THE SENSITIVITY OF NUMERICALLY SIMULATED MULTICELL CONVECTION TO GRID SPACING AND COMPUTATIONAL MIXING COEFFICIENS

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1. INTRODUCTION

Historically, grid spacings on the order of one kilometer have been used for simulating deep moist convection using a large eddy simulation (LES) model (Wilhelmson and Wicker, 2001). This standard has recently been called into question by Bryan et al. (2003) (hereafter referred to as BWF). They suggest that smaller grid spacings are necessary to adequately resolve complex cloud scale processes. BWF proposed that in the absence of a converged model solution, grid spacing should be at least two orders of magnitude smaller than the phenomena to be studied.

This study and its parent project are concerned with the regeneration characteristics of multicell updrafts that typically have a horizontal scale of a few kilometers. According to the criteria of BWF, simulations of these updrafts should have grid spacings of 100 m or less. Bryan and Rotunno (2005) showed a statistically converged solution could be attained for shallow convection with grid spacings smaller than 50 m. Computing resources are currently unavailable to conduct a large number of simulations at such a low grid spacing, as is necessary for subsequent portions of the project. Therefore, a grid spacing that is computationally feasible and satisfies the general conditions proposed by BWF is chosen.

This grid spacing should not be chosen arbitrarily. Characteristics of convection have been shown to be sensitive to grid spacing (Adlerman and Droegemeier, 2002; Weisman et al., 1997) A simple qualitative method for grid spacing selection is first examined, followed by a more objective, quantitative selection method. Lastly, the effects of computational mixing coefficients on the simulated convection are investigated using similar analysis methods.

2. METHOD

2.1 A Simple Qualitative Method

A converged solution for deep moist convection is unlikely at grid spacings larger than 50 m. The diagnosis of convergence given various grid spacings can be simple, and although solution convergence is not expected, a brief investigation can be beneficial. Cross sections of cloud water mixing ratio for six different grid spacings (Fig. 1) show that even though details of each simulation differ, the general characteristics are the same. Each simulation contains one main updraft that reaches a vertical height of approximately 10 km. Obvious differences exist (specifically in the updraft shape between the 200 m and 75 m simulations) to suggest that solution convergence has not been attained. The simulations are also similar enough to make a determination of an acceptable grid spacing difficult with only the use of fields of cloud water mixing ratio. Both the 100 m and 75 m simulations produce cloud features with more detail than their large grid spacing counterparts, making either a better choice for subsequent simulations. However, cloud water fields provide little information for choosing between the 100 m and 75 m simulations. Spectral analysis can be used to aid in this selection process (see Bryan and Rotunno, 2005; Skamarock, 2004).

2.2 Energy Spectral Method

The energy cascade in the mesoscale and convective scale of the atmosphere can be described by applying Kolgomorov theory for isotropic turbulence to the inertial subrange (Kolmogorov, 1941). The inertial subrange is an energy cascade region of the atmosphere where energy is transferred from large scales to small scales. This transfer of energy follows a $k^{-\frac{5}{3}}$ dependence, where k is wavenumber. Energy spectra derived from observations compare well with this theoretical spectrum (see Nastrom and Gage, 1985; Lindborg, 1999).

Spectral analysis is preferred on spherical domains where atmospheric fields are naturally periodic, and allow for the direct application of Fourier transforms. Methods do exist to allow aperiodic data from limited-

