

# SPACE WEATHER AND THE INCIDENCE OF REACTIVE “OFF-COST”

## OPERATIONS IN THE PJM POWER GRID

7.6

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### 1. Introduction

Forbes and St. Cyr (2004) have provided empirical evidence that adverse space weather conditions have affected the price of electricity in the PJM power grid. In this paper we examine one of the mechanisms by which space weather impacts the electricity market in the PJM power grid.

The starting point of this paper is that there are two types of power on alternating current systems: real power and reactive power (Sauer, 2003). Real power is the power that consumers need to light their lamps and run their computers and refrigerators. In contrast, reactive power maintains the voltages required for system stability and thus is critical to the delivery of real power to consumers. With respect to space weather, it is well-known that there is an increase in reactive power consumption when GICs pass through a transformer (Kappenman, 2003, p 4).

Generators in PJM are normally dispatched based on their cost with the lowest cost generators being dispatched first. One major exception to this “economic merit” or “on-cost” method of dispatch is when reactive power conditions warrant an “out of economic merit” order dispatch. In PJM, this is known as a reactive “off-cost” operation (PJM, 2005). During a reactive off-cost operation, generators are redispached so to reduce power flows across transmission lines vulnerable to voltage collapse. In this paper, we examine the effect of GICs on the incidence of these reactive “off-cost” operations in PJM using a nonparametric statistical analysis as well as an econometric model.

Before proceeding, it is worth noting that there was an abnormally high incidence of reactive off-cost operations in PJM during the “Halloween Storms” of late 2003. Specifically, over the period 29-31 October 2003, reactive off-cost operations were implemented in 18 of the 72 hours, a rate more than twice the average rate of incidence. Moreover, inspection of PJM’s emergency logs and its postings of off-cost events reveal that one four hour long reactive off-cost operation on 29 October 2003 was recognized by PJM as being space weather in origin. Interestingly,

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Pulkkinen, Viljanen, and Pirjola. (2005) have observed that these same storms led to significant operational problems for the Swedish power grid.

### 2. The PJM Electricity Grid

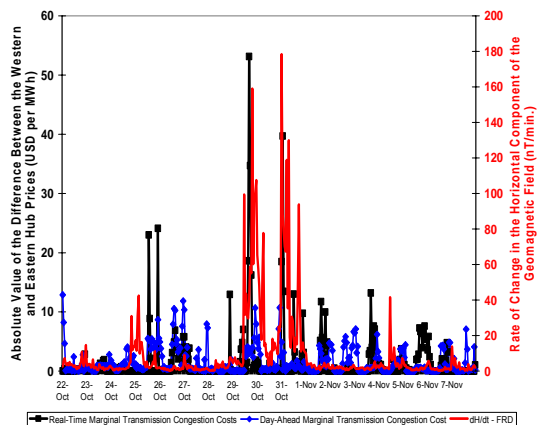
PJM Interconnection is a regional transmission organization (RTO) that as of 30 April 2004 coordinated the dispatch of 76,000 megawatts (MW) of generating capacity over 20,000 miles of transmission lines in all or parts of Delaware, Maryland, New Jersey, Ohio, Pennsylvania, Virginia, West Virginia and the District of Columbia (PJM, 2004). PJM operates both real-time and day-ahead markets for energy. Prices in these markets are reported hourly. The prices are location based which means the prices will be equal across locations when the transmission system is not congested, but can vary substantially from one location to another when there are transmission constraints.

The sample period for this study is 1 April 2002 through 30 April 2004. The starting date of the sample period represents the first day of PJM’s functional control of Allegheny Power’s five state transmission system (PJM, 2002). The ending date of the sample period is one day prior to the integration of Commonwealth Edison’s control area in northern Illinois into PJM (PJM, 2004).

PJM has a number of trading hubs whose economic function is to facilitate electricity trading. The two most important hubs over the course of the study period are its Eastern Hub and the Western Hub. These two hubs are located in eastern and central Pennsylvania, respectively. The difference in the day-ahead Hub prices reflects expected transmission constraints. The difference in the real-time Hub prices reflects actual transmission constraints. On average, because of transmission congestion largely related to terrestrial supply and demand considerations, the real-time price at the Eastern Hub over the sample period was \$1.80 per MWh higher than at the Western Hub. The differential was significantly above average during the “Halloween Storms” of October

2003 when PJM implemented a number of reactive off-cost operations (Figure 1).

