \approx -\Delta bh\sin\alpha;$$

$$\frac{\partial}{\partial t}\left(\Delta bh\right) + N^2\left(Uh\right)\sin\alpha \approx B_o. \qquad (8a,b)$$

The oscillations are expected to have a frequency of $N\sin\alpha$ with a time period of

$$T_o = 2\pi/N\sin\alpha. \qquad (9)$$

Oscillations in katabatic flows have been predicted on theoretical grounds (Fleagle 1950; McNider 1982) and have been observed in field studies (Doran & Horst 1981; Porch *et al.* 1991; Van Gorsel *et al.* 2003). Their existence, however, has been attributed to different mechanisms. For example, Fleagle's (1950) model postulates that, as air accelerates down the slope, an adverse pressure gradient is generated due to adiabatic heating of air to retard the flow. If the friction can be written as $F = -kU$, then the friction increases as air decelerates, while driving force continues to be slowly enhanced by radiational cooling, thus resulting in oscillatory flow behavior (also see McNider 1992). On the other hand, Porch *et al.* (1991) argued that oscillations in katabatic flows can arise due to the presence of cross flows, which temporarily stops drainage currents from tributaries entering nearby valleys. As cold air builds up and buoyancy forcing increases in tributaries, there will be sudden releases of colder air onto the slopes periodically. It is also interesting to note that (9) represents critical (resonant) internal waves that impinge on the slope normally, and they undergo local degeneration while creating enhanced turbulence near the boundary (De Silva *et al.* 1997); see Figure 1.

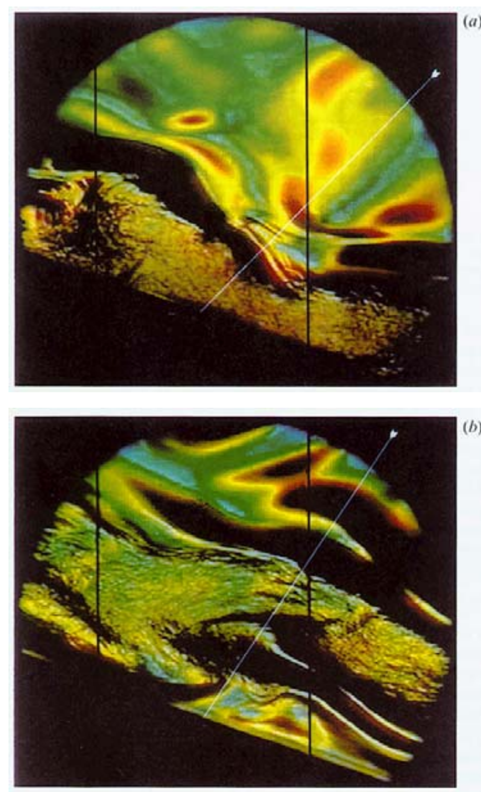

Figure 1: Schlieren video images showing the on-slope and off-slope initiation of instabilities during internal wave reflection on a slope. The oblique thin white line indicates the centre-line of the incident wave ray (From De Silva *et al.* 1997).

3. Hydraulic Adjustment

Most natural mountain slopes are not uniform, but have slope breaks, with higher (altitude) slopes being usually steeper. In order to analyze the flow, it is possible to use (1) and (2), with the simplifying assumption of unit shape factors for S_1, S_2 and S_3. Then, it is possible to write approximately (Turner 1973)

$$\frac{\partial}{\partial t}(Uh)+\frac{\partial}{\partial s}(U^2h)=-\Delta bh\sin\alpha+\frac{\partial}{\partial s}\left(\frac{1}{2}\Delta bh^2\cos\alpha\right)-C_DU^2, \qquad (10)$$

$$\frac{\partial}{\partial t}(\Delta bh)-(E\cos\alpha-\sin\alpha)N^2Uh+\frac{\partial}{\partial s}(\Delta bUh)=B_0, \qquad (11)$$

and under steady conditions (11) becomes, for small α and B_0, and $g'=-\Delta b(>0)$

$$\frac{\partial}{\partial s}(U^2h)=g'h\alpha-\frac{\partial}{\partial s}\left(\frac{1}{2}g'h^2\right)-C_DU^2, \qquad (12)$$

$$(E-\alpha)N^2Uh+\frac{\partial}{\partial s}(g'Uh)=0. \qquad (13)$$

When E is small and N = 0, say at highly stratified conditions at the flow interface but no ambient stratification, it is possible to write (12) as

$$\frac{\partial(U^2h)}{\partial s}=g'h\alpha-g'h\frac{\partial h}{\partial s}-C_DU^2,$$

$$Uh=\text{const},\ \ g'=\text{const}. \qquad (14a\text{-}c)$$

Using $Fr^2=U^2/g'h$, (14) becomes

$$\frac{\partial h}{\partial s}\left(1-Fr^2\right)=(\alpha-C_DFr^2) \qquad (15)$$