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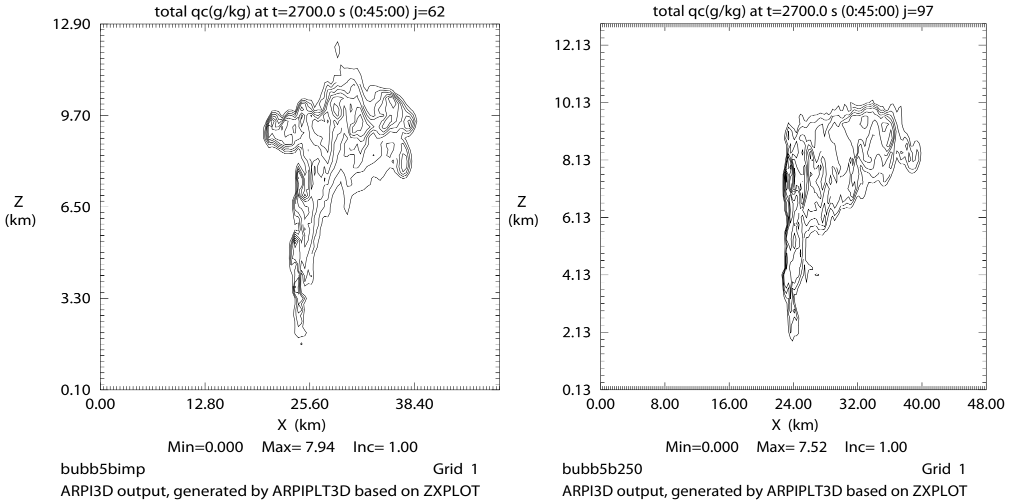
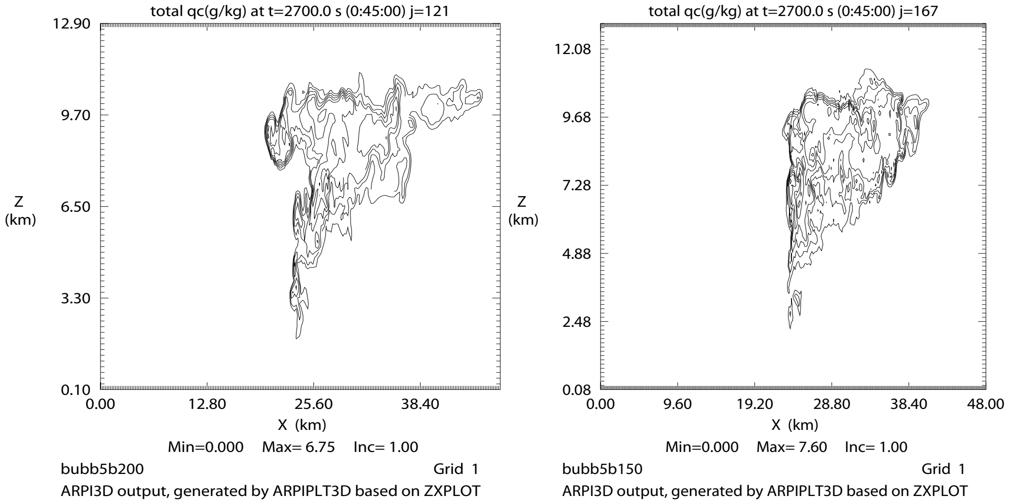


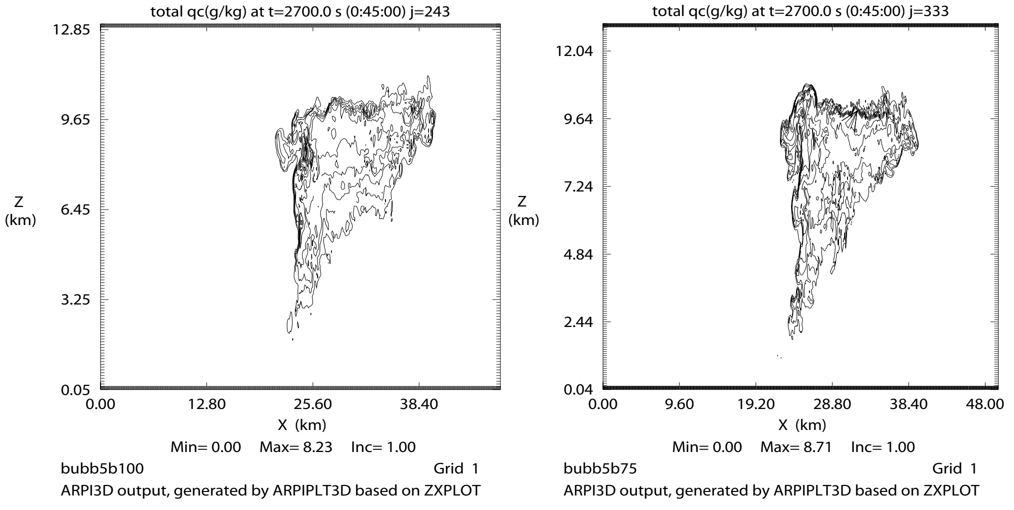
(a) 400 m

(b) 250 m



(c) 200 m

(d) 150 m



(e) 100 m

(f) 75 m

Figure 1: x-z cross-sections of cloudwater mixing ratio at 2700 s for six different grid spacings.

area domains to be transformed. This study uses the method described by Denis et al. (2002). Using mirror image symmetry, an aperiodic function is made periodic over twice the original domain. This new periodic function can be transformed using a one-dimensional discrete cosine transform (DCT). Unlike other limited-area methods, mirror image symmetry leaves the data values unchanged.