**Figure 1. The Rate of Change in the Horizontal Component of the Geomagnetic Field and Transmission Congestion Costs between PJM's Western and Eastern Hubs, 22 October – 7 November 2003**



### 3. What is Reactive Power, Why is it Important, and What is the Problem posed by GICs?

Power grids in the United States are almost exclusively alternating current systems in which voltage and current oscillate up and down 60 times per second (FERC, 2005, p. 3; Sauer, p 1). As a result, the power transmitted on single transmission line also pulsates up and down around some “average” value. This average value is a measure of “real” power (Sauer, p 1). However, because voltage and current do not necessarily peak at the same time, a form of power known as reactive power is either supplied or consumed depending on whether current peaks before or after voltage (FERC, 2005, p. 17)

Sources of reactive power include generators and capacitors. Equipment that consumes reactive power are transmission lines, transformers, and motors (FERC, 2004, p 2)

Reactive power is critical to the delivery of real power to load centers because the consumption of reactive power lowers voltage while production of reactive power by generators and capacitors increases voltage. Hence, excessive reactive power consumption has the potential to lead to voltage collapse (Sauer, p 3).

A recent report by the Federal Energy Regulatory Commission (FERC) on reactive power provides a useful analogy to describe reactive power (FERC, 2005, p 17-18). According to the authors of the report,

reactive power is analogous to the bouncing up and down that takes place when one bounces along on a trampoline. Because of the elastic nature of the trampoline, the up and down bouncing results in no net forward movement but is nevertheless necessary if one is to move across the trampoline.

Kiesling (2005) reports on an analogy authored by K. P. Spouse. In Spouse’s view, the relationship between reactive power and real power can be illustrated by the labyrinth puzzle, Labyrinthspel (Figure 2). This mechanical maze puzzle consists of a wooden box. On the top surface of the box there is a series of raised wooden rails that make up a maze. There are also a series of holes in the path of the maze. On one side of the box is a knob that causes the top surface of the box to tilt front to back. There is another knob that causes the surface to tilt left to right. The object of this game is to twist the two knobs so as to keep the ball rolling through the maze without falling into any of the holes. In the analogy, these twists are reactive power, which helps propel the real power to the consumer. Completing the analogy, without a sufficient amount of reactive power, the ball is unstable and it falls into one of the holes and the player loses the game.

**Figure 2. The Game of Labyrinthspel**



With respect to space weather, Kappenman has pointed out that there is an increase in reactive power consumption when GICs pass through a transformer (Kappenman, 2003, p. 4). In his words,

“Though these quasi-DC currents are small compared to the normal AC current flows in the network, they have very large impacts upon the operation of transformers in the network....The principal concern to network reliability is due to increased reactive power demands from transformers that can cause voltage regulation problems, a situation that can rapidly escalate into a grid-wide voltage collapse.”

Other researchers have also noted the impact of GICs on reactive power demands. For example, Molinski (2002, p 1770) has noted that there is evidence of a linear relationship between GICs levels and the reactive power consumption by high voltage transformers. According to the figure in his article, this relationship is evident even at low GIC levels.

#### 4. Data

PJM monitors reactive conditions at several interfaces within the grid (PJM, 2006, p. 52-53) Based on these readings, the system is operated reactive on-cost or off-cost. The time periods in which reactive off-cost operations were implemented are posted on the PJM web site ([www.PJM.com](http://www.PJM.com)). These data were downloaded and the data were coded as a binary variable with "1" representing the outcome when the system was operated reactive off-cost for all or part on a hour and zero otherwise.

Data on hourly load and the locational prices by hour were downloaded directly from the PJM web site. Hourly ambient temperature were obtained from the National Weather Service for the Baltimore-Washington and Philadelphia airports, both of which lie within the PJM control area.

The sample period for this study is 1 April 2002 -30 April 2004. There are 17,671 hourly observations. Over this period, there were 1,884 hours in which a reactive off-cost operation was in effect. GICs are proxied using geomagnetic data from USGS' Fredericksburg (FRD) geomagnetic observatory in Fredericksburg Virginia (<http://geomag.usgs.gov/observatories/fredericksburg/>). Specifically, for each one hour market period, the rate of the change in the horizontal component of the geomagnetic field (dH/dt) was calculated using the one minute geomagnetic data reported by the Fredericksburg observatory. The descriptive statistics are reported in Table 1.