Note that at the critical depth corresponding to Fr = 1, $\alpha_{cr} = C_DFr^2 = C_D$. This critical slope α_{cr} is determined mainly by the drag coefficient on the surface, which tends to be small $C_D = 10^{-3}$, and thus $\alpha_{cr} \approx 0.05^0$. A change from subcritical to supercritical or vice-versa needs to occur through Fr = 1 or a critical slope (Figure 2).

4. Field and Laboratory Observations

4.1 *Katabatic Flow Velocity*

Katabatic flow velocities measured during the Vertical Transport and Mixing eXperiment (VTMX) conducted in Salt Lake City, Utah, were used to evaluate the theoretical predictions given in Section 2. The Salt Lake Valley was heavily instrumented during the VTMX campaign (Doran *et al*. 2002), but only the data from two field sites dedicated for recording the vertical profiles of velocity and temperature are used in the present study: the Arizona State University Cemetery Site (ACS) operated by our group and the Slope Site (SS) instrumented by the Pacific Northwest National Laboratory (David Whiteman's group). Details of the velocity and temperature profiles and their time evolution are given in Princevac *et al*. (2005). Figure 3 shows a plot of normalized velocity $U/(\Delta bL_H\sin\alpha)^{1/2}$ according to (3b) as a function of the Richardson number Ri for ACS and SS. It was clear that λ_u becomes constant, as predicted by (3b), only at higher Ri, arguably at Ri > 1.5. The large Ri asymptote of λ_u was found to be $\lambda_u \approx 0.2$. Note that (3) is clearly unsuitable at lower Ri, possibly because of the influence of entrainment that changes the force balance substantially, as evident from (6).

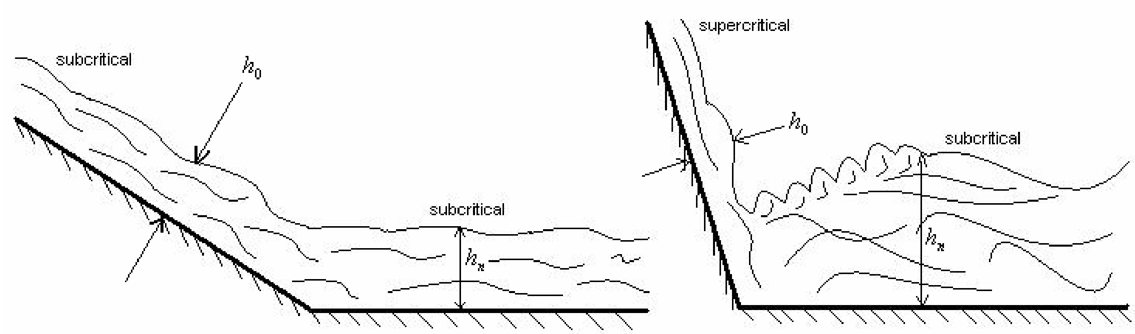

Figure 2: Flow over a slope discontinuity: supercritical and subcritical flows and their evolution -- h_0 the flow thickness and h_c is the critical thickness. (a) $h_0 > h_c$ (b) $h_0 < h_c$. (Turner 1973).

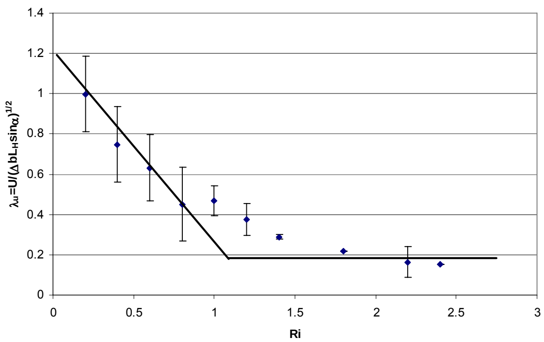

Figure 3: A plot of normalized velocity (*i.e.* λ_u) versus Richardson number *Ri*