Following the method of BWF, the square of the vertical velocity field is used in producing energy spectra. Unlike the study by BWF, only one main updraft is present within the model domain. Horizontal averaging over a constant height is used to sample the updraft region in a one-dimensional form. Horizontal averaging is performed in the *y* direction prior to the application of the DCT. When horizontal averaging is used to sample a larger portion of the updraft than a single one-dimensional strip, care must be taken to constrain the averaging to the region of maximum vertical velocity. Inclusion of lower vertical velocity data from outside the main updraft reduces the total energy of the spectrum. A median filter is used to cosmetically smooth the spectrum.

2.3 Numerical Model and Simulation Specifics

The non-hydrostatic, fully compressible cloud model, ARPI V1.4, is used for each simulation (Weber, 1997). Convection is initiated in a horizontally homogeneous base state, using a constant surface heat flux mechanism. Turbulence closure is provided using a 1.5 order TKE scheme following Sullivan et al. (1994). A fourth order computational mixing scheme is used to damp small-scale features in the model (Xue, 2000). Since this project is concerned with the environmental forcing of convection, precipitation is not included.

Each simulation is performed on a 48 km x 48 km x 20 km domain in the x, y, and z directions, respectively. All simulations have equal vertical and horizontal grid spacing, except the 400 m simulation, which provides only 50 vertical points. To increase the number of vertical points, 200 m vertical grid spacing is used for the 400 m simulation. Convection is simulated for 5400 seconds. Six values of grid spacing are used: 400 m, 250 m, 200 m, 150 m, 100 m, and 75 m. Also, four values of computational mixing coefficients are used: 0.0005 s^{-1}, 0.001 s^{-1}, 0.002 s^{-1}, and 0.004 s^{-1} (hereafter referred to as cmix1, cmix2, cmix3, and cmix4, respectively).

Horizontally averaged energy spectra are produced every two minutes for the convectively active time period, for each simulation. This convectively active period is defined as the time when convection first reaches a height of 6000 m to the time when convection no longer exists at 6000 m. This height of 6000 m was chosen to allow convection to develop vertically away from the heat source. This minimizes the footprint of the source region on the convective energy spectrum. Also, the anvil region is avoided because of cirrus and gravity wave contamination of the convective spectrum. Since grid spacing varies for each simulation, so does the horizontal averaging distance. Horizontal averaging ranges from 800 m for the 400 m simulation, to 500 m for the 75 m simulation. Each horizontally averaged spectrum is then averaged in time to produce one spectrum that represents the horizontally averaged spectrum every two minutes during the convectively active time period.

3. RESULTS

3.1 Grid Spacing Results

As stated previously, features with horizontal scales on the order of 1000 m are the interest of this project. The grid spacing selected should represent features of this scale with reasonable accuracy, according to the turbulence theory. Although grid spacings of 400 m, 250 m and 200 m reproduce the inertial subrange at wavelengths between 10000 m and 1000 m, the cascade begins to deviate from the $k^{-\frac{5}{3}}$ line before 1000 m (Fig. 3). The 150 m, 100 m and 75 m grid spacing simulations reproduce the expected energy cascade below 1000 m; the 100 m and 75 m simulations reproduce the inertial subrange for wavelengths as small as 500 m.

The 150 m simulations would require the least amount of computation time, however, the representation of the inertial subrange begins to deviate from the expected response close to 1000 m. It is possible that 150 m grid spacing would not represent the inertial subrange at 1000 m for all cases. Conversely, the 75 m simulations would likely produce the inertial subrange at 1000 m correctly for the majority of the future simulation. The 75 m simulations do have a large increase in the computational requirements (simulation time as well as storage) that make this grid spacing unattractive. The most logical choice is to use 100 m grid spacing. The 100 m simulation produces the inertial subrange well below 1000 m and would be expected to represent 1000 m features accurately for the range of future simulations. Also, the computational time as well as storage requirements for a large number of simulations is not too large for the rest of the project to be completed in a relatively short time frame.

3.2 Computational Mixing Results

By design, the computational mixing scheme strongly damps features with wavelengths of 8Δx and smaller. The shortwave end of the energy spectrum is affected