**Table 1**  
**Descriptive Statistics for dH/dt as**  
**Measured at the Fredericksburg**  
**Observatory, 1 April 2002 - 30 April 2004**

Sample Mean:	2.5 nT/minute
25 <sup>th</sup> Percentile	1.0
50 <sup>th</sup> Percentile	1.6
75 <sup>th</sup> Percentile	2.8
Maximum	178.4
Standard Deviation	4.3

#### 5. Preliminary Analysis Using a Nonparametric Test Statistic

Events  $A_i$  and  $B_j$  are statistically independent, i.e. there is no relationship between the two events, if their joint probability is equal to the product of their marginal probabilities, i.e.

$$P(A_i \cap B_j) = P(A_i)P(B_j) \quad (1)$$

In the case before us,  $A_i$  represents GIC categories and  $B_j$  represents conditions on the PJM power grid. Specifically, we denote four GIC categories: GIC1, GIC2, GIC3 and GIC4 where GIC1 represents GIC values in the first quartile, GIC2 represents the second quartile, GIC3 represents the third quartile, and GIC4 represents the fourth quartile. With respect to grid conditions, we will consider two categories: reactive off-cost (ROFF) and reactive on-cost (RON).

To test the hypothesis whether GICs and the incidence of reactive off-cost events are independent we first consider the two central errors in hypothesis testing. The first is known as a Type I error. This is when the researcher rejects the null hypothesis when in fact it is true. In this case the null hypothesis is that GIC levels and grid conditions are independent. The probability of a Type I error is known as  $\alpha$ . Researchers will not reject a null hypothesis unless the observed probability based on a test statistic that is calculated under the assumption that the null hypothesis is true is less than  $\alpha$ . Many researchers specify a value of  $\alpha$  of 0.05 or 0.01.

The second central error in hypothesis testing is a Type II error. This is when the researcher fails to reject the null hypothesis when in fact it is false. The probability of a Type II error is known as  $\beta$ . It is well established that  $\beta$  increases with lower values of  $\alpha$ . For this reason, it is generally considered undesirable for a researcher to focus exclusively on minimizing the probability of a Type I error. Readers interested in knowing more about hypothesis testing may wish to consult Greene (2003, pp. 892-896).

This study will initially test the hypothesis that GIC levels and grid conditions are related using Pearson's Chi Squared Statistic. This is a nonparametric test statistic that makes no assumption about the distribution of the frequencies. In contrast to simple correlation analysis, it does not presume that the relationship is linear.

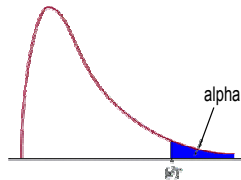
The chi squared statistic is calculated as the sum of the squared differences between actual and expected frequencies relative to expected:

$$\chi^2 = \sum \frac{(f_a - f_e)^2}{f_e} \quad (2)$$

Where  $f_a$  represents the actual frequency while  $f_e$  represents the expected frequency.

High values of the Chi Square Statistic tend to cast doubt on the null hypothesis. Specifically, the probability that  $\chi^2$  exceeds the critical value ( $\chi^2_*$ ) is equal to  $\alpha$  (Figure 3). Accordingly, a researcher will reject the null hypothesis of statistical independence if the calculated Chi Square statistic exceeds ( $\chi^2_*$ ). The exact value of ( $\chi^2_*$ ) will depend on the number of degrees of freedom which will equal the number of row categories minus one multiplied by the number of column categories. In this case there are four GIC categories and two grid categories and thus there are three degrees of freedom. Readers interested in knowing more about this test statistic may wish to consult Mood, Graybill and Boes(1974, pp. 440-461)

**Figure 3. The Chi Square Distribution and the Critical Value of the Chi Square Statistic**



To control for the possibility that the GIC/ROFF relationship, if it exists, may not be independent of system load, the sample was stratified by load quartiles. Based on the observed and expected frequencies under the null hypothesis a Chi Square statistic was calculated for each load quartile. With the exception of the first load quartile, the calculated Chi Square statistic exceeds the critical value of the statistic corresponding to the five percent level of statistical significance when there are nine degrees of freedom. Accordingly, one can reject the null hypothesis that GIC levels and incidence of off-cost

operations are statistically unrelated when system load is moderate to high.