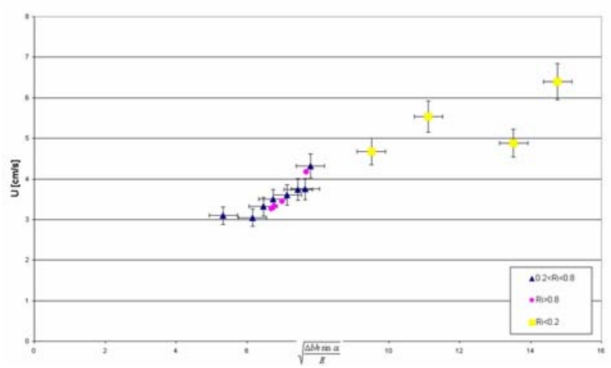

Figure 4: A plot of U versus $\sqrt{\Delta bh \sin\alpha / E}$ for three different Ri ranges

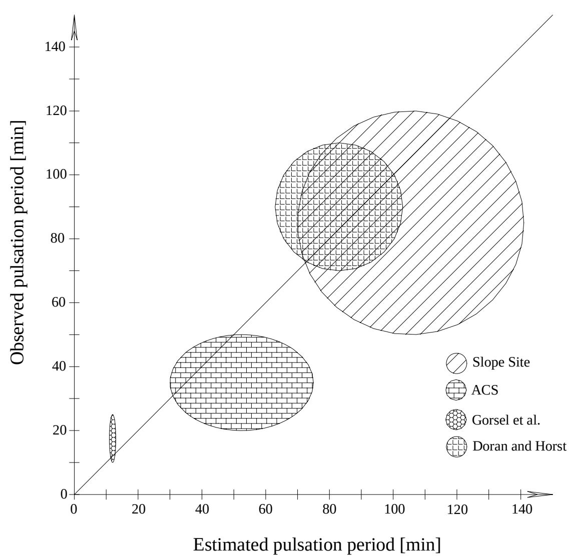

Figure 5: Observed and predicted pulsation periods at four different sloping terrains. Different shaded areas represent the variability of observed and estimated pulsation periods.

In order to study velocity scaling for a gravity current descending along a slope with substantial entrainment, a series of laboratory experiments was conducted by generating a gravity current on a uniform slope by releasing a two-dimensional stream of dense fluid (Dumitrescu 2005). The velocity and density of the fluid were determined using Particle Image Velocimetry (PIV) and Laser-Induced Fluorescence (LIF) techniques. The results are shown in Figure 4, plotted in accordance with (6). Although the results show scatter, some support for the scaling argument (6) can be seen over a wide range of *Ri*.

4.2 *Flow Oscillations*

Figure 5 shows the ranges of pulsation frequencies observed at ACS and SS during the VTMX experiment, over all days of observations (Fernando & Princevac 2004). Also included are the ranges of predictions as well as some previously reported data from Van Gorsel *et al.* (2003) taken in the Riviera valley and Doran & Horst (1981) in the Geysers area of northern California. All of these data support the notion that the observed low frequency oscillations on sloping terrain under nocturnal conditions contain a dominant frequency component $N\sin\alpha$, which is the frequency of internal waves arriving normal to the slope. As discussed, these waves are liable to breakdown via a resonance mechanism. katabatic flows, therefore, are expected to be in a state of continuous turbulence generation despite stable stratification within.

4.3 *Hydraulic adjustment*

The hydraulic analysis presented in Section 3 indicate that for non entraining flows (large *Ri*) the theoretical supercritical to subcritical transition is possible when passing through an angle as small as $\alpha \approx 0.05^0$. This estimate is expected to change if entrainment is taken into account in the analysis. In order to demonstrate that the supercritical to subcritical transition is associated with passing through a small angle, a laboratory experiment was set up using three plates, and the gravity current generator was placed at the edge of the highest slope; see Figure 6. Note that the flow is supercritical at the top and middle slopes, Fr ≈ 1.5 and 1.2, respectively, and there is no hydraulic adjustment at the top discontinuity. Note the hydraulic adjustment at the discontinuity between the horizontal plate and the second plate. A clear thickening of the flow, similar to that is occurring at

Figure 6: Hydraulic flow on three slopes with angle 0, 10, 35 degrees The source conditions at the gravity current generator are: initial buoyancy jump Δb_0 = 30 cm/s^2 and the buoyancy flux Q_0 = 45 cm^2/s.

a hydraulic jump, is clear, providing support for the notion that hydraulic jumps are possible at a slope discontinuity, when the lower slope is close to horizontal. The Froude numbers on the lower slope was found to be $Fr \approx 0.9$.

5. Conclusions

Katabatic flows in complex terrain exhibit a rich variety of flow phenomena, some of which were studied in this communication. Results of theoretical analyses and laboratory/field experiments on flow scaling, unsteady flow oscillations and hydraulic adjustments at slope discontinuities were presented. Further details of the results will be presented in future publications.

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