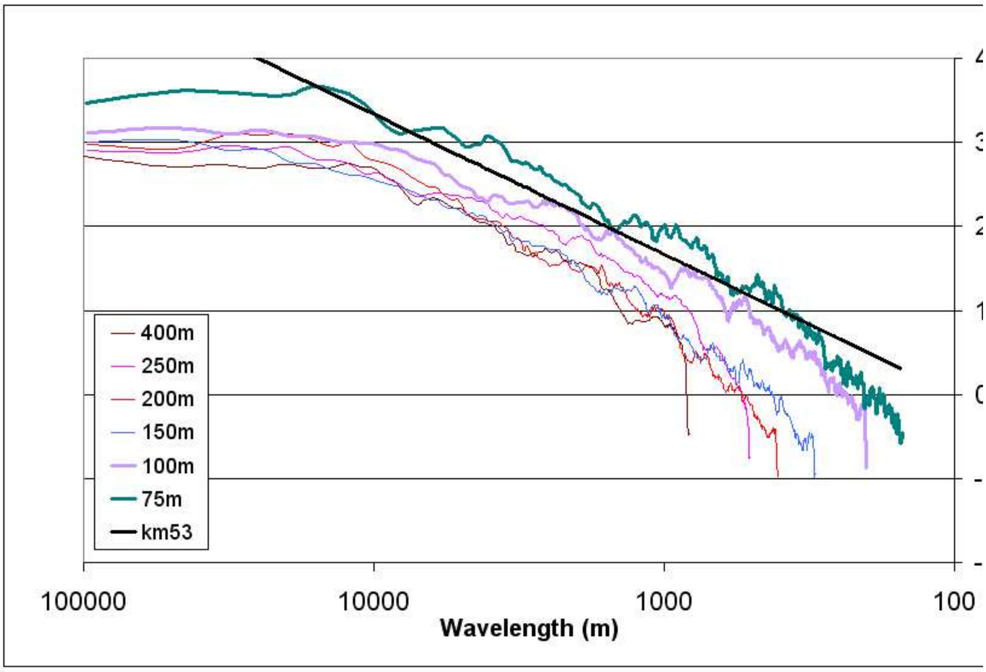


Figure 2: Energy spectra for four different computational mixing coefficients. The thick black line (km53) is the $k^{-\frac{5}{3}}$ dependence.

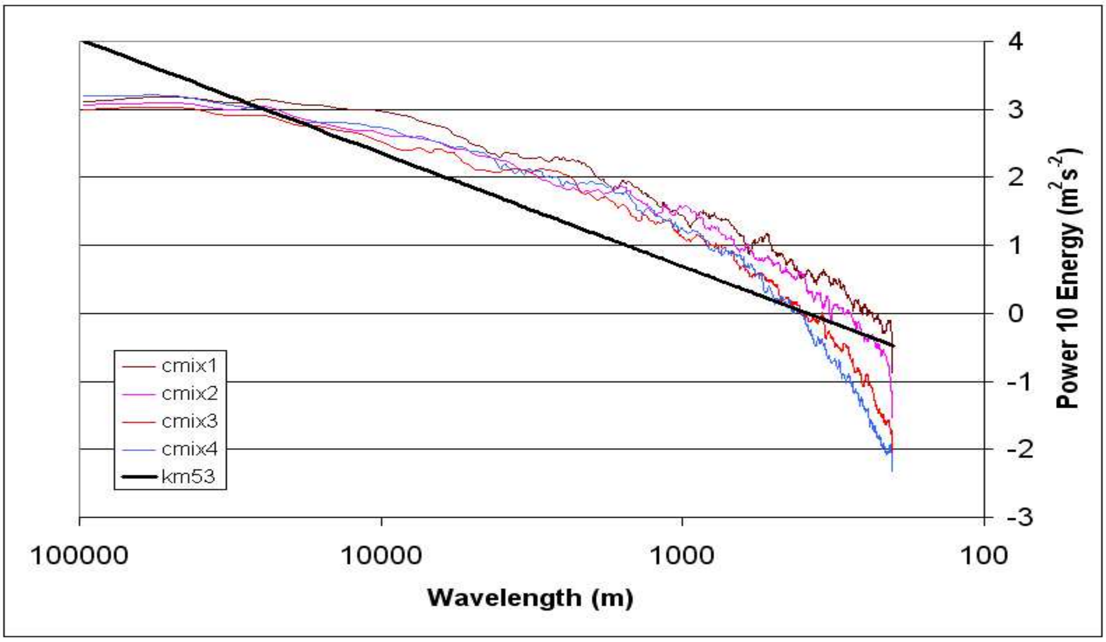
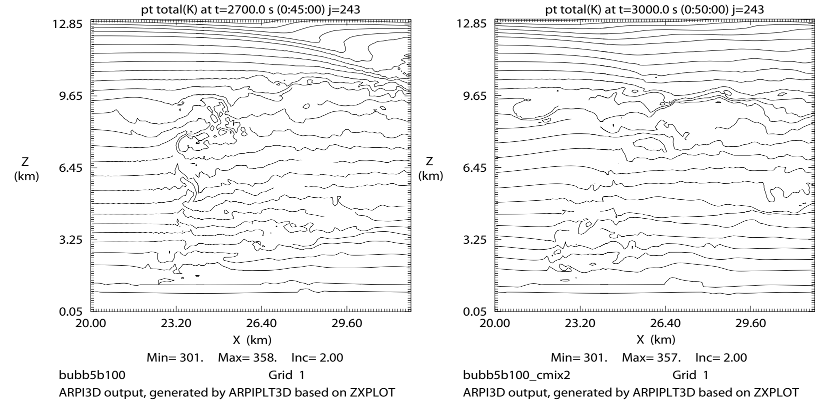
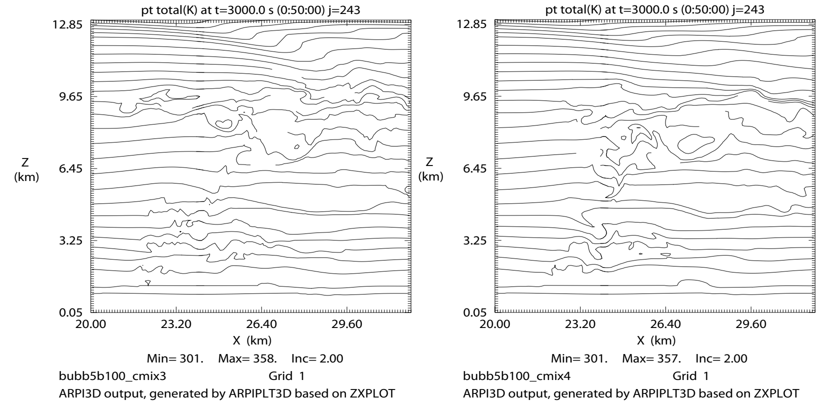