## 6. Factors that Affect the Incidence of Reactive Off-Cost Events

In addition to GICs, possible determinants of reactive off-cost operations include:

**Hourly Load.** The amount of reactive power consumed by a transmission line is related to the level of current flowing on the line (FERC, 2005, p 31). Accordingly, it is hypothesized that probability of a reactive off-cost event generally increases with system load. It is also hypothesized that the probability of a reactive off-cost event will be higher when system load is rising.

**Ambient Temperature.** Capacitors are an important source of reactive power. Unfortunately, the supply of reactive power from capacitor declines with temperature and thus it is likely that the probability of a reactive off-cost event increases with temperature.

**Preexisting Transmission Constraints.** The probability of a reactive off-cost event is likely to be a function of preexisting transmission constraints. To the extent that the wholesale electricity market is efficient, these transmission constraints will be reflected in the congestion costs implied by the day-ahead locational marginal prices.

To represent these preexisting transmission constraints, we employ various measures of day-ahead congestion costs. One easily understood measure of congestion costs is the absolute value of the difference in the prices at PJM's Eastern and Western Hubs. Another measure of congestion is the standard deviation in the day-ahead zonal prices.

**Scheduled Imports.** While PJM does occasionally export power to other grids, on average it is a net importer of electricity. It is hypothesized that the probability of a reactive off-cost event will be higher; the greater PJM's reliance on imports.

**Other Factors.** There are obviously other factors that can influence the probability of a reactive off-cost event. For example, changes in the geographical mix of base load generating capacity, investments in transmission capacity, and changes in policies by the system operator could conceivably affect the probability of congestion. Unfortunately, we do not have direct measures of these factors. We will therefore control for their influence through the use of binary variables. Specifically, the econometric model will include binary variables to each three month time

interval of the sample period. We will also include binary variables that represent the hour of the day.

## 7. Econometric Results

The binary variable representing reactive off-cost operations was regressed on the independent variables using a Probit model. This is a standard econometric procedure when the dependent variable is binary. The estimation results indicate the following:

- Increases in load increase the probability that a reactive off-cost operation will be declared.
- The probability of a reactive off-cost operation increases as ambient temperature increases.
- The probability of a reactive off-cost operation is higher, the higher the level of day-ahead congestion costs.
- The probability of a reactive off-cost operation is higher, the higher net scheduled imports are to load.
- Increases in GICs, as proxied by the rate of change in the horizontal component of the geomagnetic field, positively affect the probability that a reactive off-cost operation will be declared.

## 8. GICs and Reactive Off-Cost Operations: Causation or Mere Association?

While the results presented in the previous section are consistent with the hypothesis that GICs contribute to reactive off-cost events, they do not prove causation. It could be the case that there is merely an association between GICs and the incidence of the reactive off-cost operations.

In general, statistics is ill equipped to test for causation. One exception is the Granger Causality Test formulated by the 2003 Nobel Laureate Clive W.J. Granger. This test considers two variables X and Y. The variable X is said to Granger cause Y if past values of X are useful, in addition to past values of Y, in explaining the current value of Y. Unfortunately, this test has nothing to say about contemporaneous causality between X and Y and thus we will have to consider another approach to the issue of whether GICs contribute to the reactive off-cost events.

Our approach to the issue of causation is to replace the GIC proxy based on the FRD geomagnetic data in the regression equation with GIC

proxies for other locations. This approach is premised on the results of a study by Pulkkinen et. al. (2006) that indicates that the correspondence between GIC and  $dH/dt$  declines with distance. Based on this study, the explanatory power of a regression equation in which a GIC proxy is an explanatory variable should be higher (lower) when the GIC proxy is more (less) local in nature. This suggests that we can examine the robustness of our results by employing geomagnetic data from other geomagnetic observatories.

We therefore substitute the  $dH/dt$  variable based on data reported by FRD with  $dH/dt$  measurements based on data collected from more distant geomagnetic observatories. If the results discussed in the previous section are spurious, then one would expect that this substitution would yield a large number of cases where the data from geomagnetic observatories more distant from the PJM control area would yield regression results with equal or greater explanatory power.