Figure 3: Energy spectra for six different grid spacings. The thick black line (km53) is the theoretical $k^{-\frac{5}{3}}$ dependence.



(a) cmix1 (0.0005 s^{-1})

(b) cmix2 (0.001 s^{-1})



(c) cmix3 (0.002 s^{-1})

(d) cmix4 (0.004 s^{-1})

Figure 4: x-z cross-sections of total potential temperature for four values of computational mixing coefficients.

most by changes in computational mixing (Fig. 3). Removal of these shortwaves is important to maintain stability in the model by preventing the growth of instabilities. As expected, increasing the coefficient of mixing decreases the amount of energy in the short wavelengths. However, this analysis does not provide a clear advantage for any individual coefficient. By investigating fields of potential temperature, the ability of each coefficient to remove small scale waves can be evaluated.

The model used for this study employs a 4th order advection scheme. The leading error term associated with even-order advection schemes is wave dispersion. These dispersive waves are clearly seen near strong gradients (specifically Fig 4a). The computational mixing coefficient should remove these dispersive waves. The cmix1 simulation is the poorest of the four coefficients in removing these waves. The cmix2 simulation removes the dispersive waves better than the cmix1 value. Yet the cmix2-simulated potential temperature field still has dispersive wave features. The cmix3 simulation removes the dispersive waves completely from the potential temperature field, which suggests the cmix3 value of $0.002s^{-1}$ is the best choice. The cmix4 simulation also removes all the dispersive wave features, but produces fields that are smoothed. However, the cmix4 value has produced unstable simulations. Therefore, the cmix4 value is not considered as a viable option.

4. CONCLUSIONS

Like in the studies of Bryan et al. (2003) and Skamarock (2004), energy spectra is used to investigate the ability of a numerical model to represent the energy cascade on the mesoscale and convective scale. The use of these spectra provide valuable information in determining how accurately the model will represent features of various wavelengths. For this project, the features of interest are thunderstorm updrafts on the order of a few kilometers. Using energy spectra, the recommendations of Bryan et al. (2003) are reinforced, suggesting that a grid spacing two orders of magnitude smaller than the feature of interest is required. This study shows that grid spacings on the order of 100 m can represent the energy cascade in the inertial subrange. Energy spectra can also be used to evaluate the impact of numerical diffusion. The energy spectral method does require other types of analysis to select the best coefficient value. For this study, plots of potential temperature were used to aid in evaluating the efficiency of the numerical diffusion scheme. For the future project simulations, a grid spacing of 100 m will allow features on the order of 1000 m to be resolved adequately while producing simulations in a computationally short amount of time. A computational mixing coefficient of 0.002 s^{-1} will be used to reduce the existence of small scale numerical noise generated by the advection scheme in the vicinity of sharp gradients.

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