Geomagnetic data were collected for 40 additional geomagnetic observatories. The 40 observatories were selected for analysis on the basis of their data density as well as geographic diversity in terms of both latitude and longitude. The sample includes all the geomagnetic observatories in the Lower 48 United States operated by the USGS. The sample also includes data from observatories in Australia, New Zealand, Greenland, Sweden, Finland, France, Germany, Japan, Canada, China, South Africa, Peru, Spain, Italy, Ireland, United Kingdom and even tiny Macquarie Island (located between Tasmania and Antarctica). The econometric model was alternatively estimated using  $dH/dt$  measured using data from FRD and each of these stations. The results of this analysis strongly support the view that there is a causal relationship between GICs and the incidence of reactive off-cost operations in PJM.

## 9. Is the Market Price Impacted?

Given that cost of generation is a lower priority during a reactive off-cost operation, it is reasonable to suppose that these events would have a market impact. This section of the paper formally tests this hypothesis. Before estimating the impact, it should be noted that there can be little doubt that the estimated market impact will be modest relative to the costs imposed on consumers should an off-cost operation not be declared and the system collapses as a result. Nevertheless, an estimate of the market impact is important lest the impression be created that these events have no economic cost.

Given PJM's use of locational marginal pricing, the real-time price at PJM's Eastern Hub will be equal to the real-time price at its Western Hub in the absence of real-time transmission constraints. Likewise, the

day-ahead prices are equal when the transmission path between the two hubs is expected to be uncongested. It is therefore hypothesized that the off-cost events increase real-time congestion costs relative to day-ahead congestion costs. So as to control for other factors that can also contribute to real-time congestion costs, the estimating equation includes binary variables for the hour of the day, day-ahead congestion costs, ambient temperature, forecasted load, and measures of unexpected load.

For approximately 47 percent of the observations in the original sample, real-time congestion costs were equal to zero, i.e. the transmission path between the Western and Eastern Hubs was uncongested as reflected by the equality in the two real-time hub prices. Under these circumstances, analysis of these costs using the method of least squares can potentially lead to seriously biased estimates. To avoid this bias, the analysis employs the Tobit maximum likelihood procedure developed by the late Nobel Laureate James Tobin of Yale.

The Tobit model was developed to econometrically model relationships when the dependent variable is censored. In Tobin's original model (1958), the dependent variable was expenditures on durables by individuals. This variable is censored because expenditures below zero are not observed. The application of ordinary least squares in this case will lead to biased results because it treats all the observations in which the dependent variable is zero as identical when in fact the probability of a positive value will vary within the group. In the case before us, congestion costs are censored because they too have a lower bound of zero. See Green (1981) for a discussion of the shortcomings of ordinary least squares under these circumstances as well as how these shortcomings can be avoided through the use of the Tobit procedure.

The model was estimated using 16,394 observations. All of the coefficients have the expected signs. Almost all the coefficients are highly statistically significant. In particular, the coefficient on OFF-COST, the binary variable that represents a reactive off-cost operation, is both positive and highly statistically significant indicating that congestion costs are significantly higher than what they would be otherwise when a reactive off-cost operation is implemented.

## 10. Conclusion

This paper has examined the effect of GICs, as proxied by  $dH/dt$  based on local magnetometer data, on the incidence of a reactive off-cost operations in the PJM power grid. Both the nonparametric and econometric evidence indicate support for the hypothesis that GICs contribute to these events.

The paper has also presented evidence that the results are robust by alternatively substituting the  $dH/dt$  variable from the closest geomagnetic observatory with measurements based on geomagnetic data from 40 other locations around the world. This analysis strongly supports the existence of a causal relationship between GICs and the incidence of reactive off-cost operations in PJM. Finally, the paper has presented evidence that the reactive off-cost events have a statistically and economically significant impact on the real-time price of electricity.

## Acknowledgments

This research was made possible by a grant from the National Science Foundation (Award # ATM-0318582). We thank Cynthia Schmitt for her excellent research assistance in compiling the reactive off-cost events. We thank Michael Forbes for writing the programs that manipulated the large quantity of geomagnetic data used in this study. We also thank Antti Pulkkinen for his criticism and helpful suggestions. Any errors remain the full responsibility of the authors.